Patterns and Expressions

1. Describe the pattern using words.

2. Complete the table of the input and output values shown in the ordered pairs.

3. Describe the pattern for each group of figures. What would the next consecutive figure in the pattern look like?

4. Input Output

5. Describe each pattern using words. Draw the next figure in each pattern.

6. Answers may vary. Sample: a stack of small squares, first one square, then two squares, then 3 squares, a stack of 4 small squares

7. Input Process Output

8. Answers may vary. Sample: A black square in the lower right corner

9. The picture included with a problem is called a diagram

10. The pattern in a list of numbers is that each one is 5 more than the last one.

11. Input Output

12. Answers may vary. Sample: equilateral triangles alternating pointing up, then down; an equilateral triangle pointing up

13. Answers may vary. Sample: a black square in the center of the middle of the three consecutive blocks as pivot, each figure is obtained by moving the previous corner block up one row and right one column, shading this block and all blocks to the left of and below this new corner block.

14. Answers may vary. Sample: each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

15. Answers may vary. Sample: Draw a square. For each subsequent figure in the pattern, shade one square having vertices on the midpoints of all sides of the previous innermost square, shading all but the new innermost square.

16. Answers may vary. Sample: Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

17. Answers may vary. Sample: The graph shows the cost depending on the number of DVDs that you purchase.

18. What is the cost of purchasing 10 DVDs? $160

19. Answers may vary. Sample: Each block is one square, first one square, then two squares, then 3 squares, a stack of 4 small squares

20. The number 21 and 22 are consecutive numbers.

21. Each term is 2 less than the previous term; each term where n = 2 is less than the previous term; each term where n = 2 is less than the previous term.

22. The picture included with a problem is called a diagram

23. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

24. Each term is 2 less than the previous term; each term where n = 2 is less than the previous term where n is the difference between the previous two terms.

25. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

26. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

27. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

28. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

29. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

30. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

31. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

32. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

33. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

34. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.

35. Each term is 2 less than the previous term; each term where n is the difference between the previous two terms.
Describe each pattern using words. Draw the next figure in each pattern.

1. Answers may vary. Sample: Start with a row of two squares. Add a row of two squares below the previous figure for each new figure.

2. Answers may vary. Sample: Start with one square. Add a row of squares below the previous figure that has one more square than the row above it for each new figure.

3. Answers may vary. Sample: Start with an isosceles triangle that points upward. Rotate the triangle 90° clockwise for each new figure.

4. Make a table with a process column to represent the pattern. Write an expression for the number of circles in the nth figure. The expression for the number of circles in the nth figure is 2n.

<table>
<thead>
<tr>
<th>Figure Number (Input)</th>
<th>Process Column</th>
<th>Number of Circles (Output)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>8</td>
</tr>
<tr>
<td>n</td>
<td>10n</td>
<td>2n</td>
</tr>
</tbody>
</table>

5. Often identifying and expressing patterns using algebraic expressions is difficult because the pattern is not immediately obvious. Organizing information into a table and looking at the common differences provide a clue.

6. Write an algebraic expression for the number of circles in the nth figure. Each term is 4 times the previous term; hence, the algebraic expression will have degree 1. Degree 1 expressions will not have any exponents larger than 1. What is the algebraic expression for this pattern? (6n – 2)

7. Since the second difference is constant in the pattern above, the algebraic expression will have degree 2. In an expression of degree 2, the largest exponent is 2. Therefore, we know that the expression for this pattern must include n^2. What is the algebraic expression for the nth term in this pattern? (6n^2 + 3n)

8. Write an algebraic expression for the nth term in each pattern.
   a. – 2, 2, 12, 28, . . . (5n – 1)
   b. 2, 8, 18, 28, . . . (2n^2)
   c. 7, 26, 83, 206, . . . (n^3 + 1)
   d. 2, 5, 10, 17, . . . (n^2 + 1)
Identifying patterns is an important skill in algebra. Identify the underlying structure of a group of numbers, a set of data, or a sequence of figures to express a rule describing the relationship.

**Problem**

Look at the figures from left to right. What is the pattern? What would the next figure in the pattern look like?

First, identify the basic properties of the figure. The number of elements in each figure increases by one from each figure to the next.

Second, determine whether the figures change in size. Notice that the arrows decrease in length by about half in each step.

Third, observe whether there is displacement or rotation from one figure to the next. Each arrow is a 90° clockwise rotation from the previous arrow.

The pattern begins with an arrow pointing to the right. Each subsequent figure adds a new arrow that is shorter than the previous arrow by about half and is rotated 90° clockwise from the previous one. The next figure in the pattern is shown at the right.

**Exercises**

Describe the pattern in words and draw the next figure in the pattern.

1. Each figure is twice the size of the one before it.
2. Each figure adds a new triangle whose dimensions are half those of the dimensions of the previous smallest triangle and is inside it, rotated 180°. The color alternates white to black from biggest to smallest triangle, respectively.

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**Patterns and Expressions**

**Properties of Real Numbers**

**Think About a Plan**

Five friends each ordered a sandwich and a drink at a restaurant. Each sandwich costs the same amount, and each drink costs the same amount. What are two ways to compute the bill? What property of real numbers is illustrated by the two methods?

**Understanding the Problem**

1. There are \( \frac{5}{3} \) sandwiches and \( \frac{5}{3} \) drinks on the bill.
2. What is the problem asking you to determine?

**Planning the Solution**

3. How can you represent the cost of five sandwiches?
   
   Answers may vary. Sample: \( 5s \)

4. How can you represent the cost of five drinks?
   
   Answers may vary. Sample: \( 5d \)

5. How can you represent the cost of the items ordered by one friend?
   
   Answers may vary. Sample: \( d + s \)

**Getting an Answer**

6. Write an expression that represents the cost of five drinks and the cost of five sandwiches.
   
   Answers may vary. Sample: \( 5d + 5s \)

7. Write an expression that represents the cost of the items ordered by five friends.
   
   Answers may vary. Sample: \( 5(d + s) \)

8. What property of real numbers tells you that these two expressions are equal? Explain.

   Distributive Property: The distributive property states \( a(b + c) = ab + ac \).
19. Compare the two numbers. Use > or <.

9. $\sqrt{9} > 2$  
10. $\sqrt{5} < \sqrt{8}$  
11. $\sqrt{14} > \sqrt{12}$  
12. $\sqrt{3} < \sqrt{5}$

13. $\frac{\sqrt{8}}{\sqrt{2}} > 2$  
14. $\sqrt{56} > 7.48$  
15. $\frac{\sqrt{21}}{\sqrt{3}} < 2$

Name the property of real numbers illustrated by each equation.

17. $2(3 + 5) = 2 \cdot 3 + 2 \cdot 5$  
18. $16 = (-13) = -13 + 10$  
19. $-7 \cdot -4 = 1$  
20. $5(2 + 7) = (5 \cdot 2) \cdot 7$

Inverse Property of Multiplication  
Commutative Property of Addition  
Identity Property of Multiplication  
Associative Property of Multiplication

Estimate the numbers graphed at the labeled points.

21. point A  
22. point B  
23. point C  
24. point D

Geometry To find the length of side $b$ of a rectangular prism with square base, use the formula $b = \sqrt{s^2 + h^2}$, where $V$ is the volume of the prism and $h$ is the height. Which set of numbers best describes the value of $b$ for the given values of $V$ and $h$?

25. $V = 100, h = 5$  
26. $V = 100, h = 25$

irrational numbers  
natural numbers

27. $V = 100, h = 20$  
28. $V = 5, h = 20$

irrational numbers  
rational numbers

Write the numbers in increasing order.

29. $-2\sqrt{2}, -\frac{1}{\sqrt{5}}, -1, 0.9, -\frac{5}{2}$  
30. $-\frac{3}{4}, -\frac{1}{2}, -\sqrt{2}, -\frac{4}{3}$

Justify the equation by stating one of the properties of real numbers.

31. $(x + 37) + (-23) = x + (37 + (-23))$  
32. $x + 0 = x$

Associative Property of Addition  
Identity Property of Addition

33. $x + (37 + (-23)) = x + 0$  
34. $x + 0 = x$

Inverse Property of Addition  
Identity Property of Addition
1-2 Standardized Test Prep
Properties of Real Numbers

Multiple Choice
For Exercises 1–5, choose the correct letter.
1. Which letter on the graph corresponds to $\sqrt[4]{-1}$? F
2. Which letter on the graph corresponds to $-1.5$? F

What property of real numbers is illustrated by the equation?
3. $-6 + [6 + 5] = (-6 + 6) + 5$ D
   - Identity Property of Addition
   - Associative Property of Addition
   - Commutative Property of Addition
   - Distributive Property

4. $a + (b + c) = (a + b) + c$ D
   - Associative Property of Addition
   - Associative Property of Multiplication
   - Commutative Property of Addition
   - Distributive Property

5. Which of the following shows the numbers 13, 1.3, 12 in order from greatest to least? B
   - $13, 1.3, 12$
   - $12, 1.3, 13$
   - $13, 1.3, 12$
   - $12, 1.3, 12$

7. $a = \frac{b}{c} = \frac{1}{2}$ [2] rational numbers [1] incorrect set of numbers [0] no answer given
8. $a = \sqrt{b} = \sqrt{17}$ [2] irrational numbers [1] incorrect set of numbers [0] no answer given
9. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ D
   - Associative Property of Addition
   - Associative Property of Multiplication
   - Commutative Property of Addition
   - Commutative Property of Multiplication

10. $a + (b + c) = (a + b) + c$ D
    - Associative Property of Addition
    - Associative Property of Multiplication
    - Commutative Property of Addition
    - Commutative Property of Multiplication

Geometry: The length c of the hypotenuse of a right triangle with legs having lengths a and b is found by using the formula $c = \sqrt{a^2 + b^2}$. Which set of numbers best describes the value of $c$ for the given values of a and b?

1-2 Enrichment
Properties of Real Numbers

There are four words beginning with the letter "I" that describe certain types of operations:

a. Identity: an operation that does not change anything. For example, adding 0 to a number is an identity operation, because adding 0 does not change the original number.

b. Inverse: an operation that can be undone by another operation. For example, the operation of adding 2 to a number can be undone by subtracting 2 from the number. However, the operation of multiplying a number by 0 cannot be undone.

c. Idempotent: an operation that, when done twice, is the same as doing it once. For example, multiplying a number by 1 and then multiplying the result by 1 again has exactly the same effect as multiplying the number by 1 only once.

For each of the following operations, state which of the "I" words apply. If none apply, write none.
1. finding the absolute value of a number idempotent
2. dividing a number by 1 identity, inverse, idempotent, involutory
3. multiplying the absolute value of a number by 1 idempotent
4. finding the reciprocal of a nonzero number inverse, involutory
5. dividing a number by 1 inverse, involutory
6. multiplying a number by 0 idempotent
7. adding it to the reciprocal of a nonzero number inverse, involutory
8. multiplying the reciprocal of a nonzero number by 2 inverse, involutory
9. adding the absolute value of a nonzero number to the absolute value of its reciprocal none
10. finding the reciprocal of the absolute value of the reciprocal of a nonzero number idempotent
11. finding the absolute value of $-1$ times a nonzero number, then taking the reciprocal none

1-2 Reteaching
Properties of Real Numbers

The Properties of Real Numbers are relationships that are true for all real numbers except zero.

The additive identity for real numbers is 0. This gives the Identity Property of Addition, which states for any real number $a$:

$a + 0 = a$ and $0 + a = a$

The additive inverse of a real number $a$ is $-a$. By the Inverse Property of Addition:

$a + (-a) = 0$ and $(-a) + a = 0$

There are two similar properties for multiplication. These use the multiplicative identity and the multiplicative inverse for any nonzero real number $a$.

Identity Property of Multiplication: $a \cdot 1 = a$ and $1 \cdot a = a$

Inverse Property of Multiplication: $a \cdot \frac{1}{a} = 1$

Using the Properties of Real Numbers, what is the missing number in the equation?
a. $\frac{1}{5} + 0 = 5$
   - According to the Identity Property of Addition, the missing number is 5.

b. $7 + \frac{1}{1} = 1$
   - The Inverse Property of Multiplication shows that the product of a real number and its multiplicative inverse is 1. The missing number is the multiplicative inverse of 7, or $\frac{1}{7}$.

Exercises
Find the missing number in the equation.
1. $-5 + (-4) = -9$
2. $-2 + 7 = 5$
3. $3 \cdot -2 = -6$
4. $-1 \cdot 1 = -1$

The Commutative and Associative Properties of Addition and Multiplication are properties that help you simplify calculations.

The Commutative Property states that the order of addition or multiplication does not change the sum or product:

$a + b = b + a$ and $ab = ba$

The Associative Property states that the grouping of three or more addends or factors does not change the sum or product:

$(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

What property does the equation illustrate?
5. $5 \cdot (\frac{2}{3}) = 5 \cdot \frac{2}{3}$
   - This equation shows that the product of three numbers is the same regardless of the order of multiplication. Only the grouping of the factors is different. Therefore, the equation illustrates the Associative Property of Multiplication.

Exercises
Name the property that the equation illustrates.
5. $5 \cdot (\frac{2}{3}) = 5 \cdot \frac{2}{3}$
   - Inverse Property of Multiplication
6. $1 \cdot 85 = 85$
   - Identity Property of Multiplication

The Distributive Property combines addition and multiplication:

$a(b + c) = ab + ac$

What are the missing values in the equation?
4. $6 + (3 + 2) = 4 + \boxed{3} + 2$
   - By the Distributive Property, the sum of two numbers multiplied by a third number is equal to the sum of each multiplied by the third number. Because the third number is 4, the missing numbers are 6 and 5.

Exercises
Name the missing values in each equation.
7. $3x + (2 + 3) = \boxed{3} + \boxed{2} + 2$
8. $-2(5 + 1) = \boxed{-2} + \boxed{-2}$
9. $9y + 9 + 2 = \boxed{9} (y + 2)$
10. $8(z + 3) + 8 = \boxed{3z + 8}$

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1-3 ELL Support
Algebraic Expressions

Use the chart below to review vocabulary. These vocabulary words will help you complete this page.

<table>
<thead>
<tr>
<th>Addition (+)</th>
<th>Subtraction (−)</th>
<th>Multiplication (×)</th>
<th>Division (÷)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum</td>
<td>difference</td>
<td>product</td>
<td>quotient</td>
</tr>
<tr>
<td>increased by</td>
<td>less than</td>
<td>times</td>
<td>divided by</td>
</tr>
</tbody>
</table>

Circle the word or words in each word phrase that tell you what operations to use. Write the operation symbol word (×, ÷, +, −) next to the algebraic expression.

1. the sum of a number and 12 +
2. the product of 2 and 6 ×
3. the difference of 13 −
4. the sum of −75 and +
5. the quotient of 20 ÷

Match each word phrase in Column A with the matching algebraic expression in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. the difference of a number and 36</td>
<td>A. y = 9</td>
</tr>
<tr>
<td>7. 15 more than the number</td>
<td>B. 10(x)</td>
</tr>
<tr>
<td>8. the product of 10 and a number</td>
<td>C. q = 15</td>
</tr>
<tr>
<td>9. the total of a number and 9</td>
<td>D. p = 36</td>
</tr>
</tbody>
</table>

Match each algebraic expression in Column A with the matching word phrase in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. m + 45</td>
<td>C. 45 less than a number m</td>
</tr>
<tr>
<td>11. 2(m)</td>
<td>D. 45 times the sum of a number m and 1</td>
</tr>
<tr>
<td>12. m − 45</td>
<td>A. a number m increased by 45</td>
</tr>
<tr>
<td>13. 8(x/4 + 1)</td>
<td>B. a number m divided by 45</td>
</tr>
</tbody>
</table>

24. It will increase by $2.

25. The expression increased by fewer than 3.

26. The algebraic expression decreased by 2a models the freshman class income.

Think About a Plan
Algebraic Expressions

Write an algebraic expression to model the situation.

The freshman class will be selling carnations as a class project. What is the class’s income after it pays the florist a flat fee of $200 and sells a carnations for $2 each?

1. What does the variable represent?
   the number of carnations sold

2. How will the class’s income change for each carnation sold?
   It will increase by $2.

3. Will paying the florist increase or decrease their income? By how much?
   It will decrease by $200.

4. Will the expression include both the income for each carnation and the florist’s fee? Explain.
   Yes; the class’s profit is a function of the proceeds from carnation sales and the florist’s fee.

5. Write the expression in words.

   The income is $200 and 2 times i.

6. Write the expression using symbols.

   income = 200 + 2i

7. Check your expression by substituting 100 for the number of carnations. Does your answer make sense? Explain.
   Yes; the class has an income of $400 after they make $500 and pay the florist $200.

8. The algebraic expression 200 + 2i models the freshman class income.

1-3 Practice
Algebraic Expressions

Write an algebraic expression that models each word phrase.

1. seven less than the number t − 7

2. the sum of 11 and the product of 2 and a number n + 22

Write an algebraic expression that models each situation.

3. Arin has $250 and is earning $75 each week babysitting. $250 + 75w

4. You have 90 boxes of raisins and are eating 12 boxes each month. 50 − 12m

Evaluate each expression for the given values of the variables.

5. 4t − 3w + 2v; 4t, w = 2, v = 3 2t − 6

6. (c − a) − c2, a = 4 and c = −1 0

7. 3(5x − 3y) + 3x2 + 4yz; x = 3 and y = −7 5x2 + 2

Surface Area
The expression 6l2 represents the surface area of a cube with edges of length l. What is the surface area of a cube with each edge length?

8. 1 inch 54 in.2

9. 1.5 meters 13.5 m2

The expression 4.95 + 0.07c models a household’s monthly long-distance charges, where c represents the number of minutes of long-distance calls during the month. What are the monthly charges for each number of long-distance minutes?

10. 71 minutes $10.06

Simplify by combining like terms.

11. 2x + 2y + 10z + 3x + 5x 10x + 2y + 10z

12. 5a + 7b + 3c + 2d 5a + 7b + 3c + 2d

13. 4a − 5c + 1 + a − 5 5a − 5c + 2

14. −[2x − (y)] − [6x2 − 3y2 − 2xy + y2] −[2x − (y)] − [6x2 − 3y2 − 2xy + y2]

Open-ended
Write an example of an algebraic expression that always has the same value regardless of the value of the variable. Answers may vary. Sample: any expression that results in the variable having a coefficient of 0. Example: x − a

Match the property name with the appropriate equation.

25. Opposite of a Difference A. −[(−a) + (−b)] = −[−a − b]

26. Opposite of a Sum B. 16d + (16d + 200) = 16d + 0

27. Opposite of an Opposite C. [b − (−a)] = 10 − 5a

28. Multiplication by 1 D. −(−4 + 3x) + 1 + (−1) + (−4 + 3x) + 1

29. Multiplication by −1 E. − [−(−4 + 3x)] = −3x − 8

30. Distributive Property F. −[−(b + 2x)] = 9 − 2x

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1-3 Practice (continued) Form K

Algebraic Expressions

Write an algebraic expression to model the total score in each situation. Then evaluate the expression to find the total score.

15. In the first half, there were fifteen two-point shots and 5 one-point free throws. $T = 2w + 5f$. To start, define your variables. Let $w =$ the number of two-point shots, $r =$ the number of one-point free shots.

12. In the first quarter, there were two touchdowns and 1 extra point kick. $T = 6r + 1e$. Hint: A touchdown is worth 6 points. An extra point kick is worth 1 point.

Simplify by combining like terms.
13. $10b - 5b$ $14. 12 + 6a - 3e + 12 + 5a$
15. $3e + 2f + 6a + 3b$ $16. 5m + 2n + 6m + 4e + 15m + 6n$
17. $8r - (3s - 5f) + 13r - 3s$ $18. 2.5y - 4y - 1.5y$

The expression $13.05 + 0.05c$ models a household’s monthly Internet charges, where $c$ represents the number of online minutes during the month. What are the monthly charges for each number of online minutes?
19. 60 minutes $23.20$ $20. 120$ minutes $28.35$

Evaluate each expression for the given value of the variable.
21. $3e + (2n + 6) ; e = 2, n = 12$ $22. a = 5(x + 2) ; x = -5, 10$
23. $-r + (3r^2 + 1) ; r = 4$ $24. \sqrt{9} - 5(2x - 12) ; x = 30$

Algebraic Expressions

Write and simplify algebraic expressions for the puzzle below.

1. Write a correct expression and profit, without work shown.
2. no answer or no attempt made

Algebraic Expressions

Write an algebraic expression to model each situation. Then evaluate the expression to find the given value of the variable.

Evaluate each expression for the given value of the variable.

4. Evaluate each expression for the given values of the variables.

Write an algebraic expression that models each situation. What is the surface area of a cube with each edge length?

Evaluate each expression for the given value of the variable.

Write and simplify algebraic expressions for the puzzle below.

Algebraic Expressions

Write an algebraic expression to model each situation.

Write and simplify algebraic expressions for the puzzle below.

Algebraic Expressions

Write an algebraic expression to model each situation. Then evaluate the expression to find the given value of the variable.

Evaluate each expression for the given value of the variable.

Write and simplify algebraic expressions for the puzzle below.

Algebraic Expressions
1-3 Reteaching (continued)  
Algebraic Expressions

To simplify an algebraic expression, combine like terms using the basic properties of real numbers. Like terms have the same variables raised to the same powers.

**Problem**

What is the value of the algebraic expression $3(4x + 5y) - 2(3x - 7y)$ when $x = 3$ and $y = 5$?

Simplify the algebraic expression using the basic properties of real numbers.

$$3(4x + 5y) - 2(3x - 7y) = 12x + 15y - 6x + 14y$$

Combine like terms.

$$12x + 15y - 6x + 14y = 6x + 29y$$

Evaluate the expression, replacing $x$ with 3 and $y$ with 2 in the simplified expression.

$$6(3) + 29(2) = 18 + 58 = 76$$

**Exercises**

Simplify the algebraic expression. Then evaluate the simplified expression for the given values of the variable.

1. $4(x + 1) + 2x$; $x = 3$
2. $7y + 3z$; $y = 11$, $z = 6$
3. $7x + 2y + z$; $x = 2$, $y = 2$, $z = 1$
4. $2x + 3z$; $x = 1$, $z = 3$
5. $x^2 - 3x + 1$; $x = -2$
6. $2x^2 - 5x + 1$; $x = 3$
7. $2x^2 - 5x + 1$; $p = 4$, $q = 5$
8. $3x^2 - 2x - 4$; $x = 2$
9. $3x^2 - 4x + 5$; $x = 1$
10. $2(n - m) - \frac{3}{2} (m - a) v$; $m = 6$, $a = 2$

11. $\frac{1}{2} n + 5$; $n = 2$

12. $\frac{1}{2} m + \frac{1}{3} n - \frac{1}{2} p$; $m = 2$, $n = 3$, $p = 5$

**Geometry**

The measure of the supplement of an angle is 20° more than three times the measure of the original angle. Find the measures of the angles.

**Know**

1. The sum of the measures of the two angles is 180°.
2. What do you know about the supplementary angle?

   The sum of the measures of the two angles is 180°.

**Need**

3. To solve the problem, I need to define:

   the measure of the original angle: $x$

   We can find the measure of the supplement angle: $3x + 20$

**Plan**

4. What equation can you use to find the measure of the original angle?

   $x + (3x + 20) = 180$

5. Solve the equation.

   $x = 40$

6. What are the measures of the angles?

   original angle: 40°; supplement angle: 140°


   Yes, three times the measure of the first angle plus 20° is 140°; the sum of the measures of the angles is 180°; so they are supplementary.
ANSWERS

1-4 Practice Form G
Solving Equations

Solve each equation.

1. \(7.2 + c = 19\)    2. \(8.5 = 5p + 1.7\)
3. \(d = -31 - 124\)  4. \(x - 31 = 20.6\)     5. \(9(x - 3) = 12x - 9\)
6. \(5w + 8 = 12w - 16 - 15w + 5\)    7. \(3(x + 1) = 2\left(x + 11\right)\)

Write an equation to solve each problem.

9. Two brothers are saving money to buy tickets to a concert. Their combined savings is $55. One brother has $15 more than the other. How much has each saved? Variable may vary. Sample: \(x + x + 15 = 55\)

10. Geometry The sides of a triangle are in the ratio 5:12:13. What is the length of each side of the triangle if the perimeter of the triangle is 15 inches? Variable may vary. Sample: \(5x + 12x + 13x = 15\)

11. What three consecutive numbers have a sum of 126? Variable may vary. Sample: \(x + (x + 1) + (x + 2) = 126\)

Determine whether the equation is always, sometimes, or never true.

12. \(4(x + 1) - 25 = 3x + 5\) never 13. \(3x + 3 + 5 = 4(2y + 1) + 5\) always

14. \(S = \left(\frac{1}{2}\right)(x + y)\), for \(r = 1 - \frac{1}{2}\) 15. \(A = \frac{1}{2}w + \frac{1}{2}h,\) for \(w = \frac{5 - b}{2}\)

Solve each equation for the indicated variable.

16. \(\frac{1}{2}x + 3 = y\) 17. \(\frac{1}{2}y - x = \frac{3}{2}(y - x)\)
18. \(\frac{3}{2}x + \frac{1}{2}y + 3 = 0\)

Write an equation to solve each problem.

9. Lisa and Beth have babysitting jobs. Lisa earns $30 per week and Beth earns $25 per week. How many weeks will it take for them to earn a total of $275? Variable may vary. Sample: \(30w + 25w = 275\)

10. The angles of a triangle are in the ratio 2:12:18. The sum of all the angles in a triangle must equal 180 degrees. What is the degree measure of each angle of the triangle? Let \(x\) be the common factor. \(2x + 12x + 18x = 180\)

11. What two consecutive numbers have a sum of 53? Variable may vary. Sample: \(n + (n + 1) = 53\)

Determine whether the equation is always, sometimes, or never true.

12. \(2(3x - 4) = 6(x - 2)\) always 13. \(4(x + 3) = 2(2x + 1)\) never

1-4 Practice Form K
Solving Equations

Solve each equation.

1. \(5x + 4 = 2x + 10\) 2. \(10w - 3 = 4w + 5\)
3. \(4d - 3 = 2d\) 4. \(x + 2 = 3\) \(-6\) to subtract \(-x\) from each side.

Solve each equation. Check your answer.

5. \(5(x - 3) = 12x - 9\) 6. \(7y + 5 = 6y + 11\)
7. \(5w + 8 = 12w - 16 - 15w + 5\) 8. \(3(x + 1) = 2\left(x + 11\right)\)

Write an equation to solve each problem.

9. Lisa and Beth have babysitting jobs. Lisa earns $30 per week and Beth earns $25 per week. How many weeks will it take for them to earn a total of $275? Variable may vary. Sample: \(30w + 25w = 275\)

10. The angles of a triangle are in the ratio 2:12:18. The sum of all the angles in a triangle must equal 180 degrees. What is the degree measure of each angle of the triangle? Let \(x\) be the common factor. \(2x + 12x + 18x = 180\)

11. What two consecutive numbers have a sum of 53? Variable may vary. Sample: \(n + (n + 1) = 53\)

Determine whether the equation is always, sometimes, or never true.

12. \(2(3x - 4) = 6(x - 2)\) always 13. \(4(x + 3) = 2(2x + 1)\) never

1-4 Practice (continued) Form K
Solving Equations

Solve each equation.

20. \(0.5(x - 3) = (1.5 - x) = 5a\) 21. \(1.2(x + 5) = 1.62x + 5\)
22. \(0.5(x - 2) - 4.2x = 0.9x + 0.3\) 23. \(2.4 \cdot 5 \cdot \frac{3}{5} - 1 \cdot \frac{30}{7}\)

Solve each formula for the indicated variable.

24. \(V = \frac{4}{3} \pi r^3,\) for \(b = \frac{4V}{3\pi}\) 25. \(D = \sqrt{\frac{1}{4}(9 - \frac{4}{3})^2},\) for \(T_1, T_2, T_3 = \frac{D}{2}\)

Write an equation to solve each problem.

26. Two trains left a station at the same time. One traveled north at a certain speed and the other traveled south at twice that speed. After 4 hours, the trains were 640 miles apart. How fast was each train traveling? Variable may vary. Sample: \(640 = 2x + 5x = 14x\)

27. Geometry The sides of one cube are twice as long as the sides of a second cube. What is the side length of each cube if the total volume of the cubes is 72 cm\(^3\)? Variable may vary. Sample: \(x^3 + 2x^3 = 72\)

28. Error Analysis Rewrite and solve an equation for \(m\). Do you agree with her? Explain your answer:

No; there is an \(m\) on both sides of the equation; the correct result should be \(m = \frac{360}{5}\).

Solve each problem.

29. You and your friend left a bus terminal at the same time and traveled in opposite directions. Your bus was in heavy traffic and had to travel 20 miles per hour slower than your friend’s bus. After 3 hours, the buses were 270 miles apart. How fast was each bus going? Your bus: 35 mph; Your friend’s bus: 55 mph

30. Geometry The length of a rectangle is 5 centimeters greater than its width. What is the area of the rectangle? Width: 12 cm, \(l = 17\) cm

31. What four consecutive odd integers have a sum of 336? \(81, 83, 85, 87\)

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Solve each equation and enter your answer in the grid provided.

1. A bookstore owner estimates that her weekly profits \( p \) can be described by the equation \( p = 80 - 50b \), where \( b \) is the number of books sold that week. Last week the store's profit was \$720. What is the number of books sold?

2. What is the value of \( m \) in the equation \( 0.6m - 0.2 = 3.79 \)?

3. Three consecutive even integers have a sum of 168. What is the value of the largest integer?

4. If \( 6(x - 3) = 2(x - 2) = 11 \), what is the value of \( x \)?

5. Your long-distance service provider charges you \$0.20 per minute plus a monthly access fee of \$4.95. For referring a friend, you receive a \$10 service credit this month. If your long-distance bill is \$7.83, how many long-distance minutes did you use?

### Answers

1. 10
2. 2.5
3. 20
4. 5
5. 13

---

**Equations can be subdivided into three distinct types:**

1. **Conditional equations**, or equations that are true for some values of \( x \). For example, the equation \( x + 3 = 5 \) is true only for \( x = 2 \).

2. **Identity**, for which every possible value of the variable belongs to the solution set. For example, the equation \( x + x = 2x \) is an identity, as it is true for all values of \( x \).

3. **Impossibilities**, for which no possible value of the variable belong to the solution set. For example, the equation \( x = x + 1 \) is an imposibility, as it is never true.

For each of the following equations, find the solution if it is a conditional equation, or classify the equation as an identity or an impossibility.

1. \( x + (2a - 4) = 11 \)
2. \( x = x + 2 \) identity
3. \( x + (2a - 1) = 3a - 1 \) identity
4. \( x = (2 + 2a) \) impossibility
5. \( x - (2) = (2a + 4) = a + 1 \) impossibility
6. \( x + 2 = x + 3 \) impossibility
7. \( 2a = 3x = 0 \)
8. \( 2a = 8 - 6 - x = 2 \)
9. \( 2a - 4 = 3a - 8 = 5 \)
10. \( 2a - 5 = 5 + 2a \) identity
11. \( 2(a + 3) = 5a - (3a - 6) \) identity
12. \( x + (3a + 13) - (3a - 3) = 18 \)
13. \( (x + 3) - (3a - 3) = 2 \)
14. \( x + (3a - x) = 3 \) impossibility
15. \( 2(a + 5) - 4 = (3a + 2) - 4 \)
16. \( 1 - (x - 5) - 3 - (x - 5) \) impossibility
17. \( 4(x - 1) + 3 = 4x - (x + 1) \) 0

---

To solve an equation that contains \( x \), find all of the values of the variable that make the equation true. Use the equality properties of real numbers and inverse operations to rewrite the equation until the variable is alone on one side of the equation. Whenever the variable occurs in the other side of the equation, the solution is undefined.

<table>
<thead>
<tr>
<th>Addition Property of Equality</th>
<th>Subtraction Property of Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>To isolate ( x ) on one side of the equation, add ( b ) to each side.</td>
<td>To isolate ( x ) on one side of the equation, subtract ( b ) from each side.</td>
</tr>
<tr>
<td>( x + b = 7 )</td>
<td>( x - b = 7 )</td>
</tr>
<tr>
<td>( x + 2 = 10 )</td>
<td>( x - 2 = 10 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiplication Property of Equality</th>
<th>Division Property of Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>To isolate ( x ) on one side of the equation, multiply each side by ( b ).</td>
<td>To isolate ( x ) on one side of the equation, divide each side by ( b ).</td>
</tr>
<tr>
<td>( 2x = 10 )</td>
<td>( x = 7 )</td>
</tr>
<tr>
<td>( 2x = 30 )</td>
<td>( 2x = 10 )</td>
</tr>
</tbody>
</table>

**Exercises**

Solve each equation.

1. \( y + 12 = 8 - 4 \)
2. \( p = 9 - 12 \)
3. \( y + 2 = 20 \)
4. \( b = 8 + 15 - 2 \)
5. \( 9y = 27 - 3 \)
6. \( \frac{y}{4} = \frac{4}{2} - 24 \)
7. \( v - 2 = 25 \)
8. \( 40 - 10w = 4.9 \)
9. \( z = \frac{14}{13} \)
10. \( \frac{z}{19} = 3 - 128 \)
11. \( 27 = \frac{-5}{4} - 128 \)
12. \( 12 = 2y + 9 \)

---

**Reteaching (continued)**

To solve an equation for one of its variables, rewrite the equation as an equivalent equation with the specified variable on one side of the equation by itself and an expression not containing that variable on the other side.

**Problem**

The equation \( \frac{a}{b} = \frac{c}{d} \) represents a relationship between \( a \), \( b \), and \( c \). What is \( x \) in terms of \( a \) and \( b \)?

Use the properties of equality and the properties of real numbers to rewrite the equation as an equation of equivalent equations.

\[
\frac{a}{b} = \frac{c}{d} \Rightarrow x + 2b
\]

**Exercises**

Solve each equation for the indicated variable.

13. \( 3m = n = 2m + n \), for \( m \)
14. \( 2(a + 3b) = w = 3a \), for \( w = 2m + 2b \)
15. \( ax + bx = x + a \), for \( x = \frac{4}{5} \)
16. \( k(y + 3z) = k(2y - z) = y = \frac{x}{y + 2} \)
17. \( \frac{1}{2} + 3a = 1 \), for \( r = 2 - 6a \)
18. \( \frac{2}{x} + \frac{1}{x} = 1 - 12 \), for \( x = 3, y = 2x + 4 \)
19. \( x + \frac{2}{x} = 1 \), for \( x = 2x + 4 \)
20. \( \frac{3y}{y} + 4 = a + y \), for \( y = x + \frac{3y}{y} \)
**ANSWERS**

**1-5 ELL Support**

Solving Inequalities

To write an inequality from a sentence, first identify the operation and then identify the inequality.

**Example** What inequality represents the sentence “6 more than a number is at least 20”?

*“more than” means addition*

*“is at least” means greater than or equal to*

6 + n ≥ 20

Underline the word or words that indicate an operation.

1. the product of 12 and a number
2. 8 less than a number
3. the difference between a number and 24
4. the sum of a number and 7

Circle the word phrase that identifies the inequality to use. Then write the inequality that represents the sentence.

5. The product of 12 and a number
   12n > 140

6. 8 less than a number
   n < 8

7. The difference between a number and 24
   n > 24

8. the sum of a number and 7
   n + 7 < 25

Some word phrases are very similar, but have different meanings.

**Example** Does the sentence indicate an operation or an inequality?

A number is greater than 99.

*An inequality: n > 99

**1-5 Practice**

Solving Inequalities

Write the inequality that represents the sentence.

1. Four less than a number is greater than 28. x − 4 > 28

2. Twice a number is at least 15. 2x ≥ 15

3. A number increased by 7 is less than 5. x + 7 < 5

4. The quotient of a number and 8 is at most −6. \( \frac{n}{8} \leq -6 \)

Solve each inequality. Graph the solution.

5. 3x + 11 > 2  \( x > -3 \)

6. 5x − 22 > x + 4  \( x > 13 \)

7. \( 2(2y − 1) + 2 \leq 5(y + 3) \)  \( y \leq 17 \)

8. \( 2n − 1 \leq 3n + 7 \)  \( n \leq 22 \)

9. 5x − 22x + 23 ≤ 4  \( x \geq 1 \)

10. \( |2x − 7| + 3 ≥ w − 1 \)  \( w ≤ 6 \)

Solve each problem by writing an inequality.

11. **Geometry** The length of a rectangular yard is 30 meters. The perimeter is at most 90 meters. Describe the width of the yard. At most 15 m

12. **Geometry** A piece of rope 20 feet long is cut from a longer piece that is at least 32 feet long. The remainder is cut into four pieces of equal length. Describe the length of each of the four pieces. At least 9 ft

13. A school principal estimates that no more than 6% of this year’s senior class will graduate with honors. If 720 students graduate this year, how many will graduate with honors? No more than 43 students

14. Two sisters drove 144 miles on a camping trip. They averaged at least 32 miles per gallon on the trip. Describe the number of gallons of gas they used. At most 4.5 gal

**1-5 Think About a Plan**

Solving Inequalities

Your math test scores are 88, 78, 90, and 91. What is the lowest score you can earn on the next test and still achieve an average of at least 85?

**Understanding the Problem**

1. What information do you need to find an average of scores? How do you find an average?

2. How many scores should you include in the average?

3. You want to achieve an average that is at least 85. What score do you need to earn?

**Planning the Solution**

4. Assign a variable, x.

5. Write an expression for the sum of all of the scores, including the next test.

6. Write an expression for the average of all of the scores.

7. Write an inequality that can be used to determine the lowest score you can earn on the next test and still achieve an average of at least 85.

**Getting an Answer**

8. Solve your inequality to find the lowest score you can earn on the next test and still achieve an average of at least 85. What score do you need to earn?

**1-5 Practice [continued]**

Solving Inequalities

**Form G**

Is the inequality always, sometimes, or never true?

15. \( 3(x + 1) > 2x + 2 \)  always true

16. \( 2x − 1 < x + 2 \)  sometimes true

17. \( 7x + 2 \leq 2(2x − 4) \)  never true

18. \( 5(x − 3) = 2(x − 9) \)  sometimes true

Solve each compound inequality. Graph the solution.

19. \( 3x − 6 < 2x + 6 \)  \( x < 9 \)

20. \( 5x + 4 < 20 \)  \( x < 3 \)

21. \( 5x < 20 \)  \( x < 4 \)

22. \( 5n < 10 \)  \( n < 2 \)

23. \( 5x + 2 > 18 \)  \( x > 2 \)

24. \( 2x + 3 > x + 2 \)  \( x > 1 \)

Solve each problem by writing and solving a compound inequality.

25. A student believes she can earn between $5200 and $6250 from her summer job. She knows that she will have to buy four new tires for her car at $90 each. She estimates her other expenses while she is working at $660. How much can she earn on the next test and still achieve an average of at least 85? What score do you need to earn? At least $85

26. A chemist can combine a solution with other liquids in a laboratory at a rate of up to 4°C per hour. If the temperature is originally 0°C, how long will it take to raise the temperature at least 6.5°C?

27. The Science Club advisor expects that between 42 and 49 students will attend the next Science Club field trip. The school allows $5.50 per student for sandwiches and drinks. What is the advisor’s budget for food for the trip? Between $231 and $269.50

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Is the inequality $S$? Solve each inequality. Graph the solution.

6. $\frac{x}{4} \leq 14$ and $x < 10$.

7. $2(2x + 1) > 2x - 10$.

Solve each inequality. Graph the solution.

4. $5a - 10 \leq 5a + 5$.

5. $23 - 2y \geq 33$.

6. $-2(x + 2) + 6 \leq 16$.

Solve the following problem by writing an inequality.

8. The length of a rectangle is 4 cm less than the width. The perimeter is at most 48 cm. What are the restrictions on the dimensions of the rectangle? 

To start, record what you know. 

- Width: length $\leq 4$ 
- Perimeter: at most 48 cm
- Restrictions on the width and length of the rectangle
- The length is at most 14 cm and the width is at most 18 cm.

Is the inequality always, sometimes, or never true?

9. $5(x - 2) \geq 2x + 1$ 

10. $2x + 8 \leq 2(x + 1)$

11. $6x + 1 < 3(2x - 4)$

12. $2(3x + 3) > 2(3x + 1)$

Standardized Test Prep

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What is the solution of $4 - (3 + 2) \leq x + 74$?

A) $x \geq 2$ 
B) $x \geq 3$ 
C) $x \geq 2$ 

2. What is the solution of $-17 - 2y \leq 3|y| + 11$?

A) $r > 4$ 
B) $r < -20$ 
C) $r < -4$ 

3. Which graph best represents the solution of $|x + 4| > x + 37$?

A) $x < -1$ 
B) $x < -2$ 
C) $x < -2$ 

4. What is the solution of the compound inequality $4x - 8 \geq 0$ or $3x > 18$?

A) $x \geq 2$ 
B) $x > 2$ 
C) $x > 2$ 

5. What is the solution of the compound inequality $-2x \leq 6$ and $-3x > -27$?

A) $x \leq 3$ and $x > 9$ 
B) $x \leq 3$ and $x < -9$ 
C) $x \leq 3$ and $x > -9$ 

Short Response

6. Geometry: The lengths of the sides of a triangle are in the ratio $3:4:5$. Describe the length of the longest side if the perimeter is not more than 72 in.

7. Solve the inequality $3x + 4 < 5x + 12$ or $12x > 16$ or $x < 6$. The longest side of a triangle is $n$ in. If the triangle is a right triangle, what is the possible range of values for $n$? 

8. Between 8.5% and 3.9% of the city's population used the public transit system daily. According to the latest census, the city's population is 280,000. How many people used the transit system daily?

9. Between 56,725 and 73,790 people [1] minor incorrect misclassification of one or both values in the range [2] no answer given
1-5 Reteaching
Solving Inequalities

As with an equation, the solutions of an inequality are numbers that make it true. The procedure for solving a linear inequality is much like the one for solving linear equations. To isolate the variable on one side of the inequality, perform the same algebraic operation on each side of the inequality symbol.

The Addition and Subtraction Properties of Inequality state that adding or subtracting the same number from both sides of the inequality does not change the inequality:

If \( a < b \) then \( a + c < b + c \)

If \( a < b \) then \( a - c < b - c \)

The Multiplication and Division Properties of Inequality state that multiplying or dividing both sides of the inequality by the same positive number does not change the inequality:

If \( a < b \) and \( c > 0 \), then \( ac < bc \)

If \( a < b \) and \( c > 0 \), then \( a/c < b/c \)

The direction of the inequality changed in the last step because we divided both sides of the inequality by a negative number.

Th e Multiplication and Division Properties of Inequality also state that, when you multiply or divide each side of an inequality by an inequality by a negative number, you must reverse the inequality symbol.

If \( a < b \) and \( c < 0 \), then \( ac > bc \)

If \( a < b \) and \( c < 0 \), then \( a/c > b/c \)

Problem

What is the solution of \( 2x - 3(x - 1) < x + 5 \)? Graph the solution.

Exercises

Solve each inequality. Graph the solution.

1. \( 2x + 4(2 - x) > 4x > 2 \)

2. \( 4 - (2x - 4) > 5 - (4x + 3) x \leq -3 \)

Exercises

1-6 ELL Support
Absolute Value Equations and Inequalities

Concept List

Choose the concept from the list below that best represents the item in each box.

1. numbers more than 3 units away from zero

2. numbers three units away from zero or more than three units away from zero

3. \( |x| < 3 \)

4. numbers less than 3 units away from zero

5. numbers:3 units away from zero

6. \( |x| \geq 3 \)

7. \( |x| \\

8. numbers three units away from zero or less than three units away from zero

9. \( |x| \leq 3 \)

1-6 Think about a Plan
Absolute Value Equations and Inequalities

Write an absolute value inequality to represent the situation.

Cooking Suppose you used an oven thermometer while baking and discovered that the oven temperature varied between 5° and 3° degrees from the setting. If your oven is set to 350°, let \( t \) be the actual temperature.

1. How do you have to think to solve this problem?

If I subtract the set temperature from the real temperature, the result should be between -5° and 5°.

2. Write a compound inequality that represents the actual oven temperature \( t \).

\( 345 \leq t \leq 355 \)

3. It often helps to draw a picture. Graph this compound inequality on a number line.

4. What is the definition of tolerance?

Tolerance is the difference between a desired measurement and its
maximum and minimum allowable values. It equals half of the difference between the maximum and minimum values.

5. What is the tolerance of the oven? 5°

6. Use the tolerance to write an inequality without absolute values.

\(-5 \leq t - 350 \leq 5\)

7. Rewrite the inequality as an absolute value inequality.

\(|t - 350| \leq 5\)
19. Solve each equation. Check for extraneous solutions.
20. Solve each equation. Check for extraneous solutions.
21. Solve each equation. Check for extraneous solutions.
22. Solve each equation. Check for extraneous solutions.
23. Solve each equation. Graph the solution.
24. Solve each equation. Graph the solution.
25. Solve each inequality. Write each compound inequality as an absolute value inequality.
26. Solve each inequality. Write each compound inequality as an absolute value inequality.
27. Write an absolute value inequality to represent each situation.
28. Write an absolute value inequality to represent each situation.
29. Is the absolute value equation never; the distance between a number and 0 is never negative.
30. Is the absolute value equation never; the distance between a number and 0 is never negative.
31. Write an absolute value equation or inequality to describe each graph.
32. Write an absolute value equation or inequality to describe each graph.
33. To become a potential volume donor listed on the National Marrow Donor Program registry, a person must be between the ages of 18 and 60. Let \( a \) represent the age of a person on the registry. \( a \) for sometimes; the equation is true only for \( x \geq 0 \).
34. The diameter of a ball bearing in a wheel assembly must be between 1.758 cm and 1.817 cm. Let \( d \) represent the diameter of the ball bearing. \( d \) never; the distance between a number and 0 is never negative.
35. The outdoor temperature ranges between 37°F and 62°F in a 24-hour period. Let \( T \) represent the temperature during this time period. \( T \) for sometimes; the equation is true only for \( x \geq 0 \).
36. The outdoor temperature ranges between 37°F and 62°F in a 24-hour period. Let \( T \) represent the temperature during this time period. \( T \) never; the distance between a number and 0 is never negative.
37. The diameter of a ball bearing in a wheel assembly must be between 1.758 cm and 1.817 cm. Let \( d \) represent the diameter of the ball bearing. \( d \) never; the distance between a number and 0 is never negative.
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39. The outdoor temperature ranges between 37°F and 62°F in a 24-hour period. Let \( T \) represent the temperature during this time period. \( T \) never; the distance between a number and 0 is never negative.
ANSWERS

1-6 Standardized Test Prep
Absolute Value Equations and Inequalities

Multiple Choice
For Exercises 1–5, choose the correct letter.

1. What is the solution of \( |x| = 8 \)?
   \( x = 8 \) or \( x = -8 \)
   \( x = 8 \) or \( x = -4 \)
   \( x = -8 \) or \( x = -4 \)
   \( x = 8 \) or \( x = -1 \)
   \( x = -8 \) or \( x = -1 \)

2. What is the solution of \( |x| = 2 \)?
   \( x = 2 \) or \( x = -2 \)
   \( x = 2 \) or \( x = -4 \)
   \( x = 2 \) or \( x = -6 \)
   \( x = 2 \) or \( x = -2 \)
   \( x = -2 \) or \( x = 2 \)

3. What is the solution of \( |x + 3| = 1 \)?
   \( x = 2 \) or \( x = -4 \)
   \( x = 2 \) or \( x = -3 \)
   \( x = 2 \) or \( x = -1 \)
   \( x = 1 \) or \( x = 2 \)
   \( x = -2 \) or \( x = 1 \)

4. Which absolute value inequality is equivalent to the compound inequality?
   \( 2a + 5 \leq 8 \) or \( 2a + 5 > 11 \)
   \( 2a + 5 \leq 8 \) and \( 2a + 5 > 11 \)
   \( 2a + 5 \leq 8 \) or \( 2a + 5 > 11 \)
   \( 2a + 5 \leq 8 \) and \( 2a + 5 < 11 \)
   \( 2a + 5 > 8 \)

5. Which is the correct graph for the solution of \( 2b + 1 \geq 3 \)?
   \( b = 2 \)
   \( b = -1 \)
   \( b = 1 \)
   \( b = 2 \)
   \( b = -2 \)

Short Response
6. An employee's monthly earnings at an electronics store are based on a salary plus commissions on her sales. Her earnings range from $2500 to $3200, depending on her commission. Write a compound inequality to describe if the amount of her monthly earnings. Then rewrite your inequality as an absolute value inequality.

2. \( 2500 \leq x \leq 3200 \)

[1] Compound inequality or incorrect absolute value inequality

1-6 Reteaching
Absolute Value Equations and Inequalities

Solving absolute value equations requires solving two equations separately. Recall that for a real number \( x \), \( |x| \) is the distance from zero to \( x \) on the number line. The equation \( |x| = a \) means that either \( x = a \) or \( x = -a \), since both are \( a \) units from 0.

Problem
What is the solution set for the equation \( |2x + 1| = 4 \)?

The first step in solving an absolute value equation is to isolate the absolute value on one side of the equal sign.

\[ |2x + 1| = 4 \]

Solve the absolute value equation as two equations and solve each of them separately.

\[ 2x + 1 = 4 \text{ or } 2x + 1 = -4 \]

Add 1 to each side.

\[ 2x = 3 \text{ or } 2x = -5 \]

Simplify.

Next, solve the absolute value as two equations and solve each of them separately.

\[ 2x = 3 \text{ or } 2x = -5 \]

Addition Property of Equality

\[ x = \frac{3}{2} \text{ or } x = \frac{-5}{2} \]

Notice that the same operations are performed in the same order on each of the two equations. However, do not try to “simplify” the process by solving a single equation. This leads to errors.

The solution is \( x = \frac{3}{2} \) or \( x = \frac{-5}{2} \).

Check each solution in the original equation.

\[ 2x = 3 \text{ or } 2x = -5 \]

Exercises
1. \( |2x - 3| = 4 \) or \( |2x - 3| = -4 \)
2. \( |3x - 6| = 13 \) or \( |3x - 6| = -13 \)

To solve an absolute value inequality, keep in mind that \( |x| < a \) is the distance from zero to \( x \) on the number line. So, \( |x| < a \) means that \( x \) is less than \( a \) units from 0, so \( x < a \) and \( x > -a \).

Problem
What is the solution set for the inequality \( |2x - 3| > 11 \)?

The inequality \( |2x - 3| > 11 \) means that \( 2x - 3 \) is greater than \( 11 \) or \( 2x - 3 \) is less than \( -11 \).

Begin by rewriting the absolute value as two inequalities and solve each of them separately.

\[ 2x - 3 > 11 \text{ or } 2x - 3 < -11 \]

Solve each inequality.

\[ 2x > 14 \text{ or } 2x < -14 \]

Divide each side by 2.

\[ x > 7 \text{ or } x < -7 \]

The solution set is \( x > 7 \) or \( x < -7 \).

Exercises
3. \( |x + 2| \leq 4 \) or \( |x + 2| \geq 4 \)
4. \( |x - 3| \leq 7 \) or \( |x - 3| \geq 7 \)
5. \( |x| \leq 4 \) or \( |x| \geq 4 \)
6. What is the solution? \( 2x \leq 14 \) or \( 2x \geq -14 \)

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Do you know HOW?

1. Describe a rule for the pattern.
2. Name the integers in the list: 0, -2, 1, 7, 121, -1, -7, -12, -π.
3. Name the property of real numbers illustrated by each equation.
   a. Write an algebraic expression to model the word phrase.
   b. Justify each step by naming the property used.
   c. Describe a rule for the pattern.
   d. Solve each equation.
   e. Find the pattern and fill in the next three numbers in the pattern.
   f. Solve the inequality.
   g. Solve each equation.
   h. Solve the equation.
   i. Is there a Multiplication Property of Closure that applies to integers?
   j. Solve each equation.
   k. Solve each equation.
   l. Find the given value of the variable.
   m. Explain in words why |x| is not a negative number.

Do you UNDERSTAND?

9. Writing How can you use a graph to find a pattern? Answers may vary. Sample: Choose some points on the graph; use these points to make a table of input and output values; look for a pattern in the process column.

10. Vocabulary What is another name for an additive inverse? opposite

11. Reasoning Is there a Multiplication Property of Closure that applies to irrational numbers? Justify your answer. No. Answers may vary. Sample: \( \sqrt{2} \times \sqrt{3} = \sqrt{6} \), which is not irrational.

12. Compare and Contrast What is the difference between simplifying an expression and evaluating an expression? Answers may vary. Sample: Simplifying an expression is rewriting it using the properties of real numbers and combining like terms, resulting in a simpler expression. Evaluating an expression is substituting values for the variables, resulting in a numeric value.

Write an equation and solve the problem.

13. Two buses leave Dallas at the same time and travel in opposite directions. One bus averages 38 mi/h, and the other bus averages 52 mi/h. When will they be 363 mi apart? 3 hr 18 min later

Solve each inequality. Graph the solution.

14. \( 3e + 7 > 4 \quad m = -1 \)
15. \( 4a > 3(a - 1) \quad - \frac{7}{3} < a < 8 \)

Solve each compound inequality. Graph the solutions.

16. \( 3t - 1 \geq 5t \quad -4 \leq t < 1 \)
17. \( 7c \leq 12 \quad 2c > 6 \quad c \leq 6 \quad c < 3 \)

Solve each equation. Check for extraneous solutions.

18. \( 2b + 3 = 5 \quad a = 1 \quad -a = -4 \)
19. \( x + 6 = 2x \quad a = 6 \)

20. The temperature \( T \) of a refrigerator is at least 30°F and at most 41°F. Write a compound inequality and an absolute value inequality for the temperature of the refrigerator. \( T \geq 30 \) and \( T \leq 41 \)

Do you UNDERSTAND?

15. Writing Suppose the sum of three consecutive even integers is given. How do you find the three numbers? Answers may vary. Sample: Let the first integer be \( x \), the next integer be \( x + 2 \), and the third integer be \( x + 4 \). Therefore, \( x + (x + 2) + (x + 4) = 6x + 6 \). Solve the equation for \( x \) to get the first number. The other two numbers are \( x + 2 \) and \( x + 4 \).

16. Compare and Contrast How do the solutions to \( |x| > 1 \), \( |x| \leq 0 \), and \( |x| < -1 \) differ? Answers may vary. Sample: \( |x| \leq 0 \) has an infinite number of solutions, \( |x| < -1 \) has no solution, and \( |x| > 1 \) has a single unique solution, \( x = -1 \); and \( |x| = -1 \) has no solution.
Do you UNDERSTAND?

9. Writing Give an example of a number that is an irrational number. Explain why it is irrational.

Answers may vary. Sample: \( \sqrt{2} \) is an irrational number because 3.141592 . . . neither terminates nor repeats; it cannot be written as a quotient of integers.

10. Writing Write an inequality that has no solution. Explain why it does not have a solution.

Answers may vary. Sample: \( 6 + 1 < 3(2a - 4) \); using the Distributive Property, \( 6 + 1 < 4a - 12 \); subtract 6 from each side: \( 1 < 4a - 12 \). Since \( 1 < 12 \) is never true, the inequality has no solution.

Do you UNDERSTAND?

Do you KNOW HOW?

1. Describe a rule for the pattern.

Answers may vary. Sample: Start with a column of two circles. Add one circle to the right of the bottom row; add two circles to the right of the bottom row.

2. Identify the integers in the list: \(-1, \frac{5}{6}, 2\pi, -\sqrt{2}, 0, -\sqrt{10}, \frac{6}{5}\).

Name the property of real numbers illustrated by each equation.

3. \( (2 \cdot 5) = (5 \cdot 2) \cdot 5 \)

4. \( (x + 2) = 5 \cdot x + 5 \cdot 2 \)

Associative Property of Multiplication

Distributive Property

Write an algebraic expression to model the word phrase.

5. the sum of \( y \) and the product of 7 and \( x \)

6. eight more than the quotient of \( t \) and 2

Evaluate the expression for the given value of the variable.

7. \( 2b + 8(0 - 4); b = 2 \)

8. \( x + 2(2x - 1); x = -4 \)

Do you UNDERSTAND?

Do you KNOW HOW?

Solve each equation. Check for extraneous solutions.

9. \( |u| + 1 < 2 \)

10. \( |12 - v| = 30 \) and graph the solution.

11. \( t + 2 = 14 \) or \( 12 - t = 7 \)

Solve each equation.

12. \( 2a + 5 \leq 6a + 1 \)

13. \( 2a + 1 \geq 4a + 1 \)

Solve each compound inequality. Graph the solutions.

14. \( 3x \leq 6 \) or \( 2x + 1 \geq 3 \)

15. \( 2x + 1 < 4 \) and \( 2x - 1 < 3 \)

Solve each equation. Check for extraneous solutions.

16. \( 3x + 3 \leq 12 \) or \( 5x = -5 \) or \( x = -7 \)

17. \( |b + 2| = 2b = 2 \)

18. The weatherman announced that the temperature \( T \) over the next few weeks will be at least 60°F and at most 70°F. Write an absolute value inequality for the temperature over the next few weeks. \( |T - 71| \leq 7 \)

Do you UNDERSTAND?

19. What is another name for the multiplicative inverse? reciprocal

20. Reasoning Explain in words why \( 2|x| < -4 \) has no solution.

Answers may vary. Sample: Dividing both sides by 2 gives \( |x| < -2 \). The absolute value of any number must be nonnegative, so the inequality has no solution.

21. Open-Ended What is the difference between simplifying an expression and evaluating an expression?

Answers may vary. Simplifying an expression is rewriting it using the properties of real numbers and combining like terms, resulting in a simpler expression. Evaluating an expression is substituting values for the variables, resulting in a numerical value.
Chapter 1 Performance Tasks (continued)

Task 3
Explain how the properties of inequalities differ from the properties of equality and how the solutions of an inequality differ from the solutions of an equation. Use the following equation and inequality as part of your explanation.

\[ -3x + 10 \]

Answers may vary. Sample: When multiplying or dividing both sides of an inequality by a negative value, the inequality symbol must reverse direction. The solution of the inequality is \( x < -3 \), whereas the solution of the equation is \( x = -3 \). The solutions of the inequality do not include the solution of the equation, but consist of all numbers less than it.

[Student gives specific details of the differences and explicitly uses the examples to illustrate these differences. Steps in solving the equation and inequality are correct.]

[Student gives adequate details of the differences and makes reference to the examples. Steps in solving the equation and inequality contain minor errors.]

[Student solves the equation and the inequality but does not explain the differences. Steps in solving the equation and inequality contain errors.]

[Student does not correctly or completely solve the equation and inequality. Steps in solving the equation and inequality contain major errors.]

[Response is missing or inappropriate.]

Task 4
a. Write an inequality using an absolute value that can be rewritten as a compound inequality with or.

b. Solve and graph the inequality in part a.

c. Write an inequality using an absolute value that can be rewritten as a compound inequality with and.

d. Solve and graph the inequality in part c.

[Student correctly identifies inequalities using an absolute value. Graphs or solutions given are correct.]

[Student makes significant errors in writing or solving the inequalities. Graphs contain major errors.]

[Student does not write or solve all inequalities. Graph is incomplete or missing.]

[Response is missing or inappropriate.]

Chapter 1 Cumulative Review (continued)

15. Derek has noticed there are fewer students per class at his friend’s private school than at his public school. He talks to the principal of the private school and learns that the number of students is the same in every class. Derek surveys the classes and makes the graph at the right, comparing the number of students and the number of teachers.

a. How many teachers are required for 65 students at the private school? 5

b. How many teachers are required for 153 students? 8

16. Graph the number –3, \(-\frac{3}{2}\), and \(\frac{3}{2}\) on a number line.

17. What are the opposite and the reciprocal of \(-\frac{2}{3}\)?

Opposite: \(\frac{2}{3}\)

Reciprocal: \(-\frac{3}{2}\)

18. Write an equation to solve the problem. Find three consecutive even numbers whose sum is 144.

\[ 2x + 4 + 6 = 144 \]

19. Write the compound inequality 3.1 \% \( \leq \) g as an absolute value inequality.

\[ |x – 3.8| \leq 0.8 \]

Extended Response

20. Writing. What is the value of the expression \( 27 \) \( - \) 8? Explain.

It is undefined; because division by 0 is undefined, the entire expression is undefined.

21. [appropriate methods and explanation, but with one computational error]

22. [incorrect value OR correct value without explanation]

23. [correct value, without work shown or explanation]

24. [answer missing or no attempt made]
Chapter 1 Project: Buy the Hour

About the Project
The Chapter Project gives students an opportunity to use expressions, equations, inequalities, and graphs to model real-life situations. Students write and simplify expressions by using the Distributive Property and combining like terms, evaluate expressions, and write and solve equations and inequalities in one variable.

Introducing the Project
- Encourage students to keep all project-related materials in a separate folder.
- Ask students what the term minimum wage means.
- Have students explain the similarities and differences among the terms expression, equation, and inequality. Remind students that it is important to define the variable when writing expressions, equations, and inequalities.

Activity 1: Researching
Students research federal and state minimum wages.

Activity 2: Modeling
Students write and evaluate expressions modeling amounts of money earned based on minimum wages found in Activity 1.

Activity 3: Solving
Students write and solve equations and inequalities that model relationships between amounts of money earned. Students also graph the solutions of their inequalities.

Finishing the Project
You may wish to plan a day project on which students share their completed projects. Encourage students to explain their processes as well as their results.
- Have students review their methods for writing and evaluating expressions, for writing and solving equations, and for writing, solving, and graphing inequalities.
- Ask groups to share their insights that resulted from completing the project, such as shortcuts they found for solving equations and inequalities or for researching data.

Chapter 1 Project Teacher Notes: Buy the Hour (continued)

Activity 3: Solving
Round numbers of hours to the nearest tenth if necessary.
- Suppose that last week your employer gave you a $5.50/h raise and a $20 bonus as a reward for good work. You earned a total of $80 for the week. Let $x$ represent the number of hours you worked that week. Write an equation to model this situation. Then solve your equation and explain the meaning of your solution. Answers may vary. Sample answer: Based on a minimum wage of $7.25 effective July 24, 2009) 2.75x + 20 = 80; $x$ worked about 7.7 h last week to earn $80.
- Suppose that your friend earns the same hourly wage that you earn, but works more hours. Your friend works 40 h each week and earns $20 in tips. Write an equation to model this situation. Then solve your equation and explain the meaning of your solution. $7.25n + 20 = 7.75n$, where $n$ is the number of hours each friend worked; 40; minimum wage is $7.25, $20 in tips, and that you (earning $5.50/h more than your friend) have earned the same amount of money at the end of a week during which you worked the same number of hours as your friend. Write an equation to model this situation. Then solve your equation and explain the meaning of your solution. $7.25n + 15 = 7.75n$, $x > 11.3$; your friend must work at least 11.3 h to earn at least $95 next week.

Finishing the Project
The answers to the activities should help you to complete your project. You should prepare a presentation for the class describing your results. Your presentation should include the data you researched; the expressions, equations, and inequality you used to model the given situations; and the graph of your inequality.

Reflect and Revise
Ask a classmate to review your project. After you have reviewed each other’s presentations, decide if your work is clear, complete, and convincing. If needed, make changes to improve your presentation.

Extending the Project
Research the minimum wages set by other states. If they differ from the minimum wage set by the state in which you live, explain why you think the difference occurred. Define the variable when writing equations, expressions, and inequalities to model the relationships that determine possible factors that might contribute to the differences. Find out what conditions might exist that would allow an employer to pay an employee less than the federal minimum wage.

Chapter 1 Project Manager: Buy the Hour

Getting Started
Read the project. As you work on the project, you will need a calculator and materials on which you can record your results and make calculations. Keep all of your work for the project in a folder.

Checklist
☐ Activity 1: researching minimum wages
☐ Activity 2: writing algebraic expressions
☐ Activity 3: writing and solving equations and inequalities
☐ algebraic models

Suggestions
☐ Select a state in which you are interested.
☐ Substitute reasonable values for the variables to determine if the expressions make sense.
☐ Check that your answers are reasonable.
☐ Have you defined the variables in your expressions, equations, and inequality? How does the graph of an equation differ from the graph of an inequality? What does this mean in terms of your solution?

Scoring Rubric
4 The expressions, the equations, and the inequality are correct. The graph and all calculations are accurate. The explanations are thorough and well thought out. The presentation is clear and complete.
3 The expressions, the equations, and the inequality have minor errors. The graph and calculations are mostly correct. The explanations and presentation lack detail or contain small errors.
2 The expressions, the equations, and the inequality have major errors. The graph and calculations contain major errors. The explanations and presentation contain major inaccuracies.
1 The expressions, the equations, and the inequality are not correct. The graph is not accurate. Calculations contain major errors or are incomplete. The explanations and presentation are inaccurate or incomplete.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of the Project

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2-1  Relations and Functions

Use the vertical line test to determine whether each graph represents a function.

3.

Determine whether each relation is a function.

2.

1.

The table shows the number of gold medals won by United States athletes during the Summer Olympics.

<table>
<thead>
<tr>
<th>Year</th>
<th>Gold Medals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>23</td>
</tr>
<tr>
<td>1992</td>
<td>37</td>
</tr>
<tr>
<td>1996</td>
<td>40</td>
</tr>
<tr>
<td>2000</td>
<td>36</td>
</tr>
<tr>
<td>2004</td>
<td>38</td>
</tr>
<tr>
<td>2008</td>
<td>44</td>
</tr>
</tbody>
</table>

Represent the data using each of the following:

- a graph on the coordinate plane
- ordered pairs
- a function rule

Answers may vary. Sample: The year 1992 corresponds to the point (1992, 37), the year 2000 corresponds to the point (2000, 36), and so on.

Function notation is a way to write a function rule.

Sample: The function notation for the data is $f(x) = 2x + 5$ for the year $x$ and the corresponding number of gold medals $f(x)$.

Test your understanding by writing a function rule to represent the following data set:


Answers may vary. Sample: The function rule is $f(x) = x + 3$ for $x = 2004$ and $x = 2008$.

Understanding the Problem

1. The width of the box is $\frac{2}{3}$ in. The length of the box is $\frac{2}{3}$ in. The height of the box is $\frac{1}{3}$ in.

2. What is the problem asking you to determine?

A function that gives the surface area of a box with a length of $\frac{2}{3}$ in., a width of $\frac{2}{3}$ in., and a height of $\frac{1}{3}$ in.

Planning the Solution

3. What is the area of the top of the box? What is the area of the bottom of the box?

16 in.$^2$, 16 in.$^2$

4. What is the total area of the top and the bottom of the box?

32 in.$^2$

5. What is the area of each side of the box?

40 in.$^2$

6. What is the total area of the sides of the box?

160 in.$^2$

Getting an Answer

7. Write a function to represent the surface area of the box.

$s(x) = 16x + 32$

8. Evaluate your function for $b = 0.5$ inches.

136 in.$^2$

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2-1 Practice Form K
Relations and Functions

A motion detector tracks an egg as it drops from 10 ft above the ground. The table shows the height at various times.

1. Represent the data using each of the following:
   a. a mapping diagram
   b. ordered pairs: (0, 10), (1, 9), ..., (5, 0)
   c. a graph on the coordinate plane

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Height (Feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>0.1</td>
<td>9.8</td>
</tr>
<tr>
<td>0.2</td>
<td>9.4</td>
</tr>
<tr>
<td>0.3</td>
<td>8.6</td>
</tr>
<tr>
<td>0.4</td>
<td>7.4</td>
</tr>
</tbody>
</table>

2. What are the domain and range of this relation?
   Domain: [0, 1, 2, 3, 4]
   Range: [10, 9.8, 9.4, 8.6, 7.4]

Determine whether each relation is a function.

3. Domain: [−4, −1, 1, 3]; Range: [−5, −1, 0, 4, 6, 9, 12] yes

4. A (−4, 1), (3, 5), (−1, 0), (4, 2), (9, 5) yes

Use the vertical-line test to determine whether each graph represents a function.

5. yes

6. yes

7. no

---

2-1 Enrichment Form K
Relations and Functions

Although relations can be defined on arbitrary sets, we shall confine our attention to relations in the xy-plane. A relation is said to be reflexive if for any real number x in the domain of the relation, the point (x, x) belongs to the relation.

1. How can you tell geometrically whether a relation is reflexive?
   The graph of the relation must contain the line y = x.

2. What can you say about a relation that is both symmetric and asymmetric?
   The relation is empty.

3. Which of the following relations are reflexive, symmetric, asymmetric, transitive, or an equivalence relation?
   a. the set of all pairs of real numbers (x, y) such that x = y
   equivalence relation
   b. the set of all pairs of real numbers (x, y) such that x ≥ y
   reflexive, transitive
   c. the set of all pairs of real numbers (x, y) such that y = x
   reflexive, symmetric
   d. the set of all pairs of real numbers (x, y) such that x = y
   symmetric
   e. the set of all pairs of real numbers (x, y) such that x is an integer
   transitive
   f. the set of all pairs of real numbers (x, y) such that xy = 0
   symmetric
   g. the set of all pairs of real numbers (x, y) such that x + y = 1
   symmetric
   h. the set of all pairs of real numbers (x, y) such that x ≤ y
   reflexive
   i. the set of all pairs of real numbers (x, y) such that xy = 0
   symmetric
   j. the set of all pairs of real numbers (x, y) such that x = y
   symmetric
   k. the set of all pairs of real numbers (x, y) such that x + y = 0
   symmetric
   l. the set of all pairs of real numbers (x, y) such that xy = 0
   symmetric
   m. the set of all pairs of real numbers (x, y) such that x is an integer and y is not an integer
   asymmetric
2-1  Reteaching

Relations and Functions

- A relation is a set of ordered pairs.
- The domain is the set of first numbers in each pair, or the x-values.
- The range is the set of second numbers in each pair, or the y-values.
- A relation is a function if each input value x corresponds to exactly one output value y. In a set of ordered pairs for a function, an x-value cannot be repeated with two or more different y-values.

Problem:

Roll a number cube to find six ordered pairs. Determine whether the set of ordered pairs is a function. Find the domain and range.

{(0, 0), (A, B), (C, D), (E, F), (G, H)}

The domain is the set of first numbers in each pair: {A, C, E, G}; range: {B, D, F, H}

Yes; no

Domain: {1}; range: {2, 5}

Yes; no

Exercises

Roll a number cube to find the indicated number of ordered pairs. Determine whether each set of ordered pairs is a function. Find the domain and range of each relation.

1. 5 ordered pairs
2. 4 ordered pairs
3. 6 ordered pairs
4. 8 ordered pairs

Check students’ work.

5. (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

No; the x-value 1 is repeated with different y-values; domain: (1); range: {2, 3, 4, 5, 6}

6. (0, -1), (2, 1), (-1, -1), (-3, 2), (2, 3)

Yes; no; x-value is repeated; domain: {0, 2, -1, 3}; range: {-1, 2, 1, 0, 2, 3, 4}

7. (A, B), (C, D), (E, F), (G, H)

Yes; no; x-value is repeated; domain: {A, C, E, G}; range: {B, D, F, H}

8. (1, M), (N, P), (T, J), (L, P)

No; the x-value 1 is repeated with different y-values; domain: {1}; range: {M, N, J, L}

9. (0, 0)

Yes; no; x-value is repeated; domain: {0}; range: {0}

10. \( \{3, 1\}, \{0, 5\}, \{2, 1\} \)

No; the x-value 1 is repeated with different y-values; domain: \( \{1, 3, 4\} \); range: \( \{2\} \)

2-2  ELL Support

Direct Variation

A clerk’s weekly salary varies directly with the number of hours he works. For 12 h of work, the clerk earns $114. How much will he earn for 19 h of work? The steps to solve this problem were written on the note cards below, but they got mixed up.

Solve for x2.

Substitute \( a = 114 \), \( h_1 = 12 \), and \( h_2 = 19 \).

Write the cross products.

Let \( x = \) the salary and \( h = \) the number of hours worked.

Use the proportion \( \frac{a}{h} = \frac{a'}{h'} \) to model the situation.

Salary varies directly with hours worked, so the salary is constant.

1. First, salary varies directly with hours worked, so \( \frac{a}{h} = \frac{a'}{h'} \) is constant

2. Second, let \( x = \) the salary and \( h = \) the number of hours worked

3. Third, use the proportion \( \frac{a}{h} = \frac{a'}{h'} \) to model the situation

4. Next, substitute \( a = 114, h_1 = 12, \) and \( h_2 = 19 \)

5. Then, write the cross products

6. Finally, solve for x2

The function, \( f \) evaluated at 2 or 0:

\( f(2) = \frac{2}{2} = 1 \)

\( f(0) = \frac{0}{0} \) undefined

2-2  Think About a Plan

Direct Variation

Sports The number of rotations of a bicycle wheel varies directly with the number of pedal strokes. Suppose that in the bicycle’s lowest gear, 6 pedal strokes move the cyclist about 357 in. In the same gear, how many pedal strokes are needed to move 100 ft?

Know

1. The number of rotations of a bicycle wheel varies directly with the number of pedal strokes

2. 6 pedal strokes move the cyclist 357 in.

Need

3. To solve the problem I need to:

   Find the constant of variation by solving for \( k \) in \( d = 6p \), convert 100 ft to inches, and then use the constant of variation to find the number of pedal strokes needed to move 100 ft.

Plan

4. Write an equation of direct variation to model the situation. Find the constant of variation. 357 in. = 6 p; \( k = 59.5 \)

5. Substitute for one variable and the constant of variation in the equation of direct variation. \( 100 = 12 \cdot k \)

6. What does the solution mean?

   About 20 pedal strokes are needed to move the cyclist 100 ft.


   Yes; 100 ft is 1200 in., which is between 3 and 4 times 357 in.; 20 pedal strokes is between 3 and 4 times 6 pedal strokes.

ANSWERS

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99
14. For Exercises 10–13, determine whether two points are on the same line. If so, find the constant of variation and write the function rule.

7.

For each function, determine whether y varies directly with x. If so, find the constant of variation.

4. \( y = \frac{1}{2} x \), yes

5. \( y = -1.2x \), yes

6. \( y + 4x = 0 \), no

7. \( 3x = 1 \), no

8. \( y = 3x \), yes

9. \( y = 2 - x \), no

For Exercises 10–13, y varies directly with x.

10. If \( y = 3 \) when \( x = -3 \), find \( x \) when \( y = 5 \). 15

11. If \( y = -14 \) when \( x = -7 \), find \( x \) when \( y = 22 \). 11

12. If \( y = 5 \) when \( x = -8 \), find \( x \) when \( y = 2 \). 1.6

13. If \( y = 9 \) when \( x = 14 \), find \( x \) when \( y = -5 \). -21

14. The distance a spring stretches varies directly with the amount of weight that is hanging on it. A weight of 2.5 pounds stretches a spring 18 inches. What is the stretch of a 2.5-pound weight on a spring that is 22 inches long?

15. The amount of lemon juice in a lemonade recipe varies directly with the amount of water. The recipe calls for 8 oz of lemon juice and 32 oz of water.

How much lemon juice should you use if you start with 28 oz of water? 7 oz

16. Form G

Practice

Direct Variation

Determine whether y varies directly with x. If so, find the constant of variation.

4. \( y = \frac{1}{2} x \)

5. \( y = -1.2x \)

6. \( y + 4x = 0 \)

7. \( 3x = 1 \)

8. \( y = 3x \)

9. \( y = 2 - x \)

For Exercises 10–13, y varies directly with x.

10. If \( y = 3 \) when \( x = -3 \), find \( x \) when \( y = 5 \).

11. If \( y = -14 \) when \( x = -7 \), find \( x \) when \( y = 22 \).

12. If \( y = 5 \) when \( x = -8 \), find \( x \) when \( y = 2 \).

13. If \( y = 9 \) when \( x = 14 \), find \( x \) when \( y = -5 \).

14. The distance a spring stretches varies directly with the amount of weight that is hanging on it. A weight of 2.5 pounds stretches a spring 18 inches. What is the stretch of a 2.5-pound weight on a spring that is 22 inches long?

15. The amount of lemon juice in a lemonade recipe varies directly with the amount of water. The recipe calls for 8 oz of lemon juice and 32 oz of water.

How much lemon juice should you use if you start with 28 oz of water? 7 oz

16. Form G

Practice

Direct Variation

Write and graph a direct variation equation that passes through each point.

16. \((4, 2)\)

17. \((-4, -8)\)

18. \((-3, 15)\)

19. \((7, 3)\)

20. \((4, 12)\)

21. \((-5, -10)\)

22. \((-1, -3)\)

23. \((3, 9)\)

24. \((10, 25)\)

25. \((3, 4)\)

For Exercises 26–28, y varies directly with x.

26. If \( y = 3 \) when \( x = 2 \), find \( x \) when \( y = 15 \).

27. If \( y = 5 \) when \( x = 10 \), find \( x \) when \( y = 5 \).

28. If \( y = -4 \) when \( x = \frac{1}{2} \), find \( y \) when \( x = 2 \).

29. A new hybrid car has a 12-gallon gas tank. On one tank of gas, the owner can drive 540 miles. The number of miles traveled varies directly with the number of gallons of gas in the car.

a. Write an equation that relates the number of miles traveled with the number of gallons of gas used.

b. How many miles can the owner travel on 9 gallons of gas?

30. On a certain calling plan, a 15-minute long-distance phone call costs $0.90. The cost varies directly with the length of the call. Write an equation that relates the cost to the length of the call. How long is a call that costs $1.32?

a. $0.06 per minute

b. 22 minutes

31. How much lemon juice should you use if you start with 28 oz of water? 7 oz

32. Form G

Practice

Direct Variation

Write and graph a direct variation equation that passes through each point.

12. \((4, 2)\)

13. \((-1, -5)\)

14. \((-4, 8)\)

For each function, determine whether y varies directly with x. If so, find the constant of variation.

4. \( y = 3x = 2 \)

5. \( y = 4x + 6 \)

6. \( y = 5 \)

7. \( y = 3x = 2 \)

8. \( y = 4x \)

9. \( y = 2 - x \)

10. \( y = 12x \)

11. The length of an object’s shadow varies directly with the height of the object.

A 15-ft tree casts a 48-ft shadow.

a. Write a function rule and determine the constant of variation.

b. What length shadow would a 7-ft tree cast? 22.8 ft

12. What height tree would cast a 9-ft long shadow? 22.5 ft

13. Error Analysis: Suppose y varies directly with x. If \( y = 10 \) when \( x = 3 \), what is \( x \) when \( y = 16 \)? Suppose you say that \( x = 32 \) and your friend says \( x = 8 \). Who is correct? What mistake did you make?

Your friend is correct; you reversed your relationship between the variables. You incorrectly divided by \( k \) instead of multiplying by \( k \).

14. Suppose you drive a car 392 mi on a tank of gas. The tank holds 14 gallons.

a. Write an equation that relates miles traveled with the number of gallons of gas used.

b. Last year you drove 11,700 mi. Approximately how many gallons of gas did you use?

a. \( \frac{x}{14} = y \)

b. About 418 gallons

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100
2-2 Standardized Test Prep
Direct Variation

Multiple Choice
For Exercises 1-5, choose the correct letter.
1. If $y$ varies directly with $x$ and $y = 18$ when $x = 6$, which of the following represents this situation? B
   $y = 3x$  $y = 2x$  $y = 3x$  $y = 6x$
2. Which function best represents the relationship between the quantities in the table? C
   \[ \begin{array}{c|c}
   x & y \\
   \hline
   1 & 4 \\
   2 & 8 \\
   3 & 12 \\
   4 & 16 \\
   \end{array} \]
   $y = 2x$  $y = 3x$  $y = 2x$  $y = 4x$

3. If $y$ varies directly with $x$ and $y = 9$ when $x = 5$, which is $x$ when $y = 18$? B
   $x = 10$  $x = 15$  $x = 5$  $x = 18$

4. Which equation of direct variation has $(24, -48)$ as a solution? C
   $y = \frac{x}{6}$  $y = \frac{x}{4}$  $y = -3x$  $y = -4x$

5. Which equation does NOT represent a direct variation? C
   $y = 4x - 0$  $y = 2x$  $y = 4x = 0$  $y = -4x$  $y = -3x$

Short Response
6. You can download a 3 MB file in 2 seconds. The time it takes to download a file varies directly with the size of the file. Write an equation of direct variation to represent the situation. How long will it take to download a 3 MB file?
   $t = \frac{F}{3}$, where $t$ is in seconds.

2-2 Reteaching
Direct Variation

A direct variation is a function of the form,

$y = \frac{k}{x}$, where $k \neq 0$.

Represent the input values as $x$ and represent the output values as $y$. The ratio of any output-input pair is equal to $k$, the constant of variation.

Problem
For each function, determine whether $y$ varies directly with $x$. If so, what is the constant of variation?

### Identify direct variation from a table.
<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
</tr>
</tbody>
</table>

Find $y$ for each ordered pair.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

This is a direct variation. The constant of variation is $\frac{1}{2x}$.

### Identify direct variation from an equation.

a. $y = 2x$  $x = 1$  $y = 2$

b. $y = 2x$  $x = 1$  $y = 2$

This is a direct variation because there is a constant left when you try to put it in the form $y = ax$.

Exercises
For each function, determine whether $y$ varies directly with $x$. If so, find the constant of variation.

1. $y = 4x + 1$  yes
2. $y = 4x$  yes
3. $y = 4x$  yes
4. $y = 4x + 1$  no
5. $y = 4x$  yes
6. $y = 4x$  yes
7. $y = 4x - 5$  no
8. $y = 15x$  yes
9. $y = 17x$  yes

2-2 Enrichment
Direct Variation: Drawing Conclusions

The charts below show the amount of commission each salesperson received on a given real estate sale.

<table>
<thead>
<tr>
<th>John’s Sales</th>
<th>Commission</th>
<th>Jeff’s Sales</th>
<th>Commission</th>
</tr>
</thead>
<tbody>
<tr>
<td>$125,000</td>
<td>$4,375</td>
<td>$80,000</td>
<td>$3,200</td>
</tr>
<tr>
<td>$60,000</td>
<td>$2,800</td>
<td>$80,000</td>
<td>$3,200</td>
</tr>
<tr>
<td>$150,000</td>
<td>$5,100</td>
<td>$125,000</td>
<td>$5,200</td>
</tr>
<tr>
<td>$225,000</td>
<td>$6,700</td>
<td>$150,000</td>
<td>$6,200</td>
</tr>
</tbody>
</table>

1. Determine which salesperson received a straight commission. (A straight commission is a constant percent received based on sales.)
   John: 5%  Jeff: 6%

2. Suppose sales are made directly with $x$ and you are given a particular value of $x$. What do you need to know in order to find the corresponding value of $y$? the value of the constant of variation

3. In an electric circuit, the voltage varies directly as the electric current, measured in amperes. If the voltage is 75 volts when the current is 15 amps, find the voltage when the current is 10 amps. 50 volts
Write an equation for each line.

Find the slope and y-intercept of each line.

Graph each equation.

Find the slope and y-intercept of each line.

Think about a plan

The equation \( d = 4 - \frac{t}{15} \) represents your distance from home if for each minute of your walk:

a. If you graphed this equation, what would the slope represent? Explain.

b. Are you walking towards or away from your home? Explain.

1. What does \( d \) represent? your distance from home in miles

2. What does \( t \) represent? the number of minutes you spend walking

3. Is the equation in slope-intercept form? If not, write the equation in slope-intercept form.

4. What units make sense for the slope? Explain.

5. What does the slope represent? Explain.

6. Is your distance from home increasing or decreasing?

...
Write each equation in slope-intercept form. Then find the slope and the y-intercept of each line.

1. $m = -1$ and the y-intercept is 3
2. $m = \frac{3}{2}$ and the y-intercept is -5

Write an equation for each line.

4. $y = 4x + 3$
5. $y = -2x - 2$
6. $y = -3x + 2$
7. $y = 4x + 1$

Write each equation in slope-intercept form. Then find the slope and y-intercept of each line. To start, substitute $(-3, 2)$ and $(3, 2)$

4. $y = 4x + 3$
5. $y = -2x - 2$
6. $y = -3x + 2$
7. $y = -5x + 2$
8. $y = -5x - 2$
9. $y = 4x - 5$

Graph each equation.

11. $y = -x + 3$
12. $y = \frac{1}{2}x + 4$
13. $y = -x + 3$

Multiple Choice

For the equation $3x + 2y = 12$, which has value $m = \frac{3}{2}$, the function is said to be linear.

5. $m = \frac{3}{2}$
6. $m = -\frac{2}{3}$
7. $m = \frac{3}{2}$
8. $m = -\frac{2}{3}$

Enrichment

Linear Functions and Slope-Intercept Form

Explicit, Implicit, and Parametric Equations of Lines

When a function $y = f(x)$ is written so that you can directly compute the value of $y$ given any value of $x$, the function is said to be defined explicitly. If you have to do algebraic manipulations to find the value of $y$, the function is said to be defined implicitly. For example, the equation $2y + 4x = 10$ implicitly defines $y$ as a function of $x$. Given $x = 1$, you would have to solve the equation $2y + 4 = 10$ to find the corresponding value of $y$.

5. $y = -3x + 8$
6. $y = -3x + 8$
7. $y = -3x + 8$
8. $y = -3x + 8$
9. $y = -3x + 8$
10. $y = -3x + 8$

Sometimes it is possible to go from a parametric representation to an explicit one. In the example above, solve for $x$ as a function of $y$. $t = \frac{x}{2} + \frac{y}{4}$

1. $t = 2$ if $x = 0$
2. $t = 2$ if $x = 0$
3. $t = 2$ if $x = 0$
4. $t = 2$ if $x = 0$
5. $t = 2$ if $x = 0$
6. $t = 2$ if $x = 0$
7. $t = 2$ if $x = 0$
8. $t = 2$ if $x = 0$
9. $t = 2$ if $x = 0$
10. $t = 2$ if $x = 0$
You can use the slope-intercept form to write equations of lines.

- The slope-intercept formula is \( y = mx + b \), where \( m \) represents the slope of the line, and \( b \) represents its \( y \)-intercept. The \( y \)-intercept is the point at which the line crosses the \( y \)-axis.
- The slope of a horizontal line is always zero, and the slope of a vertical line is always undefined.

**Problem**

What is the equation of the line that contains the point \((3, 2)\) and has a slope of \(-\frac{1}{4}\)?

- \( y = mx + b \)
- \( y = -\frac{1}{4}x + b \)
- Subtract \(\frac{1}{2}\) from both sides of the equation to find \( b \).
- Add \(\frac{1}{2}\) to both sides to find \( b \).
- \( y = \frac{1}{2} \)
- Add \(\frac{1}{2}\) to each side and simplify.

**Exercises**

1. Write an equation for each line.
   - \( m = 4; \) contains \((3, 2)\)
   - \( y = 4x + 10 \)
   - \( y = -2; \) contains \((4, 7)\)
   - \( y = -2x + 15 \)
   - \( m = -1; \) contains \((-5, -2)\)
   - \( y = x - 7 \)
   - \( m = 8; \) contains \((5, 0)\)
   - \( y = 8x - 40 \)
   - \( m = -1; \) contains \((0, 4)\)
   - \( y = x + 7 \)
   - \( m = 4; \) contains \((2, 5)\)
   - \( y = 4x + 3 \)
   - \( m = 4; \) contains \((3, 2)\)
   - \( y = 4x + 2 \)
   - \( m = -1; \) contains \((2, -6)\)
   - \( y = -x - 4 \)

**Exercises**

16. \(-3x + 2y = 6\)
17. \(3y + x = 3\)
18. \(3y - x = 2\)
19. \(-2x + 4y = -3\)
20. \(y + 7 = -2x\)
21. \(2y + 6 = 0\)

**ANSWERS**

1. \( y = 4x + 10 \)
2. \( y = -2x + 15 \)
3. \( y = 8x - 40 \)
4. \( y = x + 7 \)
5. \( y = 4x + 2 \)
6. \( y = x + 7 \)
7. \( y = 4x + 3 \)
8. \( y = x + 7 \)
9. \( y = 4x + 2 \)
10. \( y = x + 7 \)
11. \( y = 4x + 3 \)
12. \( y = x + 7 \)
13. \( y = 4x + 2 \)
14. \( y = x + 7 \)
15. \( y = 4x + 2 \)
16. \(-3x + 2y = 6\)
17. \(3y + x = 3\)
18. \(3y - x = 2\)
19. \(-2x + 4y = -3\)
20. \(y + 7 = -2x\)
21. \(2y + 6 = 0\)
Write in point-slope form an equation of the line through each pair of points.

1. slope $= \frac{2}{3}$; through $(1, 2)$
   
   $y - 2 = \frac{2}{3}(x - 1)$

2. slope $= -1$; through $(2, 0)$
   
   $y - 0 = -1(x - 2)$

3. slope $= 0$; through $(-2, 3)$
   
   $y - 3 = 0(x + 2)$

Write in point-slope form an equation of the line through each pair of points.

4. slope $= \frac{2}{3}$; through $(-3, 5)$
   
   $y - 5 = \frac{2}{3}(x + 3)$

5. slope $= \frac{1}{4}$; through $(4, 3)$
   
   $y - 3 = \frac{1}{4}(x - 4)$

6. slope $= -\frac{1}{3}$; through $(0, -1)$
   
   $y + 1 = -\frac{1}{3}(x - 0)$

Write in point-slope form an equation of the line through each pair of points.

7. slope $= \frac{5}{2}$; through $(2, 9)$
   
   $y - 9 = \frac{5}{2}(x - 2)$

8. slope $= \frac{1}{2}$; through $(3, 5)$
   
   $y - 5 = \frac{1}{2}(x - 3)$

9. slope $= -\frac{5}{3}$; through $(-2, -1)$
   
   $y + 1 = -\frac{5}{3}(x + 2)$

10. slope $= \frac{3}{2}$; through $(4, 2)$
    
    $y - 2 = \frac{3}{2}(x - 4)$

11. slope $= -\frac{2}{3}$; through $(3, 29)$
    
    $y - 29 = -\frac{2}{3}(x - 3)$

Write an equation in point-slope form of the line through each pair of points.

12. slope $= \frac{2}{3}$; through $(0, 5)$
    
    $y - 5 = \frac{2}{3}(x - 0)$

13. slope $= -\frac{3}{2}$; through $(1, 3)$
    
    $y - 3 = -\frac{3}{2}(x - 1)$

14. slope $= \frac{1}{2}$; through $(-1, 4)$
    
    $y - 4 = \frac{1}{2}(x + 1)$

15. slope $= -\frac{5}{4}$; through $(2, 5)$
    
    $y - 5 = -\frac{5}{4}(x - 2)$

16. slope $= \frac{3}{4}$; through $(1, -2)$
    
    $y + 2 = \frac{3}{4}(x - 1)$

17. slope $= 0$; through $(3, 4)$
    
    $y - 4 = 0(x - 3)$

Graph each equation.

18. $y = 2x - 3$

19. $y = -x + 4$

20. $y = x - 2$

21. $y = 3x - 1$

22. $y = -2x + 5$

23. $y = 4x - 2$

24. $y = \frac{1}{2}x + 1$

25. $y = -\frac{1}{3}x - 2$

26. $y = \frac{2}{3}x - 4$

27. $y = -\frac{3}{2}x + 5$

28. $y = \frac{1}{4}x + 3$

29. $y = -\frac{3}{4}x - 2$

30. $y = \frac{2}{5}x - 3$

31. $y = -\frac{3}{5}x + 4$

32. $y = \frac{3}{5}x - 2$

33. $y = -\frac{4}{5}x + 1$

34. $y = \frac{4}{5}x - 3$

35. $y = -\frac{5}{4}x - 1$

36. $y = \frac{5}{4}x + 3$

37. $y = -\frac{5}{3}x + 2$

38. $y = \frac{5}{3}x - 1$

39. $y = -\frac{5}{2}x - 4$

40. $y = \frac{5}{2}x + 3$

41. $y = -\frac{3}{2}x - 5$

42. $y = \frac{3}{2}x + 4$

43. $y = -\frac{4}{3}x + 1$

44. $y = \frac{4}{3}x - 2$

45. $y = -\frac{5}{4}x + 3$

46. $y = \frac{5}{4}x - 2$

47. $y = -\frac{6}{5}x - 1$

48. $y = \frac{6}{5}x + 2$

49. $y = -\frac{7}{6}x + 3$

50. $y = \frac{7}{6}x - 2$

51. $y = -\frac{8}{7}x - 4$

52. $y = \frac{8}{7}x + 3$

53. $y = -\frac{9}{8}x - 5$

54. $y = \frac{9}{8}x + 4$

55. $y = -\frac{10}{9}x - 6$

56. $y = \frac{10}{9}x + 5$

57. $y = -\frac{11}{10}x - 7$

58. $y = \frac{11}{10}x + 6$

59. $y = -\frac{12}{11}x - 8$

60. $y = \frac{12}{11}x + 7$
2-4 Enrichment

More About Linear Equations

Exploring Standard Form of an Equation

You can easily find the slope and y-intercept when you are given an equation in slope-intercept form. If you are given an equation in standard form, you can change the equation to slope-intercept form in order to find the slope and y-intercept. However, you can also find the slope and y-intercept directly from an equation in standard form. The following exercises will help you discover a method to simplify finding the slope and y-intercept.

1. The standard form of an equation of a line is \(Ax + By = C\). Change this equation into slope-intercept form by solving the equation for \(y\).
   \[ y = \frac{-Ax}{B} + \frac{C}{B} \]
   2. Using your answer from Exercise 1, what is the slope of the line \(Ax + By = C\)?
   \[ \frac{-A}{B} \]
   3. Using your answer from Exercise 1, what is the \(y\)-intercept of the line \(Ax + By = C\)?
   \[ \frac{C}{B} \]

4. Use your answers from Exercises 2 and 3 to explain how you can find the slope and \(y\)-intercept for the equation \(3x + 4y = 12\).
   \[ A = 3, B = 4, \text{ and } C = 12 \]
   The slope is \(\frac{-3}{4}\) and the \(y\)-intercept is \(\frac{12}{4}\) or \(3\).

5. For what values of \(A\) and \(B\) will the slope of the line \(Ax + By = C\) be positive?
   \[ A > 0 \text{ and } B < 0 \text{ or } A < 0 \text{ and } B > 0 \]

6. For what values of \(A\) and \(B\) will the slope of the line \(Ax + By = C\) be negative?
   The slope of the line will be negative when either both \(A\) and \(B\) are positive or when both \(A\) and \(B\) are negative.

7. How does the slope change if you multiply both sides of the equation \(Ax + By = C\) by a constant \(k\)?
   The slope does not change.

8. How does the \(y\)-intercept change if you multiply both sides of the equation \(Ax + By = C\) by a constant \(k\)?
   The \(y\)-intercept does not change.
ANSWERS

2-5 ELL Support
Using Linear Models

Use a word or phrase from the list below to complete each sentence. Each word or phrase will be used twice.

correlation coefficient line of best fit scatter plot

1. A __________ relates two sets of data by plotting the data as ordered pairs.
2. The strength of the relationship between data sets is called the __________
3. The __________ is a number that shows the strength of the correlation.
4. Data that has a strong linear relationship has a __________
5. The trend line that gives the most accurate linear model for the data is called the __________
6. The __________ is a number that shows the strength of the correlation.
7. Linear regression is one way to find the __________
8. Use the STAT feature on your graphing calculator to enter data for a __________

Circle the word or phrase in column B that has the same meaning as the word in Column A.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.</td>
<td>correlation</td>
</tr>
<tr>
<td>10.</td>
<td>ordered pairs</td>
</tr>
<tr>
<td>11.</td>
<td>ordered pairs</td>
</tr>
<tr>
<td>12.</td>
<td>ordered pairs</td>
</tr>
</tbody>
</table>

13. Describe the difference between a line of best fit that has a positive correlation and a line that has a negative one. Draw an example of each.

Answers may vary. Sample: A line of best fit that has a positive correlation will have a positive slope. A line of best fit that has a negative correlation will have a negative slope.

2-5 Practice
Using Linear Models

Make a scatter plot and describe the correlation.

1. {(-1, 7), (2, 11), (3, 15), (4, 20), (5, 23)}
2. The tables show the percent of people who voted in presidential election years.

Write the equation of a trend line, if possible.

3. {(-1, 1.2), (1, 3.3), (2, 4.5), (5, 7.2), (9, 13.5)}
4. {(-2, 3.5), (-1, 1.8), (0, 0.1), (1, -0.9), (2, -3.8)}

5. The table shows the number of misdirected bags and the number of late flight arrivals by week, for one airline.

<table>
<thead>
<tr>
<th>Number of Missed Bags</th>
<th>Number of Late Arrivals</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>27</td>
<td>38</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Year</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per lb</td>
<td>5.46</td>
<td>5.41</td>
<td>5.37</td>
<td>5.15</td>
<td>5.13</td>
<td>5.07</td>
<td>5.13</td>
</tr>
</tbody>
</table>

7. Is the correlation between the number of years since 2000 and the number of years since 2000. Is a model invalid if new data does not fit its predictions? Explain.

2-5 Think About a Plan
Using Linear Models

Data Analysis: The table shows population and licensed driver statistics from a recent year.

<table>
<thead>
<tr>
<th>State</th>
<th>Population (millions)</th>
<th>Number of Drivers (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arkansas</td>
<td>6.8</td>
<td>0.7</td>
</tr>
<tr>
<td>Arizona</td>
<td>6.2</td>
<td>0.6</td>
</tr>
<tr>
<td>California</td>
<td>38.6</td>
<td>4.5</td>
</tr>
<tr>
<td>Colorado</td>
<td>5.0</td>
<td>0.6</td>
</tr>
<tr>
<td>Ohio</td>
<td>12.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Texas</td>
<td>23.9</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Know
1. The independent variable should be __________

Need
3. To solve the problem, I need to __________

Plan
4. Make the scatter plot.
5. Draw a trend line.
6. How do you find the equation of the trend line? Write the equation.

7. About how many licensed drivers lived in Michigan that year? About __________

8. What is correlation? Is the correlation between population and licensed drivers strong or weak? Explain.

Correlation is the strength of the relationship, so how close the data points are to the trend line, the correlation is strong; the points fall very close to the trend line.

2-5 Practice
Using Linear Models

8. The table shows the number of countries that participated in the Winter Olympics from 1984 to 2006.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Countries</td>
<td>12</td>
<td>18</td>
<td>22</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>38</td>
</tr>
</tbody>
</table>

9. The table shows the price per box of fresh Florida oranges from 2001 to 2006.

<table>
<thead>
<tr>
<th>Year</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per lb</td>
<td>5.46</td>
<td>5.41</td>
<td>5.37</td>
<td>5.15</td>
<td>5.13</td>
<td>5.07</td>
</tr>
</tbody>
</table>

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4. Solve each exercise and enter your answer in the grid provided.

2-5 Practice (continued)

Using Linear Models

6. A woman is considering buying a car. She researches prices for various years of the same model of car. The table below shows the data from her research.

<table>
<thead>
<tr>
<th>Car Prices by Model Year</th>
<th>Model Year</th>
<th>Price</th>
<th>Model Year</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2008</td>
<td>$12,241.20</td>
<td>2020</td>
<td>$14,876.80</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>$10,980</td>
<td>2010</td>
<td>$13,876.80</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>$9,800</td>
<td>2012</td>
<td>$12,720.80</td>
</tr>
<tr>
<td></td>
<td>2010</td>
<td>$8,720.80</td>
<td>2013</td>
<td>$11,620.80</td>
</tr>
<tr>
<td></td>
<td>2012</td>
<td>$7,620.80</td>
<td>2014</td>
<td>$10,520.80</td>
</tr>
</tbody>
</table>

a. Use a calculator to find the line of best fit. Let x = the number of years since 2000. y = 109.76x + 9895.2
b. Use your linear model to predict how much a 2007 model should cost. $14,876.80

C. Error Analysis. She predicts that a 2005 model costs $9800. Does this seem reasonable? Why or why not?

An error of $4800 could not occur; the model does not fit these data. The linear model is a rough estimate of the actual price of a car in a given year. The model does not give a situation of very low or very high prices.

Consider each situation and predict the type of correlation you might find.

Do you think that changes in the first quantity caused the changes in the second quantity?

7. A person’s weight and the size of clothing they wear strong positive correlation; as a person’s weight increases, so does the size of clothing that person wears. The first quantity causes the change in the second quantity.
8. The number of rooms in a person’s home and the number of books a person owns. These two things do not have any correlation with each other.
9. The length of time a candle has been burning and the brightness of the candle strong negative correlation; the longer a candle burns, the shorter the candle will become. The first quantity causes the change in the second quantity. For each situation, find a linear model and use it to make a prediction.
10. A 3.5-mi cab ride costs $5.25. A 3.5-mi cab ride costs $5.25. How much does a 3.5-mi cab ride cost?

y = 1.75x + 1.75

11. There are 55 blades of grass in 1 in.² of lawn. There are 230 blades of grass in 4 in.² of the same lawn. How many blades of grass are in 3 in.² of lawn?

Answers may vary. Sample: y = 58.75x + 225

12. An empty 5-gal water jug weighs 0.75 lb. With 3 c of water inside, the jug weighs 2.25 lb. Predict the weight of the jug with 5 c of water inside.

y = 0.65x + 0.75, 2.25 lb

When a cake is first removed from the oven, its temperature is 350°F. After 3 hours, its temperature is approximately 72°F, the temperature of the kitchen.

1. Use the information above to write two ordered pairs (x, y), where x represents the time (in hours) since the cake was removed from the oven and y represents the temperature (in degrees Fahrenheit) of the cake at that time. (0, 350); (3, 72)
2. Find the slope of the line through the two points identified in Exercise 1. 228°F
3. Write in slope-intercept form the equation of the line through the two points in Exercise 1. y = 228x + 350
4. Use the equation from Exercise 3 to estimate the temperature of the cake after 1 hour, after 2 hours, and after 4 hours. 250°F; 167°F; 13°F
5. Suppose that the actual temperature of the cake is about 110°F after 1 hour, about 81°F after 2 hours, and about 75°F after 4 hours. Compare these temperatures to your answer from Exercise 4. Does the equation from Exercise 3 model the temperature of the cake accurately?

No.

What happened? Linear equations are not appropriate for modeling every situation. The linear model assumes that the temperature of the cake decreases by the same number of degrees during each hour. Notice how the model fails after 4 hours, when the temperature of the cake is below room temperature, an impossibility.

The problem is that the temperature of the cake does not decrease by a constant number of degrees each hour. Its temperature decreases by a percent of the difference between its temperature and room temperature during each hour.

6. What is the difference between the temperature of the cake and room temperature when the cake is removed from the oven? 279°F
7. Find 85% of the difference you found in Exercise 6. 233.7°F
8. Subtract your answer from Exercise 7 from 350°F. How does this number compare to the temperature after 1 hour given in Exercise 5? 116.25°F; very close
2-5 Reteaching
Using Linear Models

The strength of the relationship between data sets is called a correlation.

Positive Correlation: Data points fall close to a line with a positive slope.

Negative Correlation: Data points fall close to a line with a negative slope.

No Correlation: Data points are scattered and do not fall close to any line.

Problem
The table shows nutritional data for different types of pizza. Make a scatter plot and describe the correlation.

Exercises
Make a scatter plot and describe the correlation.

<table>
<thead>
<tr>
<th>Type</th>
<th>Calories per Slice</th>
<th>Calories From Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheese</td>
<td>298</td>
<td>112</td>
</tr>
<tr>
<td>Pepperoni</td>
<td>390</td>
<td>148</td>
</tr>
<tr>
<td>Veggie</td>
<td>412</td>
<td>179</td>
</tr>
<tr>
<td>Mexican</td>
<td>274</td>
<td>137</td>
</tr>
<tr>
<td>Chicken</td>
<td>277</td>
<td>137</td>
</tr>
<tr>
<td>Works</td>
<td>94</td>
<td>45</td>
</tr>
<tr>
<td>BBQ</td>
<td>103</td>
<td>36</td>
</tr>
</tbody>
</table>

Correlation:
- Positive correlation
- Negative correlation
- No correlation

Plot the points. Use the calories per slice for the independent variable. The data points fall closely along a line with a positive slope. The data sets have a strong positive correlation.

Problem
Reteaching (continued)
Using Linear Models

A trend line is a mathematical model that shows the relationship between two sets of data. A trend line can be used to make predictions.

Problem
Use the data in the table to draw a scatter plot and a trend line. If the trend continues, about how many billion dollars will the U.S. spend on health expenditures in 2015?

Exercises
Use the data in the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Expenditure (billions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>180</td>
</tr>
<tr>
<td>2002</td>
<td>193</td>
</tr>
<tr>
<td>2004</td>
<td>182</td>
</tr>
<tr>
<td>2006</td>
<td>219</td>
</tr>
</tbody>
</table>

The model predicts expenditures of approximately $3200 billion in 2015.

Exercises
Use the data in the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population Enrolled in Medicare</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>180</td>
</tr>
<tr>
<td>2002</td>
<td>193</td>
</tr>
<tr>
<td>2004</td>
<td>182</td>
</tr>
</tbody>
</table>

3. Make a scatter plot and describe the correlation.

4. Draw a trend line and write its equation.

5. Use your model to predict the number of Medicare enrollees in 2015.

Answers may vary. Sample: Using (1, 180) and (4, 219) y = 1.4x + 160 $1.4 \times 9 = 12.6$ billion

2-6 ELL Support
Families of Functions

Use the list below to complete the web diagram.

reflection
vertical translation
vertical compression

- $f(x) \text{ where } a > 1$
- $f(x - h) - f(x)$
- $f(x) \text{ where } 0 < a < 1$

3. What quantity is represented by the independent axis in your graph?

4. What quantity is represented by the dependent axis in your graph?

Understanding the Problem
1. What is the problem asking you to determine?

- The transformation needed to change a graph of the yo-yo routine to represent the graph of a video of the routine that started 10 s earlier.

2. How is this problem related to problems about parent functions?

- This problem is like any problem that changes a parent function to another function through transformations.

Planning the Solution
3. What quantity is represented by the independent axis in your graph?

4. What quantity is represented by the dependent axis in your graph?

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2. How is this problem related to problems about parent functions?

- This problem is like any problem that changes a parent function to another function through transformations.

Planning the Solution
3. What quantity is represented by the independent axis in your graph?

4. What quantity is represented by the dependent axis in your graph?
How is each function related to its parent function? Graph the function by translating the parent function.

1. \( y = x + 2 \) translated up 2 units

2. \( y = x - 2 \) translated down 1 unit

3. \( y = (x - 5)^2 \) translated 5 units right.

4. \( y = (x + 5)^2 \) translated 5 units left.

5. \( y = x + 2.5 \) translated up 2.5 units.

6. \( y = (x - 2.5)^2 \) translated 2.5 units right.

7. \( y = (x - 2) + 1 \), reflected in the x-axis.

8. Make a table of values for \( f(x) \) after each given translation.

9. \( y = (x - 3)^2 \) translated 3 units right.

10. \( y = (x + 3)^2 \) translated 3 units left.

Write an equation for each transformation of \( f(x) \) to \( g(x) \).

11. \( f(x) = x + 3 \) translated up 3 units.

12. \( f(x) = x - 2 \) translated down 2 units.

13. \( f(x) = x + 1 \) reflected in the x-axis.

14. \( f(x) = x - 3 \) reflected in the y-axis.

15. \( f(x) = x + 1 \), reflected in the x-axis.

16. \( f(x) = x - 3 \), reflected in the y-axis.

Graph each pair of functions on the same coordinate plane. Describe a transformation that changes \( f(x) \) to \( g(x) \).

17. \( f(x) = x + 3 \), \( g(x) = x - 2 \), \( g(x) \) is \( f(x) \) translated down 5 units.

18. \( f(x) = x - 3 \), \( g(x) = x + 1 \), \( g(x) \) is \( f(x) \) reflected in the y-axis.

Write an equation for each transformation of \( f(x) \) to \( g(x) \).

19. \( f(x) = x^2 \), \( g(x) = x^2 - 2 \), \( g(x) \) is \( f(x) \) translated down 2 units.

20. \( f(x) = x^2 \), \( g(x) = x^2 + 2 \), \( g(x) \) is \( f(x) \) reflected in the x-axis.

21. \( f(x) = x^2 \), \( g(x) = x^2 + 3 \), \( g(x) \) is \( f(x) \) reflected in the y-axis.

22. \( f(x) = x^2 \), \( g(x) = x^2 - 3 \), \( g(x) \) is \( f(x) \) reflected in the x-axis and a translation of 3 units up.

23. \( f(x) = x^2 \), \( g(x) = x^2 - 3 \), \( g(x) \) is \( f(x) \) reflected in the y-axis and a translation of 3 units up.
Exercises

3. Horizontal and Vertical Translations

Which of the following is the graph of \( f(x) \) that shifts the graph of \( g(x) \) right 2 units and up 1 unit?

a) \( y = f(x - 2) \) 

b) \( y = f(x + 2) \) 

c) \( y = f(x) + 1 \) 

4. Multiple Choice

Which equation represents the horizontal translation of \( f(x) \) 3 units to the left?

a) \( y = f(x + 3) \) 

b) \( y = f(x - 3) \) 

5. Multiple Choice

Which of the following is the graph of \( f(x) \) that vertically stretches the graph of \( g(x) \) by a factor of 3 because \( a > 0 \)?

a) \( y = 3f(x) \) 

b) \( y = \frac{f(x)}{3} \) 

6. Reteaching

Families of Functions

Horizontal and Vertical Translations

If \( b \) and \( c \) are positive numbers, then

- \( g(x) = f(x - b) \) shifts the graph of \( f(x) \) right \( b \) units.
- \( g(x) = f(x + b) \) shifts the graph of \( f(x) \) left \( b \) units.
- \( g(x) = f(x) - c \) shifts the graph of \( f(x) \) down \( c \) units.
- \( g(x) = f(x) + c \) shifts the graph of \( f(x) \) up \( c \) units.

Problem

Here are some translations of \( f(x) = |x| \). Draw each graph on the grid provided. Identify the translation in terms of \( h \) and \( k \). Explain why 0 is not the answer for any equation.

\[ f(x) = |x| \] 
\[ g(x) = f(x - 2) \] 
\[ h(x) = f(x) + 1 \] 

Exercises

Identify the type of translation of \( f(x) = |x| \).

1. \( g(x) = |x - 2| \) 2. \( g(x) = |x + 1| \) 3. \( g(x) = |x - 3| \) 4. \( g(x) = |x + 3| \)

5. Graph each translation of \( f(x) = |x| \).

a) \( g(x) = f(x - 2) \) b) \( h(x) = f(x) + 3 \) c) \( k(x) = f(x) - 4 \) d) \( m(x) = f(x) + 1 \) 

ANSWERS

page 57

2-6 Standardized Test Prep

Families of Functions

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Which of the following is the graph of \( f(x) = -2x + 6 \) reflected in the \( y \)-axis and vertically compressed by a factor of \( \frac{1}{2} \)?

a) \( y = -2x + 3 \) 

b) \( y = -2x - 3 \) 

2. The graph of \( y = x + 4 \) is translated 3 units down. Which point is on the new graph?

a) (0, 4) 

b) (0, 1) 

3. The graph of \( y = f(x) \) is reflected in the \( x \)-axis and translated 3 units right. Which is the equation of the new graph?

a) \( y = -f(x - 3) \) 

b) \( y = f(-x + 3) \) 

4. Which equation represents the vertical translation of \( y = f(x) \) up 5 units?

a) \( y = -f(x) \) 

b) \( y = f(x + 5) \) 

5. Which equation represents the horizontal translation of \( y = f(x) \) to the left 3 units?

a) \( y = f(x - 3) \) 

b) \( y = f(x + 3) \) 

Short Response

6. How will a vertical compression of the parent function \( y = x \) change the graph of the function? Write a new equation that represents this transformation.

2-6 Enrichment

Families of Functions

Transformations of a Graph

You can identify translations, reflections, vertical stretches, and compressions from a given algebraic equation. You can apply transformations to a graph even when it is not easy to write an equation for the graph.

The graph at the right represents the function \( f(x) \). Describe what each change to the equation will have on the graph of \( f(x) \).

1. \( y = 3f(x) \) vertical stretch by a factor of 3

2. \( y = f(x - 1) \) translated 1 unit down

3. \( y = f(x + 2) \) translated 4 units left

Draw new graphs by applying each transformation. Apply the transformation to the endpoints and corners first, and then connect the new points to form the new graph.

4. \( y = 2f(x) \) 

5. \( y = f(x - 1) \) 

6. \( y = f(x + 2) \) 

Note: Make a new graph when all combinations of the transformations are applied together.

Graph \( f(x) \) and \( g(x) \) on the same coordinate plane.

7. \( f(x) = x^2 \) 

8. \( g(x) = (x - 2)^2 \) 

9. \( f(x) = \sqrt{x} \) 

10. \( g(x) = \sqrt{x} \)
2-7 Ell Support

Absolute Value Functions and Graphs

Circle a vertex in each figure below:
1.  
2.  
3.  

A line of symmetry is a line that divides a graph into two halves that are mirror images. Geometric figures also have lines of symmetry if the line divides the figure into two halves that are mirror images. An axis of symmetry is an axis that divides a graph into two halves that are mirror images.

4. Draw the line of symmetry in each figure below:
4.  
5.  
6.  

For Exercises 7–11, match each word in Column A with a word that has the same meaning in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. translation</td>
<td>A. change</td>
</tr>
<tr>
<td>8. compression</td>
<td>B. change</td>
</tr>
<tr>
<td>9. reflection</td>
<td>C. change</td>
</tr>
<tr>
<td>10. vertex</td>
<td>D. change</td>
</tr>
<tr>
<td>11. A unit</td>
<td>E. change</td>
</tr>
</tbody>
</table>

Graph each equation.
1. \( y = |x| - 2 \)
2. \( y = |x| + 3 \)
3. \( y = |x| - 5 \)
4. \( y = |x| - 4 \)
5. \( y = |x| - 3| + 1 \)
6. \( y = |x + 1| - 4 \)
7. \( y = |2x| \)
8. \( y = \frac{1}{2}|x| \)
9. \( y = -3|x| \)

Without graphing, identify the vertex, axis of symmetry, and transformations from the parent function \( f(x) = |x| \).

10. \( y = |x - 4| \) (4, 0); \( a = 4 \); translation of the parent function 4 units to the right
11. \( y = -|x + 2| \) (0, -2); \( a = 0 \); vertical stretch of the parent function by a factor of 3, reflected in the \( x \)-axis, and translated 2 units down
12. \( y = -3|x| + 4 \) (0, 4); \( a = 0 \); vertical stretch of the parent function by a factor of 3, reflected in the \( x \)-axis, and translated 4 units up
13. \( y = -5|x - 1| + (1, 0); \( a = 1 \); translated 1 unit right, reflected in the \( x \)-axis, and translated 5 units up

2-7 Think About a Plan

Absolute Value Functions and Graphs

Graph \( y = 4|x - 3| + 1 \). List the vertex and the \( x \) - and \( y \) -intercepts, if any.

Understanding the Problem
1. What is the problem asking you to determine?
2. What is the parent function for the function \( y = 4|x - 3| + 1 \)?

Planning the Solution
3. What do you know about the function \( y = 4|x - 3| + 1 \)?

The graph requires multiple transformations of the parent function.

4. Graph the parent function.

5. What transformations do you need to apply to the parent function to graph this function?

Translate right 3 units; stretch vertically by a factor of 4; translate up 1 unit

Getting an Answer
6. What is the vertex of the function? \((3, 1)\)
7. What are the \( x \) - and \( y \) -intercepts of the function?
8. Graph the function.

2-7 Practice Form G

Absolute Value Functions and Graphs

Graph each equation.
14. \( y = |x - 4| + 5 \) List the vertex and the \( x \) - and \( y \) -intercepts, if any.
vertex: \((4, 5); x\)-intercepts: \((-1, 0); (5, 0); y\)-intercepts: \((0, 1)\)

Graph each absolute value equation.
15. \( y = |3 - x| \)
16. \( y = 3|x + 1| \)
17. \( y = |3 - x| \)
18. \( y = -|x| + 2 \)
19. \( y = |3x - 1| - 2 \)
20. \( y = \frac{1}{2}|x + 1| \)
21. \( y = -\frac{1}{2}|x - 2| \)
22. \( y = |x + 1| - 3 \)
23. \( y = -\frac{1}{2}|x - 4| \)
24. a. Graph the equations \( y = 2|x + 4| - 1 \) and \( y = \frac{1}{2}|x - 4| + 1 \) on the same set of axes.
b. Writing Describe the similarities and differences in the graphs. The first equation is a stretch of \( y = |x| \) by a factor of 3 and the second equation is a compression of \( y = |x| \) by a factor of 2. The first equation is also translated left 4 units and down 1 unit. The second equation is also translated right 4 units and up 1 unit.
2-7 Practice

**Form K**

### Absolute Value Functions and Graphs

Make a table of values for each equation. Then graph the equation.

1. \( y = |x| + 4 \)

2. \( y = |x| - 2 \)

3. \( y = |x + 2| \)

4. \( y = |x + 1| - 3 \)

Graph each function. Then describe the transformation from the parent function \( f(x) = |x| \).

5. \( y = \frac{1}{3}|x| \)

6. \( y = -\frac{1}{2}|x| \)

7. \( y = 3|x| \)

Without graphing, identify the vertex, axis of symmetry, and transformations from the parent function \( f(x) = |x| \).

8. \( y = 2|x| + 1 \), vertex \( (1, 0) \), axis of symmetry \( x = -1 \), stretched by a factor of 2 and translated left 1 unit

9. \( y = |x + 2| + 3 \), vertex \( (2, 2) \), axis of symmetry \( x = -2 \), translated right 2 units and up 3 units

10. \( y = \frac{1}{2}|x| - 2 \), vertex \( (0, -2) \), axis of symmetry \( x = 0 \), reflected in the \( y \)-axis and compressed by a factor of 2 and translated down 2 units

---

2-7 Enrichment

### Absolute Value Functions and Graphs

#### Translations of Graphs

You have seen that the graph of \( y = |x + 2| - 1 \) is the graph of the parent function \( y = |x| \) shifted 2 units left and 1 unit down as shown in the graph to the right.

Now consider how the graph of \( y = |x + 2| - 1 \) is related to the graph of \( y = |y| \).

1. Complete the table of values for \( x = |y| \), and then graph the equation by plotting the ordered pairs from the table.

2. Complete the table of values for \( x = |x + 2| - 1 \), and then graph the equation on the same set of axes by plotting the ordered pairs from the table.

3. Complete the following sentence describing the relationship between the two graphs.

   The graph of \( x = |y + 2| - 1 \) is the graph of \( x = |y| \) shifted 2 units down and 1 unit left.

4. Complete the following statements, assuming that both \( b \) and \( k \) are positive real numbers.

   a. The graph of \( x = |y + k| \) is the graph of \( x = |y| \) shifted \( k \) units up.

   b. The graph of \( x = |y + k| \) is the graph of \( x = |y| \) shifted \( k \) units left.

   c. The graph of \( x = |y + k| \) is the graph of \( x = |y| \) shifted \( k \) units right.

   d. The graph of \( x = |y + k| \) is the graph of \( x = |y| \) shifted \( k \) units down.

Graph each of the following equations using translations of the graph of \( y = |y| \).

5. \( x = |y - 3| \)

6. \( x = |y + 2| \)

7. \( x = |y + 4| - 2 \)
2.7 Reteaching
Absolute Value Functions and Graphs

A function of the form \( y = a|x - h| + k \) is an absolute value function. The graph of \( y = a|x - h| + k \) is an angle; its vertex is located at the point \((h, k)\).

**Problem**

What is the graph of \( y = 2|x + 3| - 17 \)?

This function is in the general form \( y = a|x - h| + k \) where \( a = 2, h = -3, \) and \( k = -17 \). The vertex is \((-3, -17)\).

Now make a table showing several points on the graph. Choose values of \( x \) on both sides of the vertex.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>9</td>
<td>5</td>
<td>1</td>
<td>-1</td>
<td>-5</td>
<td>-9</td>
<td>-13</td>
</tr>
</tbody>
</table>

Plot the vertex and the points from the table.

Connect the points to graph the function.

**Exercises**

Make a table of values for each equation. Then graph the equation.

1. \( y = |3x| \)
   - Table of values may vary.

2. \( y = |3x - 1| \)
   - Table of values may vary.

3. \( y = \frac{1}{2}x - 1 \)
   - Table of values may vary.

4. \( y = |x + 1| + 3 \)
   - Table of values may vary.

5. \( y = 2|x - 1| - 5 \)
   - Table of values may vary.

6. \( y = \frac{1}{2}|x - 1| + 4 \)
   - Table of values may vary.

7. \( y = |x - 1| + 2 \)
   - (1, 2), \( a = 1 \), translated 1 unit right and 2 units up

8. \( y = |x + 1| - 3 \)
   - (0, 1), \( a = 0 \), translated 1 unit left and 3 units down

9. \( y = |2x - 1| - 3 \)
   - (0, 1), \( a = 0 \), translated 2 units right, vertically stretched by a factor of 2

10. \( y = -\frac{1}{2}|x - 1| \)
    - (0, 0), \( a = 0 \), reflected in the \( x \)-axis, vertically compressed by a factor of \( \frac{1}{2} \)

2.8 Think About a Plan
Two-Variable Inequalities

The graph at the right relates the amount of gas in the tank of your car to the distance you can drive.

a. Describe the domain for this situation.

b. Why does the graph stop?

c. Why is only the first quadrant shown?

d. Reasoning Would every point in the solution region be a solution?

5. What is the domain of a function? all possible values of the independent variable

6. What is the domain for this situation?

7. What is the upper bound of the domain in this situation?

8. Why does the graph stop?

9. What do you know about \( x \) and \( y \)-values of points in the first quadrant?

10. What does the coefficient of \( x \) represent?

11. What is the coefficient of \( x \) in the graph?

12. What is the coefficient of \( y \) in the graph?
Graph each absolute value inequality. To start, graph the boundary line.

**Graph each absolute value inequality.**

1. $y < x + 1$
2. $y > x - 1$
3. $y > x - 2$
4. $y < x - 2$
5. $y = x - 1$
6. $y = x - 2$
7. $y = x - 3$
8. $y = x - 4$

9. You have a $25 calling card. Calls made using the card within the United States cost $0.10 per minute while calls made from the US to France cost $0.25 per minute.
   a. Write an inequality that relates the number of minutes you can use for calls within the U.S. and the number of minutes you can use for calls from the U.S. to France. $0.10x + 0.25y < 25$
   b. Graph the inequality.

10. $y > |x|$
11. $y > |x + 1|$
12. $y < |x - 2|$
13. $y < |x + 2|$

14. $y = 2x - 4$
15. $y = 2x + 4$
16. $y = x - 3$

Write an inequality for each graph. The equation for the boundary line is given.

**Graph each inequality on a coordinate plane.**

17. $4x + 2y < 8$
18. $3x = 5y$
19. $y > \frac{x}{2} - 1$
20. $y > \frac{x}{2} - 3$

Write an inequality for each graph.

21. $y = \frac{x}{2} - 2$
22. $y = \frac{x}{2} - 1$
23. $y = \frac{x}{2} + 1$
24. $y < |x - 4| + 3$
25. $y > |x - 1| - 4$
26. $y > |x - 3| + 4$

27. Open-Ended Write an inequality that includes $\{0, 5, -10, 10\}, \{40, -20\}$, and $\{-20, 15\}$ in its solutions. Answers may vary. Example: $y > \frac{x}{2} - 9$

28. A salesperson sells two models of vacuum cleaners. One brand sells for $150 each and the other sells for $200 each. The salesperson has a weekly sales goal of at least $1,000.
   a. Write an inequality relating the revenue from the vacuum cleaners to the sales goal.
   b. Graph the inequality.
   c. If the salesperson sold exactly six $200 models last week, how many $150 models did she have to sell to make her sales goal?
   
   Open-Ended Write an absolute value inequality for which the boundary is dashed and the shaded region is above the boundary.
   Answers may vary. Sample: any inequality of the form $y > |x - h| + k$ where $a > 0$
For Exercises 1–4, choose the correct letter.

1. Which graph best represents the solution of the inequality \(-ax - 2y \leq 4\)?
   - A. \(y \geq 2x\)
   - B. \(y < 2x\)
   - C. \(y > 2x\)
   - D. \(y = 2x\)

2. Which ordered pair is a solution of \(2x - 2y > 8?\)
   - A. \((-2, 0)\)
   - B. \((2, -4)\)
   - C. \((0, -4)\)
   - D. \((4, 1)\)

3. Which ordered pair is not a solution of \(y \geq \frac{1}{2}x + 2?\)
   - A. \((-2, 0)\)
   - B. \((-1, -2)\)
   - C. \((0, 2)\)
   - D. \((2, 2)\)

4. The graph of which absolute value inequality has its vertex at \((1, 5)\)?
   - A. \(y > |x - 5|\)
   - B. \(y > -|x - 5|\)
   - C. \(y > |x - 1|\)
   - D. \(y > -|x - 1|\)

Short Response

5. Transportation
   The high school band is expecting to take at least 120 students to a regional band competition. The school rents some passenger vans that can transport 8 students. Other students, in groups of 4, still need to ride in personal vehicles driven by parents.
   a. Write an inequality that shows the total number of students that can ride in vans and personal vehicles.
   b. Explain in words or show work for how you determined the inequality.

(2) a. \(8x + 4y \leq 120\), where \(x\) represents the number of vans and \(y\) represents the number of personal vehicles.
   b. Because each van holds 8 students, the total number of students riding in vans is \(8x\). Because each car holds 4 students, the total number of students riding in personal vehicles is \(4y\). The total number of student passengers must be at least 120, so the sum of the number of students riding in vans and personal vehicles is \(8x + 4y\).

1 Incorrect inequality or incorrect explanation
0 Incorrect answers and no work shown OR no answers given

2-8 Reteaching

A linear inequality in two variables is an inequality whose graph is a region of the coordinate plane bounded by a line. This line is the boundary. If the boundary is included in the solution of the inequality, it is drawn as a solid line. If the boundary is not part of the solution of the inequality, it is drawn as a dashed line.

Problem
What is the graph of \(6x - 2y \leq 12?\)

\[\begin{align*}
6x - 2y &\leq 12 \\
3x - y &\leq 6
\end{align*}\]

To graph the boundary line, write the inequality in slope-intercept form as if it were an equation.

The boundary line is solid if the inequality contains \(\leq\) or \(\geq\). The boundary line is dashed if the inequality contains \(<\) or \(>\). Graph the boundary line \(y = 3x - 6\) as a solid line.

Graph the boundary line \(y = -\frac{1}{2}x + 3\).

Since the boundary line does not contain the origin, substitute the point \((0, 0)\) into the inequality.

\[\begin{align*}
0 &\geq 6 \quad \text{true} \\
0 &\geq 0 \quad \text{true}
\end{align*}\]

Shade the region that contains the origin. If the resulting inequality were false, then you would shade the region that does not contain the origin.

Exercises
Graph each inequality.

1. \(y \geq 2x\)
2. \(x + y < 4\)
3. \(y < x + 1\)
4. \(3x - 2y \geq x + y\)
5. \(x = 4\)
6. \(y \geq 5\)
7. \(y < \frac{1}{2}x\)
8. \(y = -\frac{3x}{2}\)
9. \(y > 2x - 4\)
10. \(y \leq \frac{x}{2} - 4\)
11. \(y < \frac{2x}{3} - 6\)
12. \(y < \frac{2x}{3} + 6\)

2-8 Enrichment

Consider the inequality \(|x + y| \leq 2\). This is an absolute inequality in two variables. In this case, both variables are inside the absolute value symbols. The following exercises will guide you through the steps of graphing this inequality.

1. Rewrite the inequality \(|x + y| \leq 2\) as an absolute value equation. \(x + y = 2\)
2. Rewrite the answer from Exercise 1 as two equations without absolute value symbols. \(x + y = 2\) or \(x + y = -2\)
3. Graph the two equations from Exercise 2 on the same coordinate plane bounded by a line. This line is the boundary. If the boundary is included in the solution of the inequality, it is drawn as a solid line. If the boundary is not part of the solution of the inequality, it is drawn as a dashed line.

To graph two variable absolute value inequalities, graph the boundary line. Then pick a test point and shade appropriately.

Problem
What is the graph of \(-y < |x + 2|?\)

\[\begin{align*}
-y &< |x + 2| \\
y &> -|x + 2| + 3
\end{align*}\]

To graph the boundary line, write the inequality in terms of \(y\) as if it were an equation. The boundary line is dashed because the inequality contains \(>\).

Graph the boundary line \(y = -|x + 2| + 3\).

Now pick a test point. Because the boundary line does not contain the origin, substitute the point \((0, 0)\) into the inequality.

\[\begin{align*}
0 &< 3 \\
0 &> -\sqrt{2}
\end{align*}\]

Shade the region that does not contain the origin. If the resulting inequality were true, then you would shade the region that does contain the origin.

Exercises
Graph each absolute value inequality.

1. \(y > (\frac{1}{2})x\)
2. \(y < \frac{1}{2}x\)
3. \(y < \frac{1}{2}x\)
4. \(y = \frac{1}{2}x\)
5. \(y > \frac{1}{2}x\)
6. \(y < \frac{1}{2}x\)
7. \(y < \frac{1}{2}x\)
8. \(y = \frac{1}{2}x\)
9. \(y > \frac{1}{2}x\)
10. \(y < \frac{1}{2}x\)
11. \(y > \frac{1}{2}x\)
12. \(y < \frac{1}{2}x\)
### Chapter 2 Quiz 1

#### Lessons 2-1 through 2-4

**Do you know HOW?**

Determine whether the relation is a function. Then list the domain and range.

1. \([-3, \{1, 3\}], \{(2, 4), (0, 1), \{(1, 2)\}\}
   - Function: true; Domain: \([-3, 1, 2]\); Range: \([-3, 1, 2]\)

2. \([-2, \{(2, 1), (2, 4), (1, 3)\}]\)
   - Not a function; Domain: \([-2, 1, 3]\); Range: \([-2, 1, 3]\)

Write an equation in slope-intercept form for the line passing through each pair of points.

3. \((-3, 3), (-2, -5)\)
   - \(y = \frac{1}{2}x + 1\)

4. \((0, 0), (5, 5)\)
   - \(y = x\)

Find the x- and y-intercepts of each line and write the equation in standard form.

5. \(y = \frac{x}{2} + 1\)
   - \(x - 2y = 2\)

6. \(y = -\frac{x}{2} + 6\)
   - \(2x + 2y = 12\)

7. Using standard form, write the equation of the line through \((-2, 5)\) and parallel to \(y = -2x + 5\).
   - \(y = -2x + 5\)

8. Using point-slope form, write the equation of the line through \((1, -5)\) and perpendicular to \(-3x + 2y = 12\).
   - \(y = \frac{3}{5}x - 1\)

**Do you UNDERSTAND?**

9. **Writing**
   - Describe a situation that could be represented by the direct variation \(y = 12x\).
   - Answers may vary. Sample: Kelly’s earnings of $12/h vary directly with the number of hours she works per week.

10. **Open-ended**
    - Write an equation for a line with zero slope and an equation for a line with an undefined slope. How do the lines differ?
    - Answers may vary. Sample: \(y = \frac{x}{2} + 1\); the graph of a line with zero slope is a horizontal line, whereas the graph of a line with undefined slope is a vertical line.

11. **Reasoning**
    - Is the slope of any linear function undefined? How do you know?
    - No; only vertical lines have undefined slope, but a vertical line does not represent a function because it is a relation that assigns more than one output to each input.

12. **Error Analysis**
    - A student says that if \(y = -5\), then \(y = -4\) when \(x = -2\). What is his error?
    - Answers may vary. Sample: If \(y\) varies directly with \(x\) and \(y = 9\) when \(x = 2\), then \(y = 9\) when \(x = -2\), not \(y = -4\). What is his error? Answers may vary. Sample: If \(y\) varies directly with \(x\) and \(y = 9\) when \(x = 2\), then \(y = 9\) when \(x = -2\), not \(y = -4\).

### Chapter 2 Quiz 2

#### Lessons 2-5 through 2-8

**Do you know HOW?**

1. Make a scatter plot of the data in the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Enrollment</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>1200</td>
</tr>
<tr>
<td>2004</td>
<td>1250</td>
</tr>
<tr>
<td>2005</td>
<td>1300</td>
</tr>
<tr>
<td>2006</td>
<td>1350</td>
</tr>
<tr>
<td>2007</td>
<td>1400</td>
</tr>
<tr>
<td>2008</td>
<td>1450</td>
</tr>
</tbody>
</table>

Write the equation of a trend line to model the relationship. Predict the value of \(x\) when \(y = 9\).

- 2008
- 2009
- 2010

2. Describe the transformations of \(f(x) = \sqrt{x}\) that produce \(g(x) = \sqrt{x} + 2 - 1\).

3. Describe the transformations of \(f(x) = \sqrt{x} - 3 + 2\) that produce \(g(x) = \sqrt{x} - 3 + 2\) translated up 5 units.

4. Describe the transformations of \(f(x) = \sqrt{x} - 3 + 2\) that produce \(g(x) = \sqrt{x} - 3 + 2\) reflected in the \(x\)-axis and translated down 3 units.

**Graph each inequality.**

- \(y < x + 4\)
- \(y < |x - 1| + 4\)
- \(y < 2x + 3\)
- \(y < 2x - 1\)

**Do you UNDERSTAND?**

8. **Writing**
   - Suppose you have the graph of a function \(y = f(x)\). Describe how the sign of \(k\) and the sign of \(b\) affect the graph of \(y = f(x) - k\).

13. **Open-ended**
    - Use the symbol < or > to write a linear inequality in two variables that has \((-3, -3)\) and \((3, 1)\) as solutions. Answers may vary. Sample: \(y < -x + 7\).

8. **Answers may vary.** Sample: The absolute value of \(k\) tells you how many horizontal units to translate the graph to the right or left, while the sign of \(b\) tells you how many vertical units to translate the graph. If \(k\) is positive, translate the graph down; if \(k\) is negative, translate the graph up.

### Chapter 2 Chapter Test

**Do you know HOW?**

Find the domain and range of each relation, and determine whether it is a function.

1. \[\{(2, 4), (3, 3), (4, 1)\}\]
   - Not a function; Domain: \([2, 4, 3, 4]\); Range: \([2, 4, 3, 1]\)

2. \[\{(x, y) | x = 0\}\]
   - Not a function; Domain: \([0, 0]\); Range: \([0, 0]\)

Find the constant of variation for each direct variation.

3. \(y = 2x\)
   - \(k = 2\)

4. \(y = \frac{x}{2}\)
   - \(k = \frac{1}{2}\)

Find the slope of each line.

5. \[\{(1, 2), (2, 4)\}\]
   - \(m = 1\)

6. \[\{(3, 6), (4, 8)\}\]
   - \(m = 2\)

Find the slope of each line.

7. \[\{(2, -1), (4, 1)\}\]
   - \(m = \frac{3}{2}\)

8. \[\{(3, 2), (6, 5)\}\]
   - \(m = \frac{3}{2}\)

Using standard form, write the equation of the line with the given slope through the given point.

9. Slope = \(\frac{1}{2}\), \((-2, 3)\)
   - \(y = \frac{1}{2}x + 1\)

10. Slope = \(-\frac{1}{2}\), \((2, 1)\)
    - \(y = -\frac{1}{2}x + 2\)

Using slope-intercept form, write the equation of the line through each pair of points.

11. \((-3, 2), (0, 4)\)
    - \(y = \frac{1}{2}x + 3\)

12. \((-2, 1), (2, -3)\)
    - \(y = \frac{1}{2}x - 2\)

**Do you UNDERSTAND?**

24. **Reasoning**
    - Represent the data in a mapping diagram.
    - \((1, 2), (2, 4), (3, 4), (4, 3), (5, 2), (6, 1)\)

25. **Open-ended**
    - Write an absolute value function \(g(x)\) that represents a multiple transformation of the parent function \(f(x) = |x|\). Then graph \(f(x)\) and \(g(x)\) on the same coordinate grid.

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**Answers**

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Determine whether each relation is a function.

10. Do you know HOW?

Describe each transformation of the parent function \( y = |x| \).

10. \( y = |x| + 2 \) right 1 unit, up 2 units

11. \( y = -4|x| \) reflected in the axis, stretched by a factor of 4, down 1 unit

12. \( y = \frac{1}{2} x + 7 \) left 7 units, compressed by a factor of \( \frac{1}{2} \)

Graph each equation.

11. \( y = x + 6 \)

12. \( y = \frac{3}{2} x + 3 \); \( m = \frac{3}{2} \)

13. \( y = 2x \)

Write an equation of the line in point-slope form through each pair of points.

5. \( (2, -3), (6, 2) \)

6. \( (4, -2), (0, 3) \)

7. \( (-2, 1), (0, -3) \)

8. \( (-1, 5), (2, 1) \)

9. \( (3, -4), (3, 2) \)

10. \( (1, 2), (6, 7) \)

Do you know HOW?

Find the domain and range of each function. Graph each relation.

1. \( \{(1, 3), (2, 5), (3, 7), (2, 9)\} \)

2. \( \{(2, 6), (3, 4), (4, 3), (6, 4)\} \)

Domain: \( \{-1, 2, 3, 4\} \)
Range: \( \{4, 5, 6\} \)

3. \( \{(0, 2), (3, 5), (5, 5), (4, 7)\} \)

4. \( \{(0, -1), (-3, 2), (0, 4), (2, -8)\} \)

Do you know HOW?

Find the \( x \)- and \( y \)-intercepts of each line.

5. \( 5y - x = 10 \)

\( x \)-intercept: \( (5, 0) \)
\( y \)-intercept: \( (0, 2) \)

6. \( 3x + 4 = y \)

\( x \)-intercept: \( (-\frac{4}{3}, 0) \)
\( y \)-intercept: \( (0, 4) \)

7. \( 2y + 8x = 14 \)

\( x \)-intercept: \( (-2, 0) \)
\( y \)-intercept: \( (0, -7) \)

Do you know HOW?

Write an equation of the line in standard form with the given slope through the given point.

8. \( m = -3, \) \( (2, -1) \)

9. \( m = 1, \) \( (4, -2) \)

10. \( m = -1, \) \( (3, 5) \)

Graph each equation.

11. \( y = 2x - 2 \)

12. \( y = 3x + 2 \)

13. \( y = \frac{1}{2} x + 3 \)

Do you know HOW?

Write an equation for each translation of \( y = |x| \).

14. 4 units down, 1 unit left

15. 2 units down, 6 units right

Do you know HOW?

Use the table at the right.

16. \( x \) \( y \)
0 0.03
1 0.05
2 0.07
3 0.09
4 0.11
5 0.13
6 0.15

Do you understand?

a. Model the relation with a scatter plot and a trend line. Write the equation of the trend line. \( y = 0.006x + 0.79 \)

b. Predict the value of \( y \) when \( x = 30 \). \( y = 7.89 \)

c. Reasoning. How do you know that your trend line is accurate?

All the points are very close to the line.

17. Multiple Choice. The graph of \( y = |x| \) is translated down 5 units and right 4 units. What is the equation of the new graph? \( y = |x - 4| - 5 \)

18. Suppose you manufacture and sell tarps. The table at the right displays your revenue and costs of producing tarps.

\begin{align*}
\text{Size} & \quad \text{Price} \\
100 & \quad $10 \quad (10) \\
200 & \quad $11 \quad (20) \\
300 & \quad $12 \quad (30) \\
400 & \quad $13 \quad (40) \\
500 & \quad $14 \quad (50) \\
\end{align*}

\( a \) How do you know that your trend line is accurate?

The linear regression is accurate.
Chapter 2 Performance Tasks

Give complete answers.

Task 1
Draw a line that crosses both the x- and y-axes but does not go through the origin.

a. Find the slope and y-intercept of this line.

b. Write the equation of this line in slope-intercept form.

c. Write the equation of this line in intercept form.

d. Draw a line that is perpendicular to this line. Write the equation of that line.

e. Modify and rewrite the original equation as a direct variation. Find the constant of variation.

Check students’ work.

[4] Student draws line correctly and finds slope and y-intercept. Student writes equation for the line in slope-intercept form and standard form with no mistakes. Student chooses and accurately graphs an appropriate line that is perpendicular to the original equation. Student correctly writes a direct variation using the original equation and finds the constant of variation.

[3] Student draws line correctly and finds slope and y-intercept. Student writes equation for the line in slope-intercept form and standard form with minor mistakes. Student graphs a line that is perpendicular to the original equation with a minor error in its equation. Student writes a direct variation using the original equation and finds the constant of variation.

[2] Student draws line correctly and finds slope and y-intercept. Student writes equation for the line in slope-intercept form and standard form with major mistakes. Student graphs a line that is perpendicular to the original equation with significant errors. Student writes an incorrect equation of direct variation or does not find the constant of variation.

[1] Student draws line correctly and finds slope and y-intercept. Student writes equation for the line in slope-intercept form and standard form with significant errors. Student does not draw a line that is perpendicular, and/or the equation of this line is not correct. The direct variation is either missing or written incorrectly.

[0] Student makes no attempt, or no response is given.

Task 2
Write a relation consisting of six ordered pairs.

a. Graph the relation.

b. Draw a mapping diagram for the relation.

c. Is the relation a function? Explain.

Check students’ work.

[4] Student provides a set of six ordered pairs, correctly graphs them, and draws a mapping diagram for the relation. Student correctly identifies the domain and range of the relation and whether the relation is a function. Student provides appropriate justification for the conclusion about whether the relation is a function.

[3] Student completes the tasks with only minor errors or omissions. Explanation could be more precise.

[2] Significant portions of the work are either incorrect or missing. Explanation is weak.

[1] Student makes major errors in writing and solving the inequalities. Graphs lack critical information or are missing.

[0] Student makes no attempt, or no response is given.

Chapter 2 Cumulative Review

Multiple Choice
For Exercises 1–6, choose the correct letter.

1. Which phrase could not describe a function?
   - a. rational number
   - b. whole number
   - c. none of the above

2. Which point cannot be part of a function that includes (3, –1), (4, 2), (5, 4), (–2, 0), and (6, –3)?
   - a. (6, 7)
   - b. (5, 6)
   - c. (3, 2)

3. Which value is in the solution set of –5 < x < 3 and 3 > x > 1?
   - a. 3
   - b. 2
   - c. 24

4. Which equations have graphs that are perpendicular?
   - a. y = 3x + 2
   - b. y = –x – 5
   - c. y = 3x + 1
   - d. y = 4 + \frac{2}{x}
   - e. I and III
   - f. II and IV
   - g. I and III
   - h. I and IV

5. Which relations are functions?
   - a. I
   - b. II
   - c. III
   - d. IV

6. Which line is parallel to y = \frac{2}{3}x + 7?
   - a. y = \frac{2}{3}x + 5
   - b. y = 3 – \frac{2}{3}x
   - c. y = \frac{2}{3}x + 2
   - d. y = \frac{2}{3}x + 1

Short Response
7. The graph of every direct variation passes through which point?
   - a. (0, 0)
   - b. (1, 2)

8. What are the opposite and reciprocal of \frac{3}{2}?
   - a. Opposite: –1 Reciprocal: \frac{2}{3}
   - b. Opposite: –\frac{2}{3} Reciprocal: \frac{3}{2}

9. Find the slope of each line.
   - a. y = 2x – 3
   - b. 3y + 7x = 1
   - c. 10
10. A summer carnival charges a $4 admission fee and $.50 for each ride. If you have $12 to spend, how many rides can you go on?

11. What are the x- and y-intercepts of 2x + 4y = 8?
   - a. (6, 0); (0, 2)
   - b. (3, 0); (0, 1)
   - c. (2, 0); (0, 2)

12. Suppose y varies directly with x and y = 1 when x = 3. What is the constant of variation? What is y when x = 4?
   - a. 4
   - b. 8
   - c. 16

Extended Response
13. Graph y > |x| – 2. Name the parent function of the boundary and describe the translation.

14. Open-ended
   - Write an equation that illustrates the Associative Property of Multiplication.

15. Writing
   - Let R be the relation consisting of ordered pairs (x, y) such that y is the biological mother of x. Is R a function? Explain.

16. [4] Rewrite the two units down
   - a. correct graph, but with one error in naming the parent function and describing the translation
   - b. Incorrect graph OR multiple errors in naming the parent function and describing the translation
   - c. Incorrect graph AND multiple errors in naming the parent function and describing the translation
   - d. No answers given

17. [4] Write an appropriate situation using the information represented in the graphs provided. Include details relating mathematical aspects of the graphs to your story.


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Chapter 2 Project: Time Squeeze

Beginning the Chapter Project

How long do you think it would take you to fill 1200 juice bottles by hand? Modern machines in factories can fill up to 1200 bottles per minute. Modern mass production depends on assembly lines that use highly specialized machines and production techniques.

In this project, you will apply mathematics to analyze assembly line production. You will use graphs and equations to help you make decisions about how to make production faster and more efficient. You will describe your methods and conclusions in a presentation to the class.

List of Materials
- Calculator
- Graph paper

Activities

Activity 1: Investigating
In this activity, you will work in groups to consider advantages and disadvantages of an assembly line.

- Think about a process your group could perform in an assembly line. Gather the materials. Have each person perform the entire process once. Record each person’s time. Find the average time needed to perform the process once. Time more people. If necessary, revise your average.
- Now break the procedure down into separate tasks. Assign each person in your group a task, and time how long the procedure takes. Does the average time differ? Compare your results with those of other groups.

Check students’ work.

Activity 2: Graphing
A juice bottling machine fills 1200 bottles per minute. Graph the number of bottles filled versus the time (in minutes) the machine runs.

- Is this model a direct variation? If so, what is the constant of variation?

![Graph of a direct variation](image)

Yes; 1200

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3-1 ELL Support
Solving Systems Using Tables and Graphs

Use the vocabulary above to fill in the blanks.

A grocery store has small bags of apples for $5 and large bags of apples for $8. Tickets for its winter concert. Student tickets are $5 and adult tickets are $10. If your school sells 85 tickets and makes $600, how many of each did they sell? 600 = 85(5) + 30x, so x = 25. If you want to practice for a total of 10 hours, which campus should you choose? Explain.

The second campus; after 2 hours, the cost of the first campus will always be greater than the cost of the second campus, because the slope of the equation for the first campus is greater than the slope of the equation for the second campus.

3-1 Practice
Solving Systems Using Tables and Graphs

Solve each system of equations and classify the system as consistent or inconsistent.

1. \( y = 2x - 2 \) (independent)
2. \( y = 2x - 1 \) (independent)
3. \( y = 2x + 10 \) (inconsistent)
4. \( 5y = 3x - 10 \) (consistent)
5. \( y = 2x + 9 \) (consistent)
6. \( y = 2x - 3 \) (consistent)
7. \( 3x + 2y = -10 \) (inconsistent)
8. \( y = 2x + 9 \) (independent)
9. \( y = 2x - 1 \) (independent)

10. Write and solve a system of equations for each situation. Check your answers.

Your school sells tickers for its winter concert. Student tickets are $5 and adult tickets are $10. If your school sells 85 tickets and makes $600, how many of each did they sell? 600 = 85(5) + 30x, so x = 25. If you want to practice for a total of 10 hours, which campus should you choose? Explain.

The second campus; after 2 hours, the cost of the first campus will always be greater than the cost of the second campus, because the slope of the equation for the first campus is greater than the slope of the equation for the second campus.

3-1 Open-Ended
Write a second equation for each system so that the system will have the indicated number of solutions.

10. \( x = y + 3 \) (independent)
11. \( x = y + 3 \) (inconsistent)
12. \( x = y + 3 \) (independent)
13. \( x = y + 3 \) (inconsistent)
14. \( x = y + 3 \) (independent)

3-1 Open-Ended
Write a second equation for each system so that the system will have the indicated number of solutions.

10. \( x = y + 3 \) (independent)
11. \( x = y + 3 \) (inconsistent)
12. \( x = y + 3 \) (independent)
13. \( x = y + 3 \) (inconsistent)
14. \( x = y + 3 \) (independent)

3-1 Open-Ended
Write a second equation for each system so that the system will have the indicated number of solutions.

10. \( x = y + 3 \) (independent)
11. \( x = y + 3 \) (inconsistent)
12. \( x = y + 3 \) (independent)
13. \( x = y + 3 \) (inconsistent)
14. \( x = y + 3 \) (independent)

3-1 Open-Ended
Write a second equation for each system so that the system will have the indicated number of solutions.

10. \( x = y + 3 \) (independent)
11. \( x = y + 3 \) (inconsistent)
12. \( x = y + 3 \) (independent)
13. \( x = y + 3 \) (inconsistent)
14. \( x = y + 3 \) (independent)
3-1 Practice Form K
Solving Systems Using Tables and Graphs

Solve each system by graphing or using a table. Check your answers.

1. \( \begin{align*}
  y &= 2x + 4 \\
  y &= -x - 3
\end{align*} \)

2. \( \begin{align*}
  y &= x + 2 \\
  y &= -x - 3
\end{align*} \)

3. \( \begin{align*}
  y &= -2x - 2 \\
  y &= 2x + 6
\end{align*} \)

Write and solve a system of equations for each situation. Check your answers.

4. Each morning you do a combination of aerobics, which burns about 12 calories per minute, and stretching, which burns about 4 calories per minute. Your goal is to burn 416 calories during a 60-minute workout. How long should you spend on each type of exercise to burn the 416 calories?

5. Suppose 28 members of your class went on a rafting trip. Class members could either rent canoes for $10 each or rent kayaks for $19 each. The class spent a total $469. How many people rented canoes and how many people rented kayaks?

For Exercise 6, use your graphing calculator to find a linear model for the set of data. In what year will the two quantities be equal?

6. \[
\begin{array}{c|c|c|c|c|c|c}
\hline
\text{Male Times (s)} & 60.13 & 58.62 & 56.73 & 58.23 & 56.12 & 57.72 \\
\text{Female Times (s)} & 60.42 & 59.13 & 59.26 & 59.26 & 59.13 & 58.62 \\
\end{array}
\]

8. You and your friend are both knitting a sweater for charity. You knit 8 rows each minute and already have knitted 10 rows. Your friend knits 5 rows each minute and has already knitted 15 rows. When will both you and your friend have the same number of rows?

9. The sides of an angle are two lines whose equations are \( x+y=12 \) and \( x-y=3e-2 \). An angle has its vertex at the point where the lines meet. Use a graph to determine the coordinates of the vertex. What are the coordinates of the vertex?

10. An angle has its vertex at the point where the lines meet.

Short Response

5. The sides of an angle are two lines whose equations are \( x+y=12 \) and \( x-y=3e-2 \). An angle has its vertex at the point where the lines meet. Use a graph to determine the coordinates of the vertex. What are the coordinates of the vertex?

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3-1 Reteaching
Solving Systems Using Tables and Graphs

As you solve a system of equations, remember the following ideas:

- Lines that have the same slopes but different y-intercepts are parallel and will never intersect. These systems are inconsistent.
- Lines that have both the same slopes and the same y-intercepts are the same line and will intersect at every point. These systems are dependent.
- Lines that have different slopes will intersect, and the system will have one solution.

These systems are independent:

**Problem**

Using a graph or a table, what is the solution of the system of equations?

\[ \begin{align*}
3x + y &= 8 \\
y &= -2x + 2
\end{align*} \]

**Exercises**

Using a graph or a table, what is the solution of the system of equations?

1. Solve each system by graphing or using a table. Check your answers.
   - \(3x + y = 6\)
   - \(x = 2\)
   - \(y = -2x + 8\)
   - \(y = -2x + 8\)
   - \(y = x + 2\)
   - \(y = x + 2\)

**Solving Systems Using Tables and Graphs**

Explain why the solution \((2, 4)\) of the equation \(x + 2y = 10\) is a solution of the problem.

**Problem**

Solve each system by graphing or using a table. Check your answers.

1. \(\begin{align*}
3x + y &= 6 \\
y &= 3
\end{align*}\)
2. \(\begin{align*}
2x + y &= 6 \\
x &= 1 - y
\end{align*}\)
3. \(\begin{align*}
2x + y &= 4 \\
x &= 2 - 2y
\end{align*}\)
4. \(\begin{align*}
x &= 1 \\
2x + y &= 4
\end{align*}\)
5. \(\begin{align*}
x &= 2 \\
2x + y &= 4
\end{align*}\)
6. \(\begin{align*}
x &= 3 \\
2x + y &= 4
\end{align*}\)

**Reteaching (continued)**

Solving Systems Using Tables and Graphs

**Problem**

The table shows the winning times for the Olympic 400-M dash. Use your graphing calculator to find linear models for women’s and men’s winning times. Assuming the trends in the table continue, when will the women’s winning time be equal to the men’s winning time? What will that winning time be?

<table>
<thead>
<tr>
<th>Year</th>
<th>Men’s Times</th>
<th>Women’s Times</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968</td>
<td>45.80</td>
<td>53.53</td>
</tr>
<tr>
<td>1972</td>
<td>44.68</td>
<td>51.08</td>
</tr>
<tr>
<td>1976</td>
<td>44.26</td>
<td>49.29</td>
</tr>
<tr>
<td>1980</td>
<td>44.08</td>
<td>48.88</td>
</tr>
<tr>
<td>1984</td>
<td>43.87</td>
<td>48.63</td>
</tr>
<tr>
<td>1988</td>
<td>43.55</td>
<td>48.03</td>
</tr>
<tr>
<td>1992</td>
<td>43.04</td>
<td>47.25</td>
</tr>
<tr>
<td>1996</td>
<td>42.88</td>
<td>46.11</td>
</tr>
</tbody>
</table>

**Exercise**

8. The table shows the winning times for Olympic 500-M speed skating. Assuming these trends continue, when will the women’s winning time equal the men’s winning time? What will that winning time be?

**Chemistry**

A scientist wants to make 5 milliliters of a 30% sulfuric acid solution. The solution is to be made from a combination of a 20% sulfuric acid solution and a 50% sulfuric acid solution. How many milliliters of each solution must be combined to make the 30% solution?

**Plan**

To solve the problem you need to define:

- Let \(x\) = ml of 20% solution
- Let \(y\) = ml of 50% solution

**Know**

1. The scientist will begin with 20% and 50% solutions.
2. The scientist wants to make 5 ml of 30% solution.

**Need**

3. Two equations you can write to model the situation:

\[ \begin{align*}
x + y &= 5 \\
0.2x + 0.5y &= 0.3 \times 5
\end{align*}\]

4. Which method should you use to solve the system of equations? Explain.

5. Explain why the solution of \(x + y = 5\) and \(0.2x + 0.5y = 1.5\) should be checked in both equations.


**ANSWERS**

3-1 Reteaching

Solving Systems Using Tables and Graphs

As you solve a system of equations, remember the following ideas:

- Lines that have the same slopes but different y-intercepts are parallel and will never intersect. These systems are inconsistent.
- Lines that have both the same slopes and the same y-intercepts are the same line and will intersect at every point. These systems are dependent.
- Lines that have different slopes will intersect, and the system will have one solution.

These systems are independent:

**Problem**

Using a graph or a table, what is the solution of the system of equations?

\[ \begin{align*}
x + 2y &= 8 \\
y &= 2x - 2
\end{align*}\]

**Exercises**

Using a graph or a table, what is the solution of the system of equations?

1. Solve each system by graphing or using a table. Check your answers.
   - \(3x + y = 6\)
   - \(x = 2\)
   - \(y = -2x + 8\)
   - \(y = -2x + 8\)
   - \(y = x + 2\)
   - \(y = x + 2\)

2. Circle each ordered pair that is a solution.

3. The linear model shows that if the table’s trends continue, the times for men and women will be equal about 1988 after 1988, in 2008. The winning time will be about 42 seconds.

4. What are two equations you can write to model the situation?

**Think About a Plan**

Solving Systems Algebraically

**Plan**

4. What are two equations you can write to model the situation?

\[ \begin{align*}
x + y &= 5 \\
0.2x + 0.5y &= 1.5 \times 5
\end{align*}\]

5. Which method should you use to solve the system of equations? Explain.


3-2 ELL Support

Solving Systems Algebraically

The column on the left shows the steps used to solve a problem using a system of equations. Use the column on the left to answer each question in the column on the right.

**Problem**

Solve by Setting up and Solving a System of Equations.

A girl received $100 for her birthday. At the store, she can buy 1 video game and 3 DVDs for $100. She could also buy 2 video games and 1 DVD for $100. Write and solve a system of equations to determine the cost of a video game and the cost of a DVD.

**Values**

- \(x\) = cost of one video game
- \(y\) = cost of one DVD

\[ \begin{align*}
x + 3y &= 100 \\
x + y &= 100
\end{align*}\]

**Solve**

1. What is the cost of one video game and one DVD?

2. Explain why the cost of a video game and cost of a DVD are represented by variables.

3. How can you interpret the solutions of the system in the context of the problem?
3-2 Practice
Solving Systems Algebraically

Solve each system by substitution. Check your answers.

1. \( \begin{align*}
\frac{y}{x} + \frac{1}{2} &= 2.25,
\frac{5}{x} - y &= 7
\end{align*} \)

2. \( \begin{align*}
\frac{x}{y} + y &= 2,
\frac{3}{x} + y &= 6
\end{align*} \)

3. \( \begin{align*}
y &= 2x + 3,
5x - y &= -3
\end{align*} \)

4. \( \begin{align*}
x - 3y &= -30,
2x - y &= -1
\end{align*} \)

5. \( \begin{align*}
2x - y &= -7,
3y - 2x &= 9
\end{align*} \)

6. \( \begin{align*}
x &= 4y,
y &= 2x + 27
\end{align*} \)

7. \( \begin{align*}
x + 3y &= -6,
y + x &= 0
\end{align*} \)

8. \( \begin{align*}
x + 3y &= -9,
y + x &= 3
\end{align*} \)

10. Suppose you bought eight oranges and one grapefruit for a total of $4.60. Later that day, you bought six oranges and three grapefruits for a total of $4.80. What is the price of each type of fruit? (1) Oranges cost $0.50; (2) Grapefruit costs $0.50.

Solve each system by elimination.

11. \( \begin{align*}
x + 2y &= 10,
x - y &= 2
\end{align*} \)

12. \( \begin{align*}
x + 3y &= 1,
x - 2y &= 2
\end{align*} \)

13. \( \begin{align*}
x + 2y &= 3,
y - 3x &= -1
\end{align*} \)

14. \( \begin{align*}
4x - 4y &= -2,
4x + 3y &= 14
\end{align*} \)

15. \( \begin{align*}
x + 2y &= 10,
y &= 0
\end{align*} \)

16. \( \begin{align*}
x - 3y &= 3,
y &= 0
\end{align*} \)

17. \( \begin{align*}
x + 2y &= -6,
x - y &= 5
\end{align*} \)

18. \( \begin{align*}
x + 3y &= -4,
y + x &= -4
\end{align*} \)

19. \( \begin{align*}
x &= 17,
2x + y &= 0
\end{align*} \)

20. There are a total of 15 apartments in two buildings. The difference of the two times the number of apartments in the first building and the number of apartments in the second building is 5.

a. Write a system of equations to model the relationship between the number of apartments in each building and the number of apartments in the second building: \( f = x + 5, 2f - 3s = 5 \)

b. How many apartments are in each building? 10 in the first; 5 in the second

21. \( \begin{align*}
x^2 - y^2 &= 3,
x + y &= 9
\end{align*} \)

22. \( \begin{align*}
x^2 + y^2 &= 2,
x + 2y &= 4
\end{align*} \)

23. \( \begin{align*}
x^2 - y^2 &= 3,
y &= x - 3
\end{align*} \)

24. \( \begin{align*}
x + y &= 10,
x - 3y &= 11
\end{align*} \)

25. \( \begin{align*}
x + 2y &= 10,
x + y &= 5
\end{align*} \)

26. \( \begin{align*}
x + y &= 1,
2x + y &= 5
\end{align*} \)

27. \( \begin{align*}
x + 2y &= 11,
x - 3y &= 27
\end{align*} \)

28. \( \begin{align*}
x + 2y &= 5,
x + y &= 0
\end{align*} \)

29. \( \begin{align*}
x + 2y &= 5,
x - 3y &= 11
\end{align*} \)

No solution (2.25, 0)

3.2 Practice
Solving Systems Algebraically

Solve each system by substitution. Check your answers. To start, solve one equation for \( y \) and substitute into the other equation.

1. \( \begin{align*}
x + 4y &= 0,
x + 2y &= 4
\end{align*} \)

2. \( \begin{align*}
x + 2y &= 10,
x + y &= 7
\end{align*} \)

3. \( \begin{x+2y=10\atop{y=x-3}} \)

4. \( \begin{x+y=0\atop{y=x+10}} \)

5. \( \begin{x+y=2\atop{y=x-2}} \)

6. \( \begin{x+y=2\atop{y=x-1}} \)

Your internet provider offers two different plans. One plan costs $5.02 per email plus $0.03 monthly service charge. The other plan costs $5.04 per email with no service charge.

a. Write a system of equations to model the cost of the two internet plans.

b. For how many email messages will both plans cost the same? 100 messages
c. If you send and receive about 500 email messages per month, which plan should you choose? The plan that costs $5.02 per email plus $0.03 monthly service charge.

A boat can travel 24 mi in 3 h when traveling against the current. When traveling against the current, the boat can travel only 18 mi in 4 h. Find the rate of the boat in still water.

2 mph, 6 mph

6. Writing Explain how you would solve the system \( \frac{2x + 10}{y} = \frac{10}{x} \) using substitution.

Answers may vary. Sample: Substitute \( x + 4 \) for \( x \) in the first equation and solve for \( y \). Then substitute \( x + 4 \) into either of the original equations and solve for \( y \).
3-2 Standardized Test Prep
Solving Systems Algebraically

Multiple Choice
For Exercises 1–8, choose the correct letter.

1. Use the system of equations for Exercises 1 and 2.

1. a. $2x - y = 3$
   b. $x + 2y = 5$
   c. $x = 3, y = 2$
   d. $a = -2, b = 2$

2. What is the value of $y$ in the solution?

   a. $-5$
   b. $-2$
   c. $4$
   d. $3$

3. Which of the following systems of equations has the solution $(4, -3)?$

   a. $2x - 2y = 14$
   b. $2x + 2y = 6$
   c. $3x - y = 0$
   d. $4x + 2y = 26$

4. At a bookstore, used hardcover books sell for $8 each and used softcover books sell for $2 each. You purchase 36 used books and spend $144. How many softcover books do you buy?

   a. 9
   b. 12
   c. 18
   d. 24

Extended Response
5. A local cell phone company offers two different calling plans. In the first plan, you pay a monthly fee of $30 and $.35 per minute. In the second plan you pay a monthly fee of $50 and $.10 per minute.

   a. Write a system of equations showing the two calling plans.
   b. When is it better to use the first calling plan?
   c. When is it better to use the second calling plan?
   d. How much does it cost when the calling plans are equal?

6. Solve each system by elimination.

   a. $2x - 3y = 1$
   b. $6x + 3y = 2$
   c. $2x - y = 0$
   d. $4x + 2y = 2$

7. Follow these steps when solving by substitution.

   Step 1: Solve one equation for one of the variables.
   Step 2: Substitute the expression for the first variable into the other equation.
   Step 3: Substitute the second variable’s value into either equation. Solve for the first variable.
   Step 4: Check the solution in the other original equation.

   What is the solution of the system of equations?

   a. $4x - 3y = 10$
   b. $x + y = 10$
   c. $x = 2, y = -3$
   d. $x = 5, y = -2$

8. Solve each system by substitution.

   a. $x + y = 6$
   b. $3x - 2y = 5$
   c. $x - 2y = -7$
   d. $7x + 3y = 1$

   The solution is $(2, 3)$. You can also check the solution by using a graphing calculator.
Solve each system of inequalities by graphing.

1. \[ y < x - 1 \]
   \[ y < x + 1 \]
   \[ y < 3x - 2 \]

2. \[ 2x + y < 3 \]
   \[ x - 2y < 3 \]

3. \[ 2x - y > 2 \]
   \[ x - 2y < 4 \]

4. \[ y > 3 \]
   \[ y > -3 \]

5. \[ x < 2 \]
   \[ y < 0 \]

6. \[ 3x + y > 6 \]
   \[ 3x - y > 4 \]

7. \[ x < 0 \]
   \[ y < 0 \]

8. \[ x < 0 \]
   \[ y < 0 \]

9. \[ x < 0 \]
   \[ y < 0 \]

10. \[ x < 0 \]
    \[ y < 0 \]

11. \[ x < 0 \]
    \[ y < 0 \]

12. \[ x < 0 \]
    \[ y < 0 \]

13. \[ x < 0 \]
    \[ y < 0 \]

14. \[ x < 0 \]
    \[ y < 0 \]

15. \[ x < 0 \]
    \[ y < 0 \]

16. \[ x < 0 \]
    \[ y < 0 \]

17. \[ x < 0 \]
    \[ y < 0 \]
Systems of Inequalities

5. a) For each graph, choose one point in the region. Explain why that point satisfies each of the inequalities.
b) For each graph, choose one point outside the region that satisfies one or more of the inequalities but not all of them.

6. a. Answers may vary. Sample: $x = 4, y = 6$. Substituting the values $x = 4$ and $y = 6$ into each inequality gives a true statement.
b. Answers may vary. Sample: $x = 3, y = 7$. Substituting the values $x = 3$ and $y = 7$ into each inequality gives a true statement.

7. Substitute the ordered pair into each inequality and simplify. Both inequalities are true, so the ordered pair is a solution of the system.
Solving a System by Using a Table

Problem
An English class has 4 computers for at most 18 students. Students can either use the computers in groups to research Shakespeare or watch performances. Each research group must have 4 students and each performance group must have 5 students. In how many ways can you set up the computer groups?

Step 1
Relate the unknowns and define them with variables.

- Let \( x \) = number of research groups
- Let \( y \) = number of performance groups

Two groups doing research and 0, 1, or 2 groups watching performances

\[ x = \text{number of research groups} \]
\[ y = \text{number of performance groups} \]

Number of research groups + Number of performance groups + 4 = Number of students

Number of research groups + Number of performance groups + 4 ≤ 18

Step 2
Make a table of values for \( x \) and \( y \) that satisfy the first inequality. The replacement values for \( x \) and \( y \) must be whole numbers. You cannot have more than 4 research groups or 5 performance groups with only 18 students.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

Step 3
In the table, check each pair of values to see which satisfy the second inequality. Highlight these pairs. These are the solutions of the system.

You cannot have more than 4 research groups or 5 performance groups with only 18 students.

Exercises
Find the whole number solutions of each system using tables.

1. \( x + y < 4 \)
   \( x + 2y < 10 \)
   \((0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (3, 0)\)

2. \( x + y = 3 \)
   \( 2x + 3y = 12 \)
   \((0, 3), (1, 2), (2, 1), (3, 0)\)

3. \( x + y < 5 \)
   \( x + 2y < 8 \)
   \((0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (2, 0), (2, 1), (3, 0), (3, 1), (4, 0), (4, 1)\)

4. \( x < 3 \)
   \( 4x + 2y < 12 \)
   \((0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0)\)

ELL Support
Linear Programming

Choose the word from the list that best matches each sentence.

- Constraints: feasible region
- Linear programming: objective function
- Variables: vertices

1. The limits or restrictions are also called constraints.

2. Linear programming is a method for finding a maximum or minimum value of some quantity, given a set of constraints.

3. On a graph, the feasible region contains all of the points that satisfy the constraints.

4. The objective function models a quantity that is related to the constraint variables.

5. The vertices of the feasible region are the points where the least and greatest values for the objective function occur.

Use a word from the list above to complete each sentence.

6. The inequalities \( x - 2y < 4 \) and \( 2x + y < 6 \) are the constraints.

7. The equation \( P = 5x + 10y \) is an example of an objective function where \( P \) stands for profit.

8. The intersections of the constraint graphs are the vertices of the feasible region.

9. You have to graph the inequalities to find the feasible region for a linear programming problem.

Multiple Choice
10. Which of the following is a vertex of the feasible region on the graph at the right?

- (2, 1), (3, 0), (3, 4), (3, 2), (2, 7), (3, 3), (1, 1)

11. How many constraints were graphed to form the feasible region on the graph?

- 2 constraints
- 3 constraints
- 4 constraints
- 5 constraints

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Graph each system of constraints. Name all vertices. Then find the values of $x$ and $y$ that maximize or minimize the objective function for each graph.

1. Minimum for $P = 4x + 2y \begin{cases} (0, 0) \\ (5, 0) \end{cases}$
2. Maximum for $P = x + 3y \begin{cases} (1, 0) \\ (0, 1) \end{cases}$
3. Minimum for $P = 3x + 2y \begin{cases} (0, 0) \\ (0, 5) \end{cases}$
4. Maximum for $P = 2x + y, (2, 0)$
5. Minimum for $P = x + 2y, (0, 5)$
6. Maximum for $P = 5x + y, (0, 10)$

Graph each system of constraints. Name all vertices. Then find the values of $x$ and $y$ that maximize or minimize the objective function.

7. Maximum for $P = 3x + 4y \begin{cases} (2, 1) \\ (5, 0) \end{cases}$
8. Minimum for $P = x + 3y \begin{cases} (0, 2) \\ (2, 0) \end{cases}$
9. Maximum for $P = 4x + y \begin{cases} (10, 4) \\ (5, 0) \end{cases}$

Write an objective function and constraints for a linear program that models the problem. $P = 50x + 200y$; constraints $2x + y \leq 30, 2y \leq 16, x \geq 0, y \geq 0$

10. You are going to make and sell baked goods. A loaf of Irish soda bread is made with 2 c flour and 1 c sugar. You will make a profit of $8.75 on each loaf of Irish soda bread and a profit of $5 on each Kugelhopf cake. You have 10 lb flour and 3 lb sugar.
   a. How many of each kind of baked good should you make to maximize the profit?
   b. What is the maximum profit?

11. A doctor allots 15 minutes for routine office visits and 45 minutes for full physicals. The doctor cannot do more than 10 physicals per day. The doctor has 9 available hours for appointments each day. A routine office visit costs $60 and a full physical costs $150. How many routine office visits and full physicals should the doctor schedule to maximize her income for the day? What is the maximum income?

12. A doctor allows 15 minutes for routine office visits and 45 minutes for full physicals. The doctor cannot do more than 10 physicals per day. The doctor sees 9 available hours for appointments each day. A routine office visit costs $60 and a full physical costs $150. How many routine office visits and full physicals should the doctor schedule to maximize her income for the day? What is the maximum income?

13. A doctor allows 15 minutes for routine office visits and 45 minutes for full physicals. The doctor cannot do more than 10 physicals per day. The doctor sees 9 available hours for appointments each day. A routine office visit costs $60 and a full physical costs $150. How many routine office visits and full physicals should the doctor schedule to maximize her income for the day? What is the maximum income?

14. A doctor allows 15 minutes for routine office visits and 45 minutes for full physicals. The doctor cannot do more than 10 physicals per day. The doctor sees 9 available hours for appointments each day. A routine office visit costs $60 and a full physical costs $150. How many routine office visits and full physicals should the doctor schedule to maximize her income for the day? What is the maximum income?

15. Answers may vary. Sample: You must know how to find the point of intersection to be able to determine the critical points of a system of constraints. The vertices enable you to find the $x$ and $y$ values that maximize or minimize an objective function.
Constraints

Graph each system of constraints. Name all vertices. Then find the values of $x$ and $y$ that maximize or minimize the objective function.

Exercises

1. \[
\begin{align*}
3y + 4x &\leq 35 \\
5y + x &\leq 30 \\
x &\leq 1 \\
\end{align*}
\]

2. \[
\begin{align*}
x + y &\leq 2 \\
x &\geq 0 \\
y &\leq 4 \\
\end{align*}
\]

3. \[
\begin{align*}
x + 4y &\leq 12 \\
x + 6y &\leq 30 \\
x &\leq 5 \\
\end{align*}
\]

Short Response

5. The figure at the right shows the feasible region for a system of constraints. This system includes $x \geq 0$ and $y \geq 0$. What are the remaining constraints? Show your work.

[2] Work should show the use of two points of each line to find slope and linear inequalities.

[1] correct inequalities, without work shown OR correct process with one computational error

[0] no answer given and no work shown OR no answer given

ANSWERS

Reteaching

Problem

What point in the feasible region maximizes $P$ for the objective function

\[ P = 10x + 15y \]

What point minimizes $P$?

Constraints

\[
\begin{align*}
x + y &\geq 16 \\
3x + 6y &\leq 60 \\
x &\geq 0 \\
y &\geq 0 \\
\end{align*}
\]

Step 1

Graph the constraints and shade the feasible region.

Step 2

Find the coordinates for each vertex of the region.

\[
\begin{align*}
A, (0, 10) &
B, (16, 0) &
C, (12, 4) &
D, (10, 10) &
\end{align*}
\]

Step 3

Evaluate $P$ at each vertex.

\[
\begin{align*}
P_A &= 10(0) + 15(10) = 150 \\
P_B &= 10(16) + 15(0) = 160 \\
P_C &= 10(12) + 15(4) = 180 \\
P_D &= 10(10) + 15(10) = 250 \\
\end{align*}
\]

The maximum value of the objective function is 250. It occurs when $x = 10$ and $y = 0$.

The minimum value of the objective function is 0. It occurs when $x = 0$ and $y = 0$.

Exercises

Graph each system of constraints. Name all vertices. Then find the values of $x$ and $y$ that maximize or minimize the objective function.

1. \[
\begin{align*}
3y + 4x &\leq 35 \\
5y + x &\leq 30 \\
x &\leq 1 \\
\end{align*}
\]

2. \[
\begin{align*}
x + y &\leq 2 \\
x &\geq 0 \\
y &\leq 4 \\
\end{align*}
\]

3. \[
\begin{align*}
x + 4y &\leq 12 \\
x + 6y &\leq 30 \\
x &\leq 5 \\
\end{align*}
\]

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### 3-5 Systems With Three Variables

**ELL Support**

A student solved the system of equations $2x + 3y - z = 15$ and wrote a description of each step on note cards. Write each description next to the correct step.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.</td>
<td>Write each description next to the correct step.</td>
</tr>
</tbody>
</table>

Write the two new equations as a system. Solve for $x$.

$$3x + 2y = 11$$

Solve each system by substitution. Check your answers.

1. 
   $$\begin{align*}
   x + y & = 2 \\
   x - y & = 2
   \end{align*}$$

2. 
   $$\begin{align*}
   x + y & = 4 \\
   x - y & = 1
   \end{align*}$$

3. 
   $$\begin{align*}
   x + y & = 2 \\
   x - y & = -1
   \end{align*}$$

**Form G**

### 3-5 Systems With Three Variables

**Practice (continued)**

Write and solve a system of equations for each problem.

25. The sum of three numbers is $-2$. The sum of the three numbers is equal to the second number, and the third number is $9$. The difference between the second number and the third number is $10$. Find the numbers.

26. Monica has $1, $5, and $10 bills in her wallet that are worth $90. If she had one more $1 bill, she would have just as many $1 bills as $5 and $10 bills combined. She has 23 bills total. How many of each denomination does she have?

27. Writing
   How do you decide whether substitution is the best method to solve a system in three variables? If one equation can be solved easily for one variable, then substitution is the best method to use. Is the solution correct? How can you be sure? Show your work.

The student gets a solution of $(2, 12, 3)$. Is the solution correct? How can you check your solution by substituting values for $x$, $y$, and $z$ into one of the original equations and solve for $z$.

31. Which of the following is a system with the solution $(0, -2, -3)$?

**Options**

- $2x + y + z = 5$
- $x + 4y + z = 16$
- $x + 4y + 2z = 18$
- $x + 3y + z = 9$
- $x + 3y + z = 0$
- $x + 3y + z = 0$
3-5 Practice
Systems With Three Variables

Solve each system by elimination. Check your answers. To start, pair the equations to eliminate one variable and add.

1. \[2x - y + 3z = 14\]
   \[x - 2y + z = 8\]
   \[x + 3y - 2z = 7\]

2. \[2x + y + z = 6\]
   \[x - 2y + z = 8\]
   \[2x - 3y + 2z = 27\]

-2(\[2x - y + 3z = 14\]) \[2x + y + z = 6\]
-2(\[x - 2y + z = 8\]) \[x - 2y + z = 8\]
(\[x + 3y - 2z = 7\]) \[x + 3y - 2z = 7\]

\[4z = -20\]
\[z = -5\]

\[2x - y + 3z = 14\]
\[x - 2y + z = 8\]
\[-3(\[x - 2y + z = 8\]) \[x + 3y - 2z = 7\]

\[2x - y + 3z = 14\]
\[x - 2y + z = 8\]
\[-3(\[x - 2y + z = 8\]) \[x + 3y - 2z = 7\]

\[z = -5\]

\[x + 3y - 2z = 7\]
\[x + 3y - 2z = 7\]
\[-3(\[x - 2y + z = 8\]) \[x + 3y - 2z = 7\]

\[y = -2x - z + 14\]
\[-x + 6y + 3z = -2\]
\[-x + 6y + 3z = -2\]
\[5x + 5z = 40\]
\[x + z = 8\]

Solve each system by substitution. Check your answers.

\[2x + 2y + z = 12\]
\[4x - 6y + 3z = -5\]
\[-x - 3y + 2z = -14\]
\[y = -2x - z + 14\]

\[5x + 5z = 40\]
\[-x + 6y + 3z = -2\]
\[x + z = 8\]

\[x = 2, \ y = 7, \ z = 5\]

5. You have 17 coins in pennies, nickels, and dimes in your pocket. The value of the coins is $8.47. There are four times the number of pennies as nickels. How many of each type of coin do you have?

- 12 pennies, 3 nickels, 2 dimes

6. Writing When you solve a system of equations, explain how you can determine if your solution is correct. Substitute your solution back into the original equations. If all of the equations are true, the solution is correct.

3-5 Standardized Test Prep
Systems With Three Variables

Gridded Response

Solve each exercise and enter your answer in the grid provided.

1. A change machine contains nickels, dimes, and quarters. There are 75 coins in the machine, and the value of the coins is $7.25. There are 3 times as many nickels as dimes. How many quarters are in the machine?

2. The sum of three numbers is 23. The first number is equal to twice the second number minus 7. The third number is equal to one more than the sum of the first and second numbers. What is the first number?

3. A fish’s tail weighs 9 lb. Its head weighs as much as its tail plus half its body. Its body weighs as much as its head and tail. How many pounds does the fish weigh?

4. You are training for a triathlon. In your training routine each week, you bike 5 times as far as you run and you run 4 times as far as you swim. One week you trained a total of 200 miles. How many miles did you swim that week?

5. Three multiplied by the first number is equal to the second number plus 4. The second number is equal to one plus two multiplied by the third number. The third number is one less than the first number. What is the sum of all three numbers?

Answers

3-5 Enrichment
Systems With Three Variables

The Italian Navigator Has Landed

The above phrase was one of the most important coded messages that has ever been sent. It referred to the fact that a team of physicists had managed to achieve the first successful controlled nuclear chain reaction. The physicist who directed these efforts was an accomplished theoretician and experimentalist, whose work in producing artificial radioactive elements won him the Nobel Prize in Physics in 1938.

\[ A + C - 2E = 2 \]
\[ A - C + E = 5 \]
\[ 3L + M + N = 4 \]
\[ L = M + 2N = -4 \]
\[ 3L = M + N = 17 \]
\[ A + B + O = 1 \]
\[ 3A - B + 2O = 9 \]
\[ A - B - O = -5 \]
\[ 3A - I + F = 4 \]
\[ A - 2I + F = 0 \]
\[ A + I + F = 12 \]
\[ 2I + K = 2 \]
\[ 4I + 6L = 20 \]

First solve each of the following sets of equations. For each letter with a value between 1 and 12, write that letter in its corresponding location in the puzzle.

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3-5 Reteaching
Systems With Three Variables

Problem

What is the solution of the system?

\[
\begin{align*}
1. & \quad 2x + y + z = 6 \\
2. & \quad 3x + 2y + z = 9 \\
3. & \quad x + 2y + 2z = 9 \\
\end{align*}
\]

Use elimination. The equations are numbered to make the process easy to follow.

\[
\begin{align*}
1. & \quad x + y + z = 6 \quad \text{Pair the equations to eliminate } x. \\
2. & \quad 2x + 2y + z = 12 \quad \text{Then add the two equations.} \\
3. & \quad 2x + y + 2z = 10 \quad \text{Multiply equation 3 by 2 to eliminate } x. \\
\end{align*}
\]

The solution is \((1, 2, 3)\).

Exercises

Solve each system by elimination. Check your answers.

1. \[
\begin{align*}
2x + y + z &= 10 \\
x - y + z &= 8 \\
\end{align*}
\]

2. \[
\begin{align*}
3x + 2y + z &= 15 \\
x + y + 2z &= 4 \\
\end{align*}
\]

3. \[
\begin{align*}
4x - y + z &= 5 \\
x + y + z &= 2 \\
\end{align*}
\]

4. \[
\begin{align*}
x - y + z &= 2 \\
x + y + z &= 1 \\
\end{align*}
\]

5. \[
\begin{align*}
x - y + z &= 10 \\
x + y + z &= 6 \\
\end{align*}
\]

6. \[
\begin{align*}
x - y + z &= -4 \\
x + y + z &= -6 \\
\end{align*}
\]

6-5, 2-4

3-6 ELL Support
Solving Systems Using Matrices

Choose the word from the list that best matches each sentence.

- coefficient
- column
- dimension
- matrix
- matrix element
- row

1. A rectangular array of numbers is called a(n) ______.
2. The numbers that are written horizontally in a matrix are called ______.
3. Each number in a matrix is called a(n) ______.
4. The ______ of a matrix are the number of rows and columns in the array.
5. A ______ is a matrix formed by multiplying the numbers vertically.
6. You can use a ______ to represent a system of equations.
7. Each ______ and constant in a system of equations are used to represent the systems of equations.
8. Each matrix ______ is a system of equations.
9. The last matrix column ______ the constraints to the right of the equal signs.

The size of a matrix is described by listing the dimensions of the matrix. This means listing the number of rows and columns in the matrix.

\[
\begin{align*}
\text{1 row} & : \begin{bmatrix} 3 \end{bmatrix} \\
\text{2 columns} & : \begin{bmatrix} 8 & -1 \\ 9 & 2 \end{bmatrix}
\end{align*}
\]

The number of rows is always the first number.

Write the dimensions of each matrix below:

10. \[
\begin{bmatrix}
-1 & 5 & 0 & -6 \\
3 & -3 & 8 & 2
\end{bmatrix}
\]

11. \[
\begin{bmatrix}
5 & -2 & 9 & 7 \\
0 & 6 & -3 & 1 \\
8 & 7 & -4 & 3
\end{bmatrix}
\]

12. \[
\begin{bmatrix}
-2 & 9 & 0 & -6 \\
1 & 4
\end{bmatrix}
\]

3-6 Think About a Plan
Solving Systems Using Matrices

Paint

A hardware store mixes paints in a ratio of two parts red to six parts yellow to make two gallons of pumpkin orange. A ratio of five parts red to three parts yellow makes two gallons of pepper red. Find the cost of 1 quart of red paint and the cost of 1 quart of yellow paint.

Know

1. There are 4 quarts in 1 gallon.
2. \[
\begin{align*}
2 \text{ pt red} & \leftrightarrow 5 \text{ pt yellow} \\
1 \text{ pt pumpkin orange} & = 5 \text{ pt red} \\
1 \text{ pt yellow} & = 4 \text{ pt pepper red}
\end{align*}
\]

3. 4 quarts of pumpkin orange cost $25

4. 4 quarts of pepper red cost $28

Need

4. To solve the problem you need to define:
   \( x = \text{cost of 1 qt of red paint} \), \( y = \text{cost of 1 qt of yellow paint} \)

5. To solve the problem you need to find the solution to a system of two equations.

Plan

6. What system of equations represents this situation?

\[
\begin{align*}
x + 3y &= 25 \\
x + 3y &= 56
\end{align*}
\]

7. How can you represent the system of equations with a matrix?

\[
\begin{bmatrix}
1 & 3 & 25 \\
1 & 3 & 56
\end{bmatrix}
\]

8. Solve the system of equations using the matrix.

\[
\begin{align*}
[7.75, 5.75]
\end{align*}
\]

9. How can you interpret the solutions in the context of the problem?

1 qt of red paint costs $7.75 and 1 qt of yellow paint costs $5.75.

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Solve the system of equations using a matrix.

1. Identify the indicated element.
   \[
   A = \begin{bmatrix}
   3 & 5 & 8 \\
   4 & 1 & 6
   \end{bmatrix}
   \]
   1. \(a_{12}\) 2. \(b_{12}\) 3. \(B_{2}\) 4. \(A_{11}\)

2. Write a matrix to represent each system.

3-6 Practice Solving Systems Using Matrices

5. \(A = \begin{bmatrix}
2 & 3 & 4 \\
5 & 6 & 7
\end{bmatrix}\)
6. \(A_{1} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
7. \(B_{12}\) 8. \(A_{1}\)

6. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

1. Write the system of equations represented by each matrix.

3. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

15. \(2x + y - z = 15\)
16. \(x - 2y + z = -2\)
17. \(x + 2y - z = -1\)

2. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)
3. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
4. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)

4. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)

5. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)

6. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)

7. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)

8. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)

9. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)

10. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)

11. Write the system of equations represented by each matrix.

5. \(2x + y - z = 15\)
6. \(x - 2y + z = -2\)
7. \(x + 2y - z = -1\)

10. \(A_{13} = \begin{bmatrix}
3 & 4 \\
5 & 6
\end{bmatrix}\)
11. \(A_{11} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}\)
12. \(A_{12} = \begin{bmatrix}
2 & 3 \\
4 & 5
\end{bmatrix}\)
3-6 Standardized Test Prep
Solving Systems Using Matrices

Multiple Choice
For Exercises 1–3, choose the correct letter.

1. Which system of equations is equivalent to

2. What is the solution of the system represented by the matrix

3. How many elements are in a 2 × 3 matrix?

Short Response
4. A clothing store is having a sale. A pair of jeans costs $15 and a shirt costs $8. You spend $31 and buy a total of 12 items. Using a matrix, how many pairs of jeans and shirts do you buy? Show your work.

5. Matrices may vary. Sample: 5; 5 pairs of jeans; 7 shirts
     (1) correct solution, without work shown OR correct process with one computational error
     (2) incorrect answer and no work shown OR no answer given

3-6 Reteaching
Solving Systems Using Matrices

Problem
How can you represent the system of equations with a matrix?

Step 1
Write each equation in the same variable order. Line up the variables.

Write the matrix using the coefficients and constants. Remember to enter a 1 for variables with no numeric coefficient.

Step 2
Write a matrix to represent each system.

Exercises
Write a matrix to represent each system.

3-6 Enrichment
Solving Systems Using Matrices

Well-Conditioned Systems of Linear Equations

A system of linear equations is said to be well-conditioned if a small change in the values of the coefficients produces a small change in the values of the solutions. A system is said to be ill-conditioned if a small change in the values of the coefficients produces a large change in the values of the solutions.

To determine whether a system of linear equations is well-conditioned or ill-conditioned, change each coefficient by one percent. Each time, write and solve the new system, finding the values of x and y to two decimal places by writing and solving a matrix. Then compare the new values of x and y to the solutions of the original system. A change of less than one percent in the values of x and y can be considered small.

1. Determine whether System 1 is well-conditioned or ill-conditioned by completing the following steps.

a. Find the values of x and y.

b. Change the coefficient of x in the first equation by one percent, from 1.01 to 1.001. Write the new system. Find the values of x and y.

c. Change the coefficient of y in the first equation by one percent, from 0.99 to 0.999. Write the new system. Find the values of x and y.

d. Change the coefficient of x in the second equation by one percent, from 1.02 to 1.0202. Write the new system. Find the values of x and y.

e. Change the coefficient of y in the second equation by one percent, from 0.992 to 0.99209. Write the new system. Find the values of x and y.

f. Is the system well-conditioned or ill-conditioned?

2. Determine whether System 2 is well-conditioned or ill-conditioned by completing the same steps as in Exercise 1.

System 2

a. Solve the system.

b. Solve the system with the new coefficient for y.

Well-conditioned

ill-conditioned
Solve each system of equations by graphing.

1. \( y = -2x + 5 \)  \( (1, 3) \)
2. \( 2x + y = 10 \)  \( (3, 4) \)

Solve each system of equations by substitution or elimination.

3. \( y = x \)  \( x = y \)  \( (0, 0) \)
4. \( \begin{cases} x + y = 6 \\ 2x - y = 3 \end{cases} \)
5. \( \begin{cases} x - y = 1 \\ 2x + y = 1 \end{cases} \)  \( (1, -1) \)

Do you UNDERSTAND?

b. Explain the steps you would follow to solve a system of equations using the substitution method.

6. Solve each system of inequalities by graphing.

a. \( y - x > 2 \)  \( y + x < 2 \)  \( (2, 1) \)

b. \( y - x < -2 \)  \( y + x > -2 \)  \( (5, 2) \)

7. Solve each system of equations by graphing.

a. \( x = y + 1 \)  \( x + y = -1 \)  \( (1, 0) \)

b. \( x - y = 3 \)  \( x + y = 5 \)  \( (1, 1) \)

Do you UNDERSTAND?

b. Explain how to determine if a point is a solution to a system of inequalities.

8. A plumber charges \$30 for a house call and \$35 for each hour spent on the job. Write an equation to represent each system.

a. \( P = 30 + y \)  \( P = 35 + y \)  \( P = 30 \)  \( P = 35 \)  \( y = -2 \)  \( y = 1 \)  \( y = 2 \)  \( y = 3 \)  \( y = 4 \)

b. \( P = 30 + y \)  \( P = 35 + y \)  \( y = -2 \)  \( y = 2 \)

9. Write a matrix to represent each system.

Graph A

a. \( \begin{bmatrix} x \\ y \end{bmatrix} \)  \( \begin{bmatrix} 1 \\ 2 \end{bmatrix} \)

b. \( \begin{bmatrix} x \\ y \end{bmatrix} \)  \( \begin{bmatrix} 2 \\ 1 \end{bmatrix} \)

10. Write a matrix to represent each system.

Graph B

a. \( \begin{bmatrix} x \\ y \end{bmatrix} \)  \( \begin{bmatrix} 1 \\ 2 \end{bmatrix} \)

b. \( \begin{bmatrix} x \\ y \end{bmatrix} \)  \( \begin{bmatrix} 2 \\ 1 \end{bmatrix} \)

Do you UNDERSTAND?

b. Explain how to determine if a point is a solution to a system of equations.

11. A fruit market is selling oranges in 5 lb bags for \$6 and 10 lb bags for \$10. You spend \$60 and buy a total of 8 bags of oranges. Using a matrix, how many 5 lb bags and 10 lb bags of oranges did you buy?

Three 5 lb bags; five 10 lb bags; 65 lb
Do you know HOW?

7. You have 13 bills in your wallet in $1, $5, and $10 bills. There are twice as many $1 bills as $5 bills. The number of $10 bills is one more than the number of $5 bills. How many of each bill do you have? How much money do you have?

Divide each system by substitution or elimination.

Graph the solution of each system.

Graph the system of constraints. Identify all vertices. Find the values of \(x\) and \(y\) that maximize or minimize the objective function. Then find the maximum or minimum value.

Do you UNDERSTAND?

8. Reasoning: Is it possible for a dependent linear system to consist of two equations with different slopes? If so, a dependent system has two lines whose graphs are the same. If the lines have different slopes, then their graphs are not the same.

Answers may vary. Sample: $61

6 six $1 bills, three $5 bills, four $10 bills

Your friend picked Matrix B. Which of you is correct? What mistake was made?

C You are correct; your friend used 1 as the coefficient of a missing variable instead of 0.

8. The sum of these numbers is 21. The second number is two more than twice the first number. The second number is three times the third number. What are the numbers? First number: 5, second number: 12, third number: 4

Writing Explain how you determine whether a system of linear equations is independent, dependent, or inconsistent without graphing the lines.

Mechanic A charges $40 for car repairs and $40 for each hour spent on your car.

Mechanic B charges $10 for repairs and $60 for each hour spent on your car.

a. If your car takes 3 hours to repair, which mechanic charges the least money? Mechanic B

b. How much will it cost you to have the work done by the less expensive mechanic? $300

12. At a bookstore, you spend $76 on 11 books and magazines. Books cost $8 each and magazines cost $5 each. Write a matrix that represents this system. How many books and how many magazines did you buy?

13. Reasoning: The sum of these numbers is 15. The second number is twice the third number. Do you have enough information to determine the three numbers? If so, what are the three numbers? If not, what information do you still need? No; you need a third equation that defines another relationship between two or three of the numbers.

Do you UNDERSTAND?

Chapter 3 Quiz 2

Do you know HOW?

Find the values of \(x\) and \(y\) that maximize or minimize the objective function for each graph.

1. \(2x + 3y = 9\)

2. \(2y = x + 5\)

3. \(x + 2y = 7\)

Maximum for \(P = 2x - 3y\)

Minimum for \(P = x + 2y\)

Solution each system by substitution or elimination.

4. \(\begin{array}{c}2x - y = 1 \\
 \end{array}\)

5. \(\begin{array}{c}x = 2y = 0 \\
 \end{array}\)

6. \(\begin{array}{c}4x + 3y = 2 \\
 \end{array}\)

(0, 4)

(0, -3)

(2, -2)

Do you UNDERSTAND?

7. Error Analysis: To represent the system \(x + 2y - 6z = -11\), you picked Matrix A.

\[
\begin{pmatrix}
2 & 1 & -1 \\
1 & 2 & 0 \\
3 & 0 & -1
\end{pmatrix}
\]

You are correct; your friend used 1 as the coefficient of a missing variable instead of 0.

8. The sum of these numbers is 21. The second number is two more than twice the first number. The second number is three times the third number. What are the numbers? First number: 5, second number: 12, third number: 4

Chapter 3 Test [continued]
Chapter 3 Performance Tasks

Task 1
Make three systems of two equations/inequalities from any in the box.

\[\begin{align*}
    y &= 2x + 1 \\
    2x + 2y &= 2 \\
    x - y &= 4 \\
    y &= 6x - 3 \\
    y &= -2x + 4 \\
    x + y &= 1
\end{align*}\]

Use each method to solve one system. Show your work.

(a) graphing 
(b) substitution 
(c) elimination

Then find a system that has each of the following.

d. coincident lines  
e. intersecting lines  
f. parallel lines  
g. perpendicular lines

Explain your reasoning. Your models should present situations in which you make comparisons and draw conclusions.

(4) Check student’s work.

(3) Student writes, solves, and graphs systems and clearly demonstrates an in-depth understanding of the mathematical principles involved. A comparison is made, and the answer fully supports the conclusion.

(2) Student shows a solid understanding of the mathematical principles. A comparison is made, but further detail or more clarity is needed.

(1) Student shows a limited understanding of the mathematical principles involved. Situation does not present a comparison or draw a conclusion.

(0) Student makes no attempt, or no response is given.

Task 2
Your art club wants to sell greeting cards using members’ drawings. Small blank cards cost $10 per box of 25. Large blank cards cost $15 per box of 20. You make a profit of $3.20 per box of small cards and $4.00 per box of large cards. The club can buy no more than 250 total cards and spend no more than $350.

(a) How can the art club maximize its profit?
(b) The card company has a minimum order requirement of 5 boxes of each size per order. How can the art club maximize its profit?

Task 3
In the sale package, they are $1.10.

(a) Write a matrix to represent the system and solve. What does this solution tell you?
(b) How many packages of each kind of bead should you buy to make 20 necklace kits? Is $138 enough money to buy this many packages of beads?
(c) The first equation shows the number of beads needed for each necklace; the second equation shows that there are 4 times as many black beads as silver beads; no, this many packages costs $144.40.
(d) The second equation shows the cost per bead for each kind of bead and the amount to spend for each necklace; the third equation shows that there are 4 times as many black beads as crystal beads in each necklace.

(4) a. The first equation shows the number of beads needed for each necklace; the second equation shows the cost per bead for each kind of bead and the amount to spend for each necklace; the third equation shows that there are 4 times as many black beads as crystal beads in each necklace.

(3) Student’s explanation of each method is accurate and contains sufficient detail to indicate a clear understanding of the method. Student solves system correctly using all three methods. Steps could be explained more clearly or have more detail.

(2) Student’s explanation does not explain in detail each method. Student solves system correctly using only two methods. Explanation of the steps is not clear or missing important details.

(1) Student makes no attempt, or no response is given.

Task 4
What is the solution of the inequality 2(3 – x) < 10?

(a) Reflected across the y-axis, shift right one unit and down three units.

(4) a. The solution of the inequality 2(3 - x) < 10 is x > 2.

(3) 0.25x + 0.15y + 5.00x = 22

(2) a. Yes, she can make the purchase

(1) b. No, this does not meet the minimum order requirements

(0) c. All packages of silver beads, 7 packages of black beads, 5 packages of crystal beads

(4) a. The first equation shows the number of beads needed for each necklace; the second equation shows the cost per bead for each kind of bead and the amount to spend for each necklace; the third equation shows that there are 4 times as many black beads as crystal beads in each necklace.

(3) 0.25x + 0.15y + 5.00x = 22

(2) a. No, this does not meet the minimum order requirements

(1) b. No, this does not meet the minimum order requirements

(0) c. No, this does not meet the minimum order requirements

Chapter 3 Performance Tasks (continued)

Task 3
Explain in detail how to solve a system of equations with three variables by each method.

(a) graphing 
(b) substitution 
(c) elimination

Using each method, solve the following system.

\[\begin{align*}
    f + 7 &= 0 \\
    y - 3z &= 10 \\
    y &= 2z - 1
\end{align*}\]

(0) no answers given

(1) correct answers, but with one computational error

(2) incorrect equations OR incorrect matrix OR multiple computational errors

(3) Student began explaining equations OR began writing matrix, but did not solve or show understanding.

(4) Student made no effort to solve the problem.
Chapter 3 Project Teacher Notes: Hot! Hot! Hot!

About the Project
The Chapter Project gives students an opportunity to simulate a successful business by minimizing costs, maximizing profits, and establishing a process for filling orders promptly. Students research costs, study profit margins, and establish selling prices. Students track inventory by designing a spreadsheet.

Introducing the Project
• Encourage students to keep all project-related materials in a separate folder.
• Ask students to graph a system of inequalities. Investigate how solutions to this system could be used to maximize profits.
• Discuss the information needed to keep track of a company’s filled orders and available stock, and the best way to organize that information on a spreadsheet.

Activity 1: Graphing
Students write and graph a system of inequalities. Then they determine how many pints of each sauce they can make.

Activity 2: Analyzing
Students use their answers from Activity 1 to determine how much of each sauce they should produce to maximize profit. Students use this information to find the maximum profit.

Activity 3: Researching
Students visit a grocery store to estimate the cost of each ingredient of the sauces. Students use this information to find the cost to produce one pint of each type of sauce. Then students determine the selling price that will maintain the same profit margin.

Activity 4: Organizing
Students determine a production schedule to meet demand given that the company will produce the same amount of sauce each week. Students prepare a spreadsheet to keep track of filled orders and available stock.

Finishing the Project
You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results.

Reflection and Revise
Present your analysis to a small group of classmates. After you have heard their analysis and presented your own, decide if your work is complete, clear, and convincing. If needed, make changes to improve your presentation.

Extending the Project
Are there other expenses you could expect in addition to those you have already considered? Estimate them. Modify your recommendations if necessary.

Chapter 3 Project: Hot! Hot! Hot!

Getting Started
Read the project. As you work on the project, you will need a calculator, graph paper, materials to record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist
☐ Activity 1: setting up graphing inequalities
☐ Activity 2: maximizing profit
☐ Activity 3: determining selling price
☐ Activity 4: designing production schedule
☐ profit analysis

Scoring Rubric
4 Correct inequalities are written and graphed. Calculations and graphs are accurate. The folder is well organized and provides useful information.
3 Minor errors are made. Reasoning and explanations are essentially correct. Graphs contain minor errors in scale or are labeled inaccurately. The folder provides useful information, but needs to be better organized.
2 Inaccurate inequalities are written and graphed. The folder lacks organization. Graphs could be neater and more accurate. Explanations lack detail.
1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project:
Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of the Project

Chapter 3 Project Manager: Hot! Hot! Hot!

Activity 1: Graphing
To fill an order for Sizzlin’ Sauce sauces, you bought 1050 green peppers and 1200 hot chili peppers.
• Write and graph a system of inequalities to represent the number of pints of each kind of sauce you can make. Refer to the recipes above.
• Select one solution of the system and determine how many peppers you will have left over.

Red Hot Sauce
Scorchin’ Hot Sauce

Yield: 1 pint
1 pt tomato sauce with onions
5 green peppers, diced
5 hot chili peppers, seeded and diced
1 pt tomato sauce with onions
4 green peppers, diced
5 hot chili peppers, seeded and diced

Answers may vary. Sample: If 150 pt of each sauce are produced, there will be 150 green peppers left over.

Activity 2: Analyzing
Suppose you make $1.20 profit on 1 pint of Red Hot Sauce and $1.00 profit on 1 pint of Scorchin’ Hot Sauce. Using the restrictions from Activity 1, decide how much of each sauce you should make and sell to maximize your profit. What is the maximum profit?

Red Hot Sauce: 150 pt, Scorchin’ Hot Sauce: 75 pt; $225 profit

Activity 3: Researching
Visit a local grocery store to estimate the cost of each sauce ingredient. Remember that buying in large quantities can save you money.
• What selling price will you set for each sauce to maintain your profit?
• Find the cost of producing 1 pt of each type of sauce.

Answers may vary. Sample: If 100 pt of each sauce are produced, there will be 100 green peppers left over.

Activity 4: Organizing
Students use their answers from Activity 1 to determine how much of each sauce they should produce to maximize profit. Students use this information to find the cost of producing 1 pt of each type of sauce. Then students determine the selling price that will maintain the same profit margin.

Answers may vary. Sample: If 100 pt of each sauce are produced, there will be 100 green peppers left over.

... (Continued)
4-1 ELL Support
Quadratic Functions and Transformations

Concept List
Choose the concept from the list below that best represents the item in each box.

<table>
<thead>
<tr>
<th>Axis of symmetry</th>
<th>Parabola</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value</td>
<td>Parent quadratic function</td>
<td>Vertex form</td>
</tr>
<tr>
<td>Minimum value</td>
<td>Quadratic function</td>
<td>Vertex of the parabola</td>
</tr>
</tbody>
</table>

1. $y = ax^2 + bx + c$
2. A line that divides a parabola into two mirror images
3. $y = x^2$
4. $(a, k)$ where $y = ax^2 + bx + k$
5. The $y$-value of the vertex when the parabola opens up
6. $y = x^2$
7. The $y$-value of the vertex when the parabola opens down
8. $y = (x - h)^2 + k$
9. A shift of the graph horizontally or vertically translated

4-1 Practice
Quadratic Functions and Transformations

Graph each function.

1. $y = x^2$
2. $f(x) = -x^2$
3. $y = \frac{1}{2}x^2$
4. $f(x) = \frac{1}{3}x^2$
5. $f(x) = \frac{4}{3}x^2$
6. $f(x) = \frac{1}{4}x^2$
7. $f(x) = x^2 + 4$
8. $f(x) = (x - 3)^2$
9. $y = (x - 2)^2 + 3$
10. $f(x) = -0.5x^2 + 3x + 2$
11. $y = (x + 2)^2 - 1$
12. $y = -4x - 3x^2 + 2$

Write a quadratic function to model each graph.

13. $y = (x - 2)^2 + 3$
14. $y = 2x^2 + 3x - 1$

4-1 Practice (continued)
Quadratic Functions and Transformations

Describe how to transform the parent function $y = x^2$ to the graph of each function below. Graph both functions on the same axes.

15. $y = 3(x + 2)^2$
16. $y = -(x + 5)^2 + 1$
17. $y = \frac{1}{2}x - 4x^2 - 2$
18. $y = -0.04(x - 0.46)^2 + 2$

19. Write the equation of each parabola in vertex form.
   a. vertex $(3, -2)$, point $(2, 3)$
   b. vertex $(1, -3)$, point $(3, 4)$
   c. vertex $(-1, 3)$, point $(2, 8)$
20. $y = x^2 + 4x + 4$
21. $y = -4x - x^2 + 1$
22. $y = 2x + 12x^2 + 35.5$
23. The amount of cloth used to make four curtains is given by the function $A = -4x^2 + 4x$, where $x$ is the width of one curtain in feet and $d$ is the total area in square feet. Find the width that maximizes the area of the curtain. What is the maximum area? 5 ft; 100 ft²
24. The diagram shows the path of a model rocket launched from the ground. It reaches a maximum altitude of 396 ft when it is above a location 10 ft from the launch site. What quadratic function models the height of the rocket? $A = -1.5x^2 - 10x + 364$
25. To make an enclosure for chickens, a rectangular area will be fenced next to a house. Only one side will need to be fenced. There is 120 ft of fencing material.
   a. What quadratic function represents the area of the rectangular enclosure, where $x$ is the distance from the house? $A = -2x^2 + 120x$
   b. What dimensions will maximize the area of the enclosure? 30 ft; 600 ft²

4-1 Think About a Plan
Quadratic Functions and Transformations

Write a quadratic function to represent the areas of all rectangles with a perimeter of 36 ft. Graph the function and describe the rectangle that has the largest area.

1. Write an equation that represents the area of a rectangle with a perimeter of 36 ft. Let $x$ = width and $y$ = length. $2x + 2y = 36$
2. Solve your equation for $y$. $y = 18 - x$
3. Write a quadratic function for the area of the rectangle.

Answers may vary. Sample: Read the $x$-coordinate of the vertex from the graph and then substitute that value into the quadratic function to get the $y$-value. $(9, 81)$

7. Describe the rectangle that has the largest area. What is its area?
   The rectangle that has the largest area is a square with length 9 ft; 81 ft²
10. What are the vertex, the axis of symmetry, the maximum or minimum value, the domain, and the range of each function?

For Exercises 1-4, choose the correct letter.

Multiple Choice

1. What is the vertex of the function $f(x) = 3x^2 - 7x + 4$?
   - A. $(7, -4)$
   - B. $(-7, 4)$
   - C. $(7, -4)$
   - D. $(7, 4)$

2. Which is the graph of the function $f(x) = -2(x - 3)^2 + 51$?
   - A. Graph A
   - B. Graph B
   - C. Graph C
   - D. Graph D

3. Which of the following best describes how to transform $y = x^2$ to the graph of $y = 4x^2 - 25x - 37$?
   - A. Translate 2 units left, stretch by a factor of 4, translate 3 units down.
   - B. Translate 2 units right, stretch by a factor of 4, translate 3 units down.
   - C. Stretch by a factor of 4, translate 2.5 units left and 3 units down.
   - D. Stretch by a factor of 4, translate 2.5 units left and 3 units down.

4. What is the equation of the parabola with vertex $(-4, 6)$ passing through the point $(-2, -27)$?
   - A. $y = -2(x + 4)^2 - 6$
   - B. $y = 2(x - 4)^2 + 6$
   - C. $y = -2(x - 4)^2 + 6$
   - D. $y = 2(x + 4)^2 + 6$

Short Response

5. A baseball is hit so that its height $h$ above ground is given by the equation $h = -16t^2 + 160t + 64$, where $h$ is the height in feet and $t$ is the time in seconds after it is hit. Show your work.

a. How long does it take the baseball to reach its highest point?
   - $t = \frac{-b}{2a}$

b. How high will it go?
   - $h = \frac{-b^2}{4a}$

6. Write an equation expressing the relationship between $x$ and $y$.

Write an equation expressing the relationship between $u$ and $v$.

7. Write an equation expressing the relationship between $x$ and $y$.

8. Use these relationships to write an equation of the parabola $y = av^2$ in terms of $u$ and $v$. $x = u + b(v - k)^2$

9. Expand and simplify your equation to express $y$ as a quadratic function of $v$.

If we let $h = 2at^2 + vt + c$, the parabola represented by the quadratic equation $y = av^2$ is equivalent to the parabola $y = av^2 + h$ in the $y = ax^2$ system.

10. In the $x = av^2$ system, express the coordinates of the vertex of this parabola in terms of $a$, $b$, and $c$. What is the equation of its axis of symmetry?
4-1 Reteaching
Quadratic Functions and Transformations

Parent Quadratic Function
The parent quadratic function is \( y = x^2 \).

Substitute 0 for \( x \) in the function to get \( y = 0 \). The vertex of the parent quadratic function is \((0, 0)\).

A few points near the vertex are:

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

The graph is symmetrical about the line \( x = 0 \). This line is the axis of symmetry.

Vertex Form of a Quadratic Function
The vertex form of a quadratic function is \( y = a(x - h)^2 + k \).

The graph of this function is a transformation of the graph of the parent quadratic function \( y = x^2 \). The vertex of the graph is \((h, k)\).

Problem
What is the greatest area the town can fence in using 100 ft of fencing?

Step 1 Write the function in vertex form.

Step 2 Find the vertex. \( h = 3 \), \( k = 2 \). The vertex is \((-3, 2)\).

Step 3 Find the axis of symmetry. Since the vertex is \((-3, 2)\), the graph is symmetrical about the line \( x = -3 \). The axis of symmetry is \( x = -3 \).

Step 4 Because \( a = 1 \), you can graph this function by sliding the graph of the parent function 3 units left and 2 units up along the \( x \)-axis and 2 units up along the \( y \)-axis. Plot a few points near the vertex to help you sketch the graph.

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4-2 ELL Support
Standard Form of a Quadratic Function

Standard Form of a Quadratic:
The standard form of a quadratic is \( f(x) = ax^2 + bx + c \), where \( a \neq 0 \).

1. Which function is written in standard form? \( f(x) = 4x - 3y - 1 \) \( f(x) = 3a^2 - x + 4 \)

2. Which has an \( x \)-coordinate of \(-5\)? \((-5, 2)\) \((2, -5)\)

3. Which quadratic has a \( y \)-intercept of \(-2\)? \( f(x) = -x^2 + 5y + 2 \) \( f(x) = -2y + 5x + 2 \)

4. Which parabola opens downward? \( f(x) = -4x^2 - 1y^2 + 5 \) \( f(x) = 4x^2 + 1 \)

Vertex Form of a Quadratic:
The vertex form of a quadratic is \( f(x) = a(x - h)^2 + k \), where \( a \neq 0 \).

5. Which function is written in vertex form? \( f(x) = -2(x - 3y)^2 + 3 \) \( f(x) = -x^2 + 2x + 6 \)

6. Which quadratic has a vertex of \((-2, 3)\)? \( f(x) = x^2 - 2x + 4 \) \( f(x) = x^2 + 3y + 4 \)

7. Which parabola opens upward? \( f(x) = 2(x - 3y)^2 + 1 \) \( f(x) = -4x^2 - 1y^2 - 9 \)

Write the vertex form of the quadratic written in standard form below.

8. \( f(x) = x^2 - 6x + 12 \) \( f(x) = (x - 3)^2 + 3 \)

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4-1 Reteaching (continued)
Quadratic Functions and Transformations

If \( a \neq 1 \), the graph is a stretch or compression of the parent function by a factor of \(|a|\).

| \( a \) | \( |a| > 1 \) | \( |a| < 1 \) 
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>The graph is a horizontal compression of the parent function.</td>
<td>The graph is a horizontal stretch of the parent function.</td>
<td></td>
</tr>
</tbody>
</table>

Problem
What is the graph of \( y = 2(x + 3)^2 + 2 \)?

Step 1 Write the function in vertex form: \( y = 2(x + 3)^2 + 2 \)

Step 2 The vertex is \((-3, 2)\).

Step 3 The axis of symmetry is \( x = -3 \).

Step 4 Because \( a = 2 \), the graph of this function is a horizontal compression by 2 of the parent function. In addition to sliding the graph of the parent function 3 units left and 2 units up, you must change the shape of the graph. Plot a few points near the vertex to help you sketch the graph.

Page 10

4-2 Think About a Plan
Standard Form of a Quadratic Function

Landscape: A town is planning a playground. It wants to fence in a rectangular space using an existing wall. What is the greatest area it can fence in using 100 ft of donated fencing?

Understanding the Problem
1. Write an expression for the width of the playground. Let \( l \) be the length of the playground.

2. Do you know the perimeter of the playground? Explain?

3. What is the problem asking you to determine?

Planning the Solution
4. Write a quadratic equation to model the area of the playground.

5. What information can you get from the equation to find the maximum area? Explain.

6. What is the width of the largest rectangle that can be enclosed by 100 ft of fence on three sides and an unknown length of wall on the fourth side?

Getting an Answer
6. What is the length of the fence that produces the maximum area? 25 ft

7. What is the greatest area the town can fence in using 100 ft of fencing? 1250 ft^2

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13. What is the vertex, the axis of symmetry, the maximum or minimum value, and the range of each parabola?

1. \( y = -x^2 + 4x + 5 \)
   - Vertex: \((2, -3)\), axis of symmetry: \(x = 2\), range: \(y \leq -3\)
   - Vertex is the minimum; \(y\) goes to \(-\infty\) as \(x\) goes to \(-\infty\) or \(+\infty\)

2. \( y = -x^2 + 2x + 3 \)
   - Vertex: \((1, 3)\), axis of symmetry: \(x = 1\), range: \(y \geq 3\)
   - Vertex is the maximum; \(y\) goes to \(-\infty\) as \(x\) goes to \(-\infty\) or \(+\infty\)

3. \( y = -2x^2 + 4x - 3 \)
   - Vertex: \((1, 2)\), axis of symmetry: \(x = 1\), range: \(y \leq 2\)
   - Vertex is the minimum; \(y\) goes to \(-\infty\) as \(x\) goes to \(-\infty\) or \(+\infty\)

4. \( y = x^2 + 1x + 2 \)
   - Vertex: \((-0.5, 0.75)\), axis of symmetry: \(x = -0.5\), range: \(y \leq 0.75\)
   - Vertex is the minimum; \(y\) goes to \(-\infty\) as \(x\) goes to \(-\infty\) or \(+\infty\)

5. \( y = x^2 + 3x - 5 \)
   - Vertex: \((-1.5, 22.25)\), axis of symmetry: \(x = -1.5\), range: \(y \geq 22.25\)
   - Vertex is the minimum; \(y\) goes to \(-\infty\) as \(x\) goes to \(-\infty\) or \(+\infty\)

Graph each function.

7. \( y = x^2 + 2x - 5 \)
8. \( y = -x^2 + 3x + 1 \)
9. \( y = 2x^2 + 4x - 4 \)
10. \( y = -\frac{1}{2}x^2 - 3x + 3 \)
11. \( y = 3x^2 + 8x \)
12. \( y = -3x^2 + 18x - 27 \)

Write each function in vertex form.

13. \( y = x^2 - 8x + 9 \)
   - \(y = (x - 4)^2 + 5\)

14. \( y = x^2 - 2x - 6 \)
   - \(y = (x - 1)^2 - 7\)

15. \( y = x^3 + 3x \)
   - \(y = x(x + 2)(x - 1)\)

16. \( y = 2x^2 + x + 1 \)
   - \(y = 2(x + 0.5)^2 - 0.25\)

17. \( y = 2x^2 - 12x + 11 \)
   - \(y = 2(x - 3)^2 - 4\)

18. \( y = -2x^2 + 4x + 4 \)
   - \(y = -2(x - 1)^2 + 6\)

19. The Gateway Arch in St. Louis was built in 1965. It is the tallest monument in the United States. The arch can be modeled with the function \( y = -0.00315x^2 + 4x \) where \(y\) is in feet.
   a. How high above the ground is the tallest point of the arch? 630 ft
   b. How far apart are the legs of the arch at their bases? 630 ft

20. Sketch each parabola using the given information.

   a. vertex \((-2, 1)\), \(y\)-intercept \(4\)
   b. vertex \((3, -5)\), \(y\)-intercept \((4, 1)\)
   c. vertex \((-1, -7)\), point \((-6, 1)\)
4-2 Standardized Test Prep

Standard Form of a Quadratic Function

Multiple Choice
For Exercises 1–6, choose the correct letter.
1. What is the vertex of the parabola \( y = x^2 + 8x + 57 \)?
   \( (-4, 11) \) \( (-4, -11) \) \( (-4, 5) \) \( (4, 5) \)

2. What is the minimum value of the function \( f(x) = -3x^2 + 12x - 8 \)?
   \( -8 \) \( 0 \) \( 2 \) \( 4 \)

3. Which function has the graph shown at the right? \( \star \)
   \( y = -x^2 + 5x + 1 \) \( y = 2x^2 - 5x - 1 \) \( y = 2x^2 + 5x - 1 \) \( y = -2x^2 - 5x - 1 \)

4. What is the vertex form of the function \( f(x) = 3x^2 - 12x + 17 \)? \( \star \)
   \( f(x) = 3(x - 2)^2 + 5 \) \( f(x) = 3(x - 2)^2 + 17 \) \( f(x) = 3(x + 2)^2 + 5 \) \( f(x) = 3(x + 2)^2 + 17 \)

5. What is the equation of the parabola with vertex \((3, -2)\) and that passes through the point \((7, 12)\)? \( \star \)
   \( y = -(x - 3)^2 + 2 \) \( y = 2(x - 3)^2 - 2 \) \( y = -(x + 3)^2 + 2 \) \( y = 2(x + 3)^2 - 2 \)

6. For the function \( f(x) = -5x^2 - 10x + c \), the vertex is \((-1, 8)\). What is \( c \)? \( \star \)
   \( -15 \) \( -5 \) \( -3 \) \( 13 \)

Short Response
7. To increase revenue, a county wants to increase park fees. The overall income will go up, but there will be expenses involved in collecting the fees. For a \( p \% \) increase in the fees, this cost \( C \) will be \( C = 0.6p^2 - 7.2p + 48 \), in thousands of dollars. What percent increase will minimize the cost to the county? Show your work.

8. A parabola is described by \( f(x) = 2x^2 - 4x - 1 \). What is the standard form of \( f(x) \)?

9. The standard form of a quadratic function, \( y = ax^2 + bx + c \), is useful, but it has the disadvantage that only one of the three constants has a simple geometrical interpretation.
   1. Which of the constants in the equation \( y = ax^2 + bx + c \) can be interpreted geometrically?
   2. What is it geometrical interpretation? The \( \star \)

A more intuitive equation is expressed in terms of constants that have geometrical interpretations. For instance, if \( f(x) = a(x - p)^2 + q \), the vertex \((p, q)\) as \( (p, q) \) is \( (p, q) \). See \( (p, q) \). Decribe a parabola with an axis of symmetry parallel to the \( y \)-axis. Using this equation, write the equation of the following parabolas in vertex form.

10. \( y \)-intercept \( \star \) vertex
   3. \(-2\) \((1, 2)\) \( y = x^2 - 4x - 4 \)
   4. \(-3, 8\) \( y = x^2 + 2x + 8 \)

The equation \( y = \frac{1}{2}(x - 3)^2 \) describes a parabola with an axis of symmetry parallel to the \( y \)-axis and passes through the point \((1, 2)\). Write the equation of the following parabolas.

11. \( y \)-intercepts \( \star \) vertex
   5. \(-4, 0\) \((1, 0)\) \( y = x^2 + 4x - 6 \)
   6. \(-4, 2\) \((2, 4)\) \((0, 0)\) \( y = x^2 - 4x - 6 \)

12. Why is this a special case of the previous equation? Answers may vary. Sample: It is the previous equation with \( p = 0 \).

4-2 Enrichment

Standard Form of a Quadratic Function

The standard form of a quadratic function, \( y = ax^2 + bx + c \), is useful, but it has the disadvantage that only one of the three constants has a simple geometrical interpretation.

1. Which of the constants in the equation \( y = ax^2 + bx + c \) can be interpreted geometrically?

A more intuitive equation is expressed in terms of constants that have geometrical interpretations. For instance, if \( f(x) = a(x - p)^2 + q \), the vertex \((p, q)\) is \( (p, q) \). Describe a parabola with an axis of symmetry parallel to the \( y \)-axis. Using this equation, write the equation of the following parabolas in vertex form.

13. \( y \)-intercept \( \star \) vertex
   3. \(-2\) \((1, 2)\) \( y = x^2 - 4x - 4 \)
   4. \(-3, 8\) \( y = x^2 + 2x + 8 \)

The equation \( y = \frac{1}{2}(x - 3)^2 \) describes a parabola with an axis of symmetry parallel to the \( y \)-axis and passes through the point \((1, 2)\). Write the equation of the following parabolas.

14. \( y \)-intercepts \( \star \) vertex
   5. \(-4, 0\) \((1, 0)\) \( y = x^2 + 4x - 6 \)
   6. \(-4, 2\) \((2, 4)\) \((0, 0)\) \( y = x^2 - 4x - 6 \)

Once the equation to vertex form is complete, check by multiplying.

15. \( y = 3(x - 4)^2 + 1 \) \( y = x^2 - 4x + 1 \) \( y = x^2 - 4x + 10 \)

The result is the standard form of the equation.

Exercises

Write each function in vertex form. Check your answers.

16. \( y = x^2 - 4x + 10 \) \( y = 2(4x - 1)^2 - 3 \) \( y = x^2 - 4x + 1 \) \( y = x^2 - 6x + 10 \)
A football player kicks a football and records the height of the ball at different times. When kicked, it was at a height of 2 ft above the ground. One second later the ball was 28 ft above the ground, and 2 seconds after being kicked the ball was 20 ft above the ground. When will the ball hit the ground?

You were given these steps to solve the problem on note cards, but you got mixed up.

1. Substitute the x and y values into the standard form of a quadratic function.
2. Solve the system of three linear equations.
3. Substitute the values of a, b, and c into the standard form of a quadratic function.
4. Use the quadratic model to determine when the ball hits the ground.

Use the note cards to write the steps in order.

1. First, substitute the x and y values into the standard form of a quadratic function.
2. Second, solve the system of three linear equations.
3. Next, substitute the values of a, b, and c into the standard form of a quadratic function.
4. Finally, use the quadratic model to determine when the ball hits the ground.

4-3 Practice
Modeling With Quadratic Functions

Find an equation in standard form of the parabola passing through the points.

1. (1, 0), (2, 0), (3, 0), (4, 0)
   \[y = x^2 - 4x + 5\]
   
2. (1, 2), (2, 3), (3, 4), (4, 5)
   \[y = x^2 - 6x + 7\]
   
3. (1, -1), (2, -2), (3, -3), (4, -4)
   \[y = x^2 - 2x + 4\]
   
4. (1, -1), (2, -2), (3, -3), (4, -4), (5, -5)
   \[y = x^2 - 3x + 2\]
   
5. (1, -1), (2, -4), (3, -9), (4, -16)
   \[y = x^2 - 5x + a\]
   
6. (1, -1), (2, -4), (3, -9), (4, -16), (5, -25)
   \[y = x^2 - 6x + b\]
   
7. (1, 1), (2, 4), (3, 9), (4, 16)
   \[y = x^2 - 2x + 1\]
   
8. (1, 1), (2, 4), (3, 9), (4, 16), (5, 25)
   \[y = x^2 - 3x + 2\]
   
9. (1, 2), (2, 3), (3, 4)
   \[y = x^2 - 5x + 2\]
   
10. (1, 2), (2, 3), (3, 4), (4, 5)
    \[y = x^2 - 6x + 1\]

11. (1, 1), (2, 2), (3, 3)
    \[y = x^2 + 1\]

12. (1, 2), (2, 3), (3, 4)
    \[y = 2x - 3\]

13. The table shows the number n of tickets to a school play sold 5 days after the tickets went on sale, for several days.
   a. Find a quadratic model for the data.
   b. Use the model to find the number of tickets sold on day 7.
   c. When was the greatest number of tickets sold? day 6

14. The table gives the number of pairs of skis sold in a sporting goods store for several months last year.
   a. Find a model for the data, using January as month 1, February as month 2, and so on. \(x = 10, 20 \Rightarrow y = 100, 200 + 100\)
   b. Use the model to predict the number of pairs of skis sold in November.
   c. In what month was the fewest skis sold? July
4-3 Practice (continued)  Form K

Modeling With Quadratic Functions

10. The table at the right shows average retail gasoline prices. Find a quadratic model for each data range: 1976 to 1980, 1980 to 1985, 1985 to 2000, and 2000 to 2004. Then determine whether a quadratic model exists for each set of values. If so, write the model.

$\begin{array}{ccc}
\text{Year} & \text{Price per gallon (cents)} \\
1976 & 34.4 \\
1980 & 39.7 \\
1985 & 41.3 \\
2000 & 170.07 \\
2004 & 182.140 \\
\end{array}$

Determine whether a quadratic model exists for each set of values. If so, write the model.

b. Use the model to estimate the average retail gasoline price in 2000. 170.07 cents

11. f(0) = 5, f(4) = 13, f(2) = 7
   a. y = x^2 + 2x + 5
   b. No, y = x^2 - 8x - 2

12. f(1) = 1, f(-1) = -19, f(3) = -9
   a. y = x^2 + 5
   b. No

13. f(6) = 0, f(1) = 2, f(2) = 4
   a. No
   b. y = x^2 + 3x - 2

14. f(-5) = 0, f(-2) = 6, f(0) = -2
   a. No
   b. y = x^2 - 3x - 2

15. The table at the right shows in thousands how many people in the U.S. subscribe to a cellular telephone.

<table>
<thead>
<tr>
<th>Year</th>
<th>Telephone Subscribership (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>340</td>
</tr>
<tr>
<td>1990</td>
<td>528</td>
</tr>
<tr>
<td>1995</td>
<td>1,280</td>
</tr>
<tr>
<td>2000</td>
<td>3,607</td>
</tr>
<tr>
<td>2004</td>
<td>182,140</td>
</tr>
</tbody>
</table>

Determine whether a quadratic model exists for each set of values. If so, write the model.

a. y = x^2 + 5
   b. No

16. Error Analysis. In Exercise 13 part (c), your friend said that the range was equal to all real numbers. Why is this incorrect?
   Because this is a real situation, you cannot have a negative number of subscribers. Therefore, the range must be greater than or equal to 0. (It cannot be negative.)

17. Reasoning. Explain how you know your answer to Exercise 15 part (b) is reasonable.
   The number of subscribers found in 1995 is reasonable because it is in between the values for 1990 and 2000 from the table.

4-3 Practice  Form K

Modeling With Quadratic Functions

4-3 Standardized Test Prep  Modeling With Quadratic Functions

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Which parabola passes through the points (1, -2), (4, 1), and (5, -2)?
   a. $y = x^2 + 2x + 5$
   b. $y = x^2 - 8x - 2$
   c. $y = x^2 - 3x - 2$

2. Which parabola passes through the points in the table at the right?
   a. $y = x^2 + 2x + 5$
   b. $y = x^2 - 8x - 2$
   c. $y = x^2 - 3x - 2$

3. A baseball coach records the height at every second of a ball thrown in the air. Some of the data appears in the table below.

   a. $y = x^2 + 2x + 5$
   b. $y = x^2 - 8x - 2$
   c. $y = x^2 - 3x - 2$

4. The accountant for a small company studied the amount spent on advertising and the company’s profit for several years. He made the table below. What is a quadratic model for the data? Show your work.

   a. $y = x^2 + 2x + 5$
   b. $y = x^2 - 8x - 2$
   c. $y = x^2 - 3x - 2$

5. Which of the following sets of values cannot be modeled with a quadratic function?
   a. (0, 0), (1, 0), (2, 7)
   b. (0, 0), (1, 0), (2, 4), (3, 2), (4, 3)
   c. (0, 0), (1, 0), (2, 4), (3, 4), (4, 3)

6. If $(f(-1) = -2, f(0) = 3)$, which of the following could be the vertex of the parabola?
   a. $(0, 3)$
   b. $(1, 3)$
   c. $(2, 3)$

7. A player hits a tennis ball across the court and records the height of the ball at different times, as shown in the table.

   a. $y = x^2 + 2x + 5$
   b. $y = x^2 - 8x - 2$
   c. $y = x^2 - 3x - 2$

8. Reasoning. Explain why the parabola model only works up to 4.5 seconds — that is, the ball hits the ground in 4.5 seconds. After it hits the ground, the ball cannot go any lower.

9. The table at the right shows the height of the tide measured at the Santa Monica Municipal Pier in California. Tides are measured from 0.00 at midnight.

   a. Find a quadratic model for this data using quadratic regression. $y = 0.00067x^2 + 0.624x + 4.1$
   b. Use the model to predict the lowest tide height. 2.5 ft
   c. When does the lowest tide occur? 4:10 after midnight

10. Determine whether a quadratic model exists for each set of values. If so, write the model.

   a. $y = x^2 + 5$
   b. No

11. Which parabola passes through the points in the table at the right?

   a. $y = x^2 + 2x + 5$
   b. $y = x^2 - 8x - 2$
   c. $y = x^2 - 3x - 2$


   a. $y = x^2 + 5$
   b. No


   a. $y = x^2 + 5$
   b. No


   a. $y = x^2 + 5$
   b. No

15. The table at the right shows in thousands how many people in the U.S. subscribe to a cellular telephone.

   a. Find a quadratic model for the data. $y = x^2 + 5$
   b. Use the model to estimate the number of subscribers in 1995. 182,140

16. Error Analysis. In Exercise 13 part (c), your friend said that the range was equal to all real numbers. Why is this incorrect?

   a. $y = x^2 + 5$
   b. No

17. Reasoning. Explain how you know your answer to Exercise 15 part (b) is reasonable.

   a. $y = x^2 + 5$
   b. No

4-3 Enrichment  Modeling With Quadratic Functions

Baseballs in Flight

When baseballs are shot out of a cannon, their flight through the air depends on both the angle at which the cannon is set and the initial velocity of the baseball.

The equation $y = 0.5x - 0.12x^2$ represents the parabolic flight of a certain baseball shot at an angle of 20° with the horizon and at an initial velocity of 25 meters per second. In this equation, $y$ is the height of the baseball, in meters, and $x$ is the horizontal distance traveled, in meters. The graph of the equation is shown to the right.

1. Given that the points (10, 4) and (40, 4) lie on the parabola, at what $x$-coordinate must the vertex lie? $x = 25$

2. Use the equation and your answer to question 1 to find the maximum height of the baseball. 62.5 m

3. Use the point (9, 0) and the location of the vertex to find the total horizontal distance that the baseball will travel. 50 m

4. When the angle of the cannon is decreased, the baseball will travel in a different flight. The parabolic flight of the baseball is shown to the right, with the vertex labeled.

5. What is the total horizontal distance that this baseball will travel? 25 m

Using the same angle, the initial velocity of the baseball is increased to produce the graph of the flight shown to the right. The point shown represents the total horizontal distance the baseball will travel.

6. How far will the baseball travel horizontally before it reaches its maximum height? 25 m

4-3 Reteaching
Modeling With Quadratic Functions

These non-collinear points, no two of which are in line vertically, are on the graph of exactly one quadratic function.

Problem
A parabola contains the points $(0, -2)$, $(1, 5)$, and $(2, 2)$. What is the equation of this parabola in standard form?

If the parabola contains the given points, what is its equation?

To solve the problem I need to:

1. Set up a system of equations to find $a$, $h$, $k$.
2. Substitute the values into the standard form $y = a(x-h)^2 + k$ to write a system of equations.
3. Solve the system to find $a$, $h$, and $k$.
4. Substitute these values into the standard form $y = a(x-h)^2 + k$. Use elimination to solve the system and obtain $a = 3$, $b = -4$, and $c = -2$. Substitute these values into the standard form $y = ax^2 + bx + c$.

The equation of the parabola that contains the given points is $y = 3x^2 - 4x - 2$.

Exercises
Find an equation in standard form of the parabola passing through the given points.

1. $(0, 1), (1, 4), (3, 22)$
2. $(0, 2), (1, 5), (2, 1)$

4-4 ELL Support
Factoring Quadratic Expressions

Choose the word from the list that best matches each sentence.

<table>
<thead>
<tr>
<th>factoring</th>
<th>greatest common factor</th>
<th>perfect square</th>
<th>trinomial</th>
<th>difference of two squares</th>
</tr>
</thead>
</table>

1. the expression $a^2 - b^2$ is a difference of two squares.
2. rewriting an expression as a product of its factors
3. a trinomial that is the square of a binomial
4. a common factor in each term of the expression

Choose the word from the list that best matches each sentence.

<table>
<thead>
<tr>
<th>factoring</th>
<th>greatest common factor</th>
<th>perfect square</th>
<th>trinomial</th>
<th>difference of two squares</th>
</tr>
</thead>
</table>

5. 10 is the greatest common factor of the expression $20x^2 - 50$.
6. An example of a perfect square trinomial is $x^2 - 8x + 16$.
7. When factoring $x^2 + 8x + 15$, find numbers with product 15 and sum 8.
8. The difference of two squares will always be a binomial.

Multiple Choice
9. Which of the following is a perfect square trinomial?
   - $2x^2 - 7$
   - $9x^2 - 6x + 1$
   - $4x^2 - 25$
   - $9x^2 - 4x$

10. Which of the following is a difference of perfect squares?
    - $2x^2 - 7$
    - $9x^2 - 6x + 1$
    - $4x^2 - 25$
    - $9x^2 - 4x$

ANSWERS
Factor each expression.

1. \(x^2 + 11x + 28\)  
2. \(x^2 + 11x + 24\)  
3. \(x^2 + 13x + 42\)  
4. \(x^2 + 21x + 56\)  
5. \(x^2 + 8x + 15\)  
6. \(x^2 + 12x + 32\)  
7. \(-x^2 - 9x - 18\)  
8. \(-x^2 - 12x - 35\)  
9. \(-x^2 - 3x - 14\)  

Find the GCF of each expression. Then factor the expression.

11. \(2x^2 - 3\)  
12. \(-10x^2 + 8\)  
13. \(2x^2 + 22x + 60\)  
14. \(5x^2 + 25x + 70\)  
15. \(2x^2 - 11x + 4\)  
16. \(-7x^2 + 7x - 14\)  
17. \(10x^2 - 11x + 3\)  
18. \(10x^2 + 13x - 3\)  
19. \(2x^2 - 16 - 12x\)  
20. \(2x^2 - 12x + 36\)  
21. \(9x^2 - 6x + 1\)  
22. \(6x^2 + 12x + 9\)  
23. \(5x^2 + 30x - 50x + 15\)  

Find the area of a rectangular field is \(x^2 - x - 72\) m². The length of the field is \(x + 8\) m. What is the width of the field in meters?  

Factor each expression.

24. \(x^2 + 6x + 9\)  
25. \(x^2 + 12x + 35\)  
26. \(x^2 + 21x + 56\)  
27. \(-x^2 + 3x + 10\)  
28. \(-x^2 - 12x - 35\)  
29. \(-x^2 - 10x - 24\)  
30. \(-x^2 + 2x + 30\)  

Find the GCF of each expression. Then factor the expression.

31. \(10x^2 + 12x + 12\)  
32. \(-5x^2 + 30\)  
33. \(5x^2 + 10x + 8\)  
34. \(10x^2 + 12x + 8\)  
35. \(15x^2 + 30x + 15\)  
36. \(10x^2 + 100x\)  
37. \(10x^2 + 30x + (x - 1)\)  
38. \(-5x^2 - 30x + (-1)\)  

Writing  

When you factor a quadratic expression, explain what it means when  

\(c < 0\) and \(b > 0\).  

When \(c < 0\), one factor is positive and the other is negative and when \(b > 0\), the factor with the greater absolute value is positive.

Error Analysis  

You factored \(-x^2 + 14x - 24\) as \((-x - 6)(x - 4)\). Your friend factored it as \((-x - 12)(x - 2)\). Which of you is correct? What mistake was made?  

You are correct; your friend forgot to factor out \(-1\).

Multiple Choice  

What is the factored form of \(16x^2 - 42x - 20\)?  

A. \(-2x^2 - 6x - 10\)  
B. \(-2x^2 - 6x - 10\)  
C. \(-2x^2 - 6x - 10\)  
D. \(-2x^2 - 6x - 10\)  

Reasoning  

The area of a carpet is \((x^2 - 11x + 28)\) m². What are the length and the width of the carpet?  

11x - 7 m and \(x - 4\) m.
4-4 Enrichment  
Factoring Quadratic Expressions

To factor a quadratic expression of the form \(ax^2 + bx + c\), break the middle term of the expression into two terms and use common factors to complete the factoring.

1. \(2x^2 - 3x - 5\)
2. \(2x^2 + 2x - 5x = 5\)
3. \(2x + 1 - 5x + 5\)
4. \(2x + 5\)  
5. \(3x - 5(4x + 1)\)

This same method can be used to factor polynomials with more than three terms.

1. \(x^2 + 3x + 4x + 4\)
2. \(x^2 + 4x + 4\)
3. \(x^2 + 4x + 4\)
4. \(x^2 + 2x - 3x - 2\)
5. \(x^2 + 2x - 4\)
6. \(x^2 + 2x - 6\)
7. \(x^2 + 2x - 10\)
8. \(x^2 + 2x - 12\)

4-4 Reteaching (continued)  
Factoring Quadratic Expressions

What is \(25x^2 - 20x + 4\) in factored form?

There are three terms. Therefore, the expression may be a perfect square trinomial.

\(a^2 = 25x^2\) and \(b^2 = 4\)

Check that the choice of \(a\) and \(b\) gives the correct middle term.

\(2ab = 2 \times 5\times 2 = 20x\)

Write the factored form.

\(a^2 - 2ab + b^2 = (a - b)^2\)

Check the factors in expanded form.

\(25x^2 - 20x + 4 = (5x - 2)^2\)

Exercises

Factor each expression.

1. \(x^2 + 6x + 8\)
2. \(x^2 + 4x + 4\)
3. \(x^2 + 3x + 2\)
4. \(x^2 + 2x + 1\)
5. \(x^2 - x - 2\)
6. \(x^2 - 3x - 4\)
7. \(x^2 - 4x + 2\)
8. \(x^2 - 5x + 2\)
9. \(x^2 + x - 2\)
10. \(x^2 + 3x - 1\)
11. \(x^2 + 4x + 4\)
12. \(x^2 + 5x + 6\)
13. \(x^2 + 6x + 5\)
14. \(x^2 + 6x + 9\)
15. \(x^2 + 5x + 2\)
16. \(x^2 + 2x + 2\)
17. \(x^2 + 6x + 12\)
18. \(x^2 + 4x - 2\)
19. \(x^2 + 5x - 2\)
20. \(x^2 + 2x - 3\)
21. \(x^2 + 6x + 5\)

16. \(x^2 + 4x + 4\)

Exercises

Factor each expression.

1. \(x^2 + 6x + 8\)
2. \(x^2 + 4x + 4\)
3. \(x^2 + 3x + 2\)
4. \(x^2 + 2x + 1\)
5. \(x^2 + x - 2\)
6. \(x^2 + 3x - 1\)
7. \(x^2 + 4x + 4\)
8. \(x^2 + 5x + 6\)
9. \(x^2 + 6x + 5\)
10. \(x^2 + 6x + 9\)
11. \(x^2 + 5x + 2\)
12. \(x^2 + 2x + 2\)
13. \(x^2 + 6x + 12\)
14. \(x^2 + 4x - 2\)
15. \(x^2 + 5x - 2\)
16. \(x^2 + 2x - 3\)
17. \(x^2 + 6x + 5\)

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What are the solutions of the quadratic equation $2x^2 - 4x = 6$?

<table>
<thead>
<tr>
<th>Explain</th>
<th>Work</th>
<th>Justify</th>
</tr>
</thead>
<tbody>
<tr>
<td>First, write the equation.</td>
<td>$2x^2 - 4x = 6$</td>
<td>Original equation</td>
</tr>
<tr>
<td>Second, subtract 6 from each side to set equal to 0.</td>
<td>$2x^2 - 4x - 6 = 0$</td>
<td></td>
</tr>
<tr>
<td>Next, factor out the GCF, 2.</td>
<td>$2(x^2 - 2x - 3) = 0$</td>
<td>Distribution Property</td>
</tr>
<tr>
<td>Then, factor the trinomial.</td>
<td>$2(x - 3)(x + 1) = 0$</td>
<td>Factor the quadratic expression</td>
</tr>
<tr>
<td>Then, use the Zero-Product Property.</td>
<td>$x - 3 = 0$ or $x + 1 = 0$</td>
<td>Zero-Product Property</td>
</tr>
<tr>
<td>Finally, solve for $x$.</td>
<td>$x = 3$ or $x = -1$</td>
<td>Addition Property of Equality</td>
</tr>
</tbody>
</table>

**Exercise**

What are the solutions of the quadratic equation $3x^2 - 6x = -37$?

<table>
<thead>
<tr>
<th>Explain</th>
<th>Work</th>
<th>Justify</th>
</tr>
</thead>
<tbody>
<tr>
<td>First, write the equation.</td>
<td>$3x^2 - 6x = -37$</td>
<td>Original equation</td>
</tr>
<tr>
<td>Second, add 3 to each side to set equal to 0.</td>
<td>$3x^2 - 6x + 3 = 0$</td>
<td>Addition Property of Equality</td>
</tr>
<tr>
<td>Next, factor out the GCF 3.</td>
<td>$3(x^2 - 2x + 1) = 0$</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>Then, factor the binomial.</td>
<td>$3(x - 1)(x - 1) = 0$</td>
<td>Factor the quadratic expression</td>
</tr>
<tr>
<td>Then, use the Zero-Product Property.</td>
<td>$x - 1 = 0$ or $x - 1 = 0$</td>
<td>Zero-Product Property</td>
</tr>
<tr>
<td>Finally, solve for $x$.</td>
<td>$x = 1$</td>
<td>Addition Property of Equality</td>
</tr>
</tbody>
</table>

**Answers**

1. $x = 2$ or $x = -1.5$  
2. $x = 2$ or $x = 2$  
3. $x = 2$ or $x = 2$  
4. $x = 4$ or $x = 2$  
5. $x = 4$ or $x = 2$  
6. $x = 4$ or $x = 2$  
7. $x = 4$ or $x = 2$  
8. $x = 4$ or $x = 2$  
9. $x = 4$ or $x = 2$  
10. $x = 4$ or $x = 2$  
11. $x = 4$ or $x = 2$  
12. $x = 4$ or $x = 2$  
13. $x = 4$ or $x = 2$  
14. $x = 4$ or $x = 2$  
15. $x = 4$ or $x = 2$  
16. $x = 4$ or $x = 2$  
17. $x = 4$ or $x = 2$  
18. $x = 4$ or $x = 2$  
19. $x = 4$ or $x = 2$  
20. $x = 4$ or $x = 2$  
21. A woman drops a front door key to her husband from their apartment window several stories above the ground. The function $h = -16t^2 + 64$ gives the height $h$ of the key in feet, $t$ seconds after she releases it.
   a. How long does it take the key to reach the ground? 2 s
   b. What are the reasonable domain and range for the function $h^2$? 0 ≤ $t$ ≤ 2; 0 ≤ $h$ ≤ 64
Solve each equation by factoring. Check your answers. To start, factor the quadratic expression.

1. \( x^2 - 10x = 0 \)

2. \( x^2 = 2x - 7 \)

3. \( x^2 - 10x = 9 \)

4. \( x^2 - 5x = 0 \)

5. \( 10x = 24 = x^2 \)

6. \( x^2 = -12x \)

Solve each equation using tables. Give each answer to two decimal places. To start, enter the equation in Y1. Make a table and look for where the \( y \)-values change sign.

7. \( x^2 + x = 12 \)

8. \( 10x = 22x + 60 = 0 \)

9. \( 22x^2 + 11x = 0 \)

10. \( 2x^2 - 13x + 18 = 0 \)

11. \( 2x^2 = 10y \)

12. \( 5x^2 - 8 = 0 \)

Write a quadratic equation with the given solutions.

13. \( 4 \) and \(-5\)

14. \(-6\) and \(0\)

15. \(3\) and \(0\)

16. Writing Explain when you would prefer to use factoring to solve a quadratic equation and when you would prefer to use tables.

When \( a = 1 \) and a factor easily into integers, it is better to solve by factoring. When \( c \) doesn’t factor easily, it is better to solve using tables.

17. A parallel jogging path intersects both ends of a street. The path has the equation \( x^2 - 25x = 0 \). If one end of the street is considered to be \( x = 0 \) and the street lies on the \( x \)-axis, where else does the path intersect the street?

at \( x = 25 \)

Solve each of the following problems by using a quadratic equation.

18. The area of a rectangular field is 1200 yd\(^2\). Two parallel sides are fenced with aluminum at $20/yd. The remaining two sides are fenced with steel at $10/yd. The total cost of the fencing is $2200.

a. What is the length of each side fenced with aluminum?

b. What is the length of each side fenced with steel?

19. The area of a rectangular field is 1000 yd\(^2\). Two parallel sides are fenced with steel costing $10/yd. The perimeter of the field is 150 yd. The total cost of the fencing is $2250.

a. What is the length of each side fenced with steel?

b. What is the length of the side fenced with aluminum?

c. What is the area of the field?

20. The area of a rectangular field is 1800 yd\(^2\). Two parallel sides are fenced with wood costing $5/yd. The remaining two sides are fenced with steel at $10/yd. The total cost of the fencing is $1700.

a. What is the length of each side fenced with steel?

b. What is the length of the side fenced with aluminum?

c. What is the area of the field?

21. The area of a rectangular field is 1500 yd\(^2\). Two parallel sides are fenced with wood at $10/yd. The remaining two sides are fenced with steel at $15/yd. The total cost of the fencing is $2200.

a. What is the length of the side fenced with steel?

b. What is the length of the side fenced with wood?

c. What is the area of the field?

22. The area of a rectangular field is 1250 yd\(^2\). Two parallel sides are fenced with wood at $15/yd. The remaining two sides are fenced with steel at $20/yd. The total cost of the fencing is $2350.

a. What is the length of each side fenced with steel?

b. What is the length of the side fenced with wood?

c. What is the area of the field?

23. The area of a rectangular field is 1000 yd\(^2\). Two parallel sides are fenced with wood costing $5/yd. The remaining two sides are fenced with aluminum at $15/yd. The total cost of the fencing is $1520.

a. What is the length of each side fenced with aluminum?

b. What is the length of each side fenced with wood?

c. What is the area of the field?

24. The student in Mr. Wilson’s Physics class is making gold ball canopies. The flight of a group of K-balls is modeled by the equation \( y = -0.0144x^2 + 0.08x \), where \( x \) is the ball’s distance from the catapult. The units are in feet.

a. How far did the ball fly? about 48.6 feet

b. How high above the ground did the ball fly? about 8.3 feet

c. What is a reasonable domain and range for this function?

Answers may vary. Sample domain: \( 0 \leq x \leq 48.6 \); range: \( 0 \leq y \leq 8.3 \)

25. A rectangular pool is 20.8 yd wide and 50.8 yd long. The pool is surrounded by a walkway. The walkway is the same width all the way around the pool. The total area of the walkway is 450 square yd. How wide is the walkway? 5 yd

26. Reasoning The equation used to solve Exercise 25 has two solutions. Why is only one solution used to answer the question?

One of the solutions is negative, and the walkway cannot have a negative width.
There are several ways to solve quadratic equations. If you can factor the quadratic expression in a quadratic equation written in standard form, you can use the Zero-Product Property.

Problem
What are the solutions of the quadratic equation \(2x^2 + x - 15 = 0\)?

- \(2x^2 + x - 15 = 0\) Write the equation.
- \(2x^2 + 5x - 4x - 15 = 0\) Rewrite the quadratic expression in the exercise, add.
- \((2x - 3)(x + 5) = 0\) Factor the quadratic expression in the exercise, subtract.
- \(2x = 3\) or \(x = -5\) Solve for \(x\).
- \(x = \frac{3}{2}\) or \(x = -5\)

Check the solutions:
\[
\begin{align*}
\text{If } x & = \frac{3}{2}, \text{ then } 2x^2 + x - 15 = 0.5(\frac{3}{2})^2 + \frac{3}{2} - 15 = 0.5(\frac{9}{4}) + \frac{3}{2} - 15 = 0.5(2.25) + 1.5 - 15 = 1.125 + 1.5 - 15 = -12.375 \neq 0 \quad \text{(Not a solution)}
\end{align*}
\]
\[
\begin{align*}
\text{If } x & = -5, \text{ then } 2x^2 + x - 15 = 2(-5)^2 + (-5) - 15 = 2(25) - 5 - 15 = 50 - 5 - 15 = 30 \neq 0 \quad \text{(Not a solution)}
\end{align*}
\]

Both solutions check: the solutions are \(x = \frac{3}{2}\) and \(x = -5\).

Exercises
Solve each equation by factoring. Check your answers.

1. \(x^2 + 10x + 16 = 2x + 2\) or \(x^2 + 9x - 10 = 0\)
2. \(x^2 - 2x - 15 = 0\) or \(x^2 - 3x - 10 = 0\)
3. \(x^2 - 5x + 6 = 0\) or \(x^2 - 7x + 12 = 0\)
4. \(x^2 + x - 20 = 0\) or \(x^2 - 2x - 15 = 0\)
5. \(x^2 - x - 12 = 0\) or \(x^2 + x - 6 = 0\)
6. \(x^2 - 10x + 9 = 0\) or \(x^2 - 5x + 6 = 0\)
7. \(x^2 - 10x + 25 = 0\) or \(x^2 - 6x + 8 = 0\)
8. \(x^2 - 2x + 1 = 0\) or \(x^2 - 4x + 4 = 0\)
9. \(x^2 + 3x + 2 = 0\) or \(x^2 + 5x + 6 = 0\)
10. \(x^2 + 2x - 3 = 0\) or \(x^2 - 3x + 2 = 0\)

4-5 Reteaching

4-6 ELI Support

Completing the Square

Problem
Solve \(2x^2 + 12x = 2 = 0\) by completing the square. Justify your steps.

\[
\begin{align*}
2x^2 + 12x - 2 &= 0 & \text{Write the original equation.} \\
2x^2 + 12x &= 2 & \text{Divide each side by 2 so the coefficient of } x^2 \text{ will be 1.} \\
2\left(x^2 + 6x\right) &= 1 & \text{Simplify.} \\
2\left(x^2 + 6x + 9\right) &= 1 + 9 & \text{Add } \frac{9}{2} \text{ to each side.} \\
\left(x + 3\right)^2 &= \frac{10}{2} & \text{Factor the trinomial.} \\
x + 3 &= \pm \sqrt{5} & \text{Get square roots.} \\
x &= -3 \pm \sqrt{5} & \text{Solve for } x.
\end{align*}
\]

Exercise
Solve \(3x^2 + 24x - 9 = 0\) by completing the square. Justify your steps.

\[
\begin{align*}
3x^2 + 24x &= 9 & \text{Write the original equation.} \\
3\left(x^2 + 8x\right) &= 9 & \text{Divide each side by 3 so the coefficient of } x^2 \text{ will be 1.} \\
\left(x^2 + 8x + 16\right) &= 16 & \text{Simplify.} \\
(x + 4)^2 &= 16 & \text{Complete the square.} \\
x + 4 &= \pm 4 & \text{Get square roots.} \\
x &= \pm 4 \pm \sqrt{16} & \text{Solve for } x.
\end{align*}
\]

4-5 Reteaching

Some quadratic equations are difficult or impossible to solve by factoring. You can use a graphing calculator to find the points where the graph of a function intersects the \(x\)-axis. At these points, \(f(x) = 0\), so \(x\) is a zero of the function.

The zeros \(x_1\) and \(x_2\) are the zeros of the function \(f(x) = (x - x_1)(x - x_2)\). The graph of the function intersects the \(x\)-axis at \(x = x_1\) or \((x_1, 0)\), and \(x = x_2\) or \((x_2, 0)\).

Problem
What are the solutions of the equation \(3x^2 + 2x - 7 = 0\)?

- **Step 1**: Write the equation in standard form, \(ax^2 + bx + c = 0\).
- **Step 2**: Enter the equation as \(Y_1\) in your calculator.
- **Step 3**: Graph \(Y_1\) and find the zeros of the function; \(Y_1\) is visible on the screen. If the zeros are not real, zoom out and determine a better viewing window. In this case, the zeros are visible in the standard window.
- **Step 4**: Use the zero option in the CBL2 feature. For the first zero, choose bounds of \(-2\) and \(-1\) and a guess of \(-1.5\).
- **Step 5**: The screen displays the first zero as \(x = \pm 2.24\). Similarly, the second zero, \(x = \pm 3.24\). The solutions to two decimal places are \(x = -2.24\) and \(x = 3.24\).

Exercises
Solve the equation by graphing. Give each answer to at most two decimal places.

- **13**: \(x^2 = 25\), \(x = \pm 5\), \(x = 5\) or \(x = -5\)
- **14**: \(x^2 = 25\), \(x = \pm 5\), \(x = 5\) or \(x = -5\)
- **15**: \(x^2 = 1/2\), \(x = \pm \sqrt{1/2}\), \(x = \sqrt{2}/2\) or \(x = -\sqrt{2}/2\)
- **16**: \(x^2 = 1/2\), \(x = \pm \sqrt{1/2}\), \(x = \sqrt{2}/2\) or \(x = -\sqrt{2}/2\)
- **17**: \(x^2 = 1/2\), \(x = \pm \sqrt{1/2}\), \(x = \sqrt{2}/2\) or \(x = -\sqrt{2}/2\)
- **18**: \(x^2 = 1/2\), \(x = \pm \sqrt{1/2}\), \(x = \sqrt{2}/2\) or \(x = -\sqrt{2}/2\)

4-6 Think About a Plan

Completing the Square

Geometric Reasoning
The table shows some possible dimensions of rectangles with a perimeter of 100 units. Copy and complete the table.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>48</td>
<td>96</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>225</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>400</td>
</tr>
</tbody>
</table>

1. What points should you plot? Plot the points on the graph: \((1, 49), (2, 96), (3, 141), (4, 184), (5, 225)\).
2. Use your graphing calculator to find a model for the data set. \(R = -0.1x + 50\).
3. What is another point in the data set? Use it to verify your model.
4. What does the domain of your function represent? The width of the rectangle.
5. The domain must be greater than 0 and less than 50.
6. A relevant domain is: all real numbers greater than 0 and less than 50.
7. Write the vertex form of your function. \(A = -(x - 25)^2 + 625\).
8. The maximum possible area is \(625\) square units. The dimensions of this rectangle are \(25\) units by \(25\) units.
9. If the width of the rectangle is \(x\), then the length is \(50 - x\).
10. The equation is the equation 9 the same as your model in Exercise 27? Explain.
11. Yes, the functions are the same.
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ANSWERS

4-6 Practice Form G

Completing the Square

Solve each equation by finding square roots.

1. \(3x^2 - 14x + 84 = 0\)
2. \(2x^2 + 4x = 9\)
3. \(2x^2 + 8x + 12 = 0\)
4. \(3x^2 + 6x - 7 = 0\)
5. \(5x^2 - 10x + 4 = 0\)
6. \(x^2 - 2x + 1 = 0\)

7. A box is 4 in. high. Its length is 1.5 times its width. The volume of the box is 1150 in.\(^3\). What are the width and length of the box? (20 in., 22.5 in.)

Solve each equation.

8. \(x^2 + 6x + 9 = 16\)
9. \(2x^2 + 3x - 2 = 0\)
10. \(3x^2 - 2x + 1 = 0\)
11. \(x^2 - 4x = 8\)
12. \(2x^2 - 5x + 3 = 0\)

Complete the square.

13. \(x^2 + 4x + 3 = 0\)
14. \(x^2 - 6x + 2 = 0\)
15. \(x^2 + 8x + 15 = 0\)

Solve each quadratic equation by completing the square.

16. \(x^2 + 6x = 7\)
17. \(x^2 - 4x = 10\)
18. \(x^2 + 8x = 1\)

4-6 Practice Form K

Completing the Square

Solve each equation by finding square roots. To start, remember to isolate \(x^2\).

1. \(x^2 - 9 = 0\)
2. \(x^2 + 4 = 20\)
3. \(x^2 + 15 = 16\)
4. \(x^2 - 6 = 10\)
5. \(x^2 + 100 = 0\)
6. \(x^2 = 25\)

You are painting a large wall mural. The wall length is 3 times the height. The area of the wall is 367 ft\(^2\).

a. What are the dimensions of the wall? (height = 10 ft; length = 30 ft)
b. If each can of paint covers 22 ft\(^2\), will 12 cans be enough to cover the wall? No; 12 cans will only cover 264 ft\(^2\).

c. The lengths of the sides of a carpet have the ratio of 4:4 to 1. The area of the carpet is 1554.7 ft\(^2\). What are the dimensions of the carpet? (12 ft by 21 ft)

d. A packing box is 4 ft deep. One side of the box is 1.5 times longer than the other. The volume of the box is 24 ft\(^3\). What are the dimensions of the box? (2 ft by 6 ft)

Solve each equation. To start, factor the perfect square trinomial.

10. \(x^2 + 6x + 9 = 1\)
11. \(x^2 + 8x + 4 = 0\)
12. \(x^2 - 12x + 9 = 0\)

Write the following equations in vertex form.

23. \(y = x^2 + 10x + 9\)
24. \(y = x^2 + 4x + 3\)
25. \(y = x^2 + 12x + 23\)

28. \(y = x^2 + 14x + 52\)
29. \(y = x^2 + 18x + 38\)
30. \(y = x^2 + 32x + 8\)

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4-6 Standardized Test Prep
Completing the Square

Multiple Choice
For Exercises 1–4, choose the correct letter.
1. What are the solutions of the equation $3x^2 - 12x + 1 = 47$?
   - A $x = 4, 8$
   - B $x = -\frac{1}{2}, \frac{1}{2}$
   - C $x = -4, -8$
   - D $x = 4, 8$
2. What are the solutions of the equation $2x^2 + 10x + 28 = 0$?
   - A $x = -4, 7$
   - B $x = 4, -7$
   - C $x = -4, 9$
   - D $x = 4, -9$
3. Which value completes the square for $x^2 - 3x$?
   - A $\frac{9}{4}$
   - B $-\frac{9}{4}$
   - C $\frac{9}{2}$
   - D $-\frac{9}{2}$
4. Which value for $k$ would make the left side of $x^2 + kx + \frac{9}{4}$ a perfect square trinomial?
   - A $\frac{3}{2}$
   - B $\frac{3}{4}$
   - C $\frac{2}{3}$
   - D $\frac{4}{3}$
5. What are the solutions of the equation $x^2 = 9x - 11$?
   - A $x = -4, 7$
   - B $x = 4, -7$
   - C $x = -4, 7$
   - D $x = 4, -7$
6. Which equation is the vertex form of $y = -3x^2 + 12x - 7$?
   - A $y = -3(x - 2)^2 - 5$
   - B $y = -3(x - 2)^2 + 5$
   - C $y = 3(x - 2)^2 - 5$
   - D $y = 3(x - 2)^2 + 5$

Short Response
7. The equation $p = -x^2 + 8x + 5$ gives the price $p$, in dollars, for a product where $x$ million units are produced.
   a. What are the solutions of the equation $-x^2 + 8x + 5 = 0$?
   b. What is the positive solution to part (a) rounded to two decimal places?
   c. What does this solution mean in terms of this problem?
   (2) $x = 4 \div 2 \pm \frac{1}{2}$; the price will be 55 when 0.5 million units are produced.
   (1) incorrect solution or incorrect interpretation
   (0) no answers given

4-6 Enrichment
Completing the Square
You can approximate square roots with an interesting quadratic relationship.

1. Multiply $(a + \frac{b}{2})^2 = a^2 + b^2$ and $\frac{b}{2}$.
2. What happens to the value of a fraction as the denominator gets larger?

As the denominator gets larger, the fraction becomes smaller.

If $a$ is much greater than $b$, the value of the fraction $\frac{b}{a}$ is very small and has little effect in the expression $a^2 + b^2 \approx \frac{b^2}{a}$. So when $a$ is much larger than $b$, you can write
   $a + \frac{b}{2} \approx a + \frac{b^2}{2a}$
   $a = \frac{b}{\sqrt{a} + \frac{b}{2a}}$

If you can write a number as $a^2 + b$ where $a > b$, then an approximate value of its square root is $a + \frac{b}{2a}$.

For example, to approximate $\sqrt{143}$, let $a = 14$ and $b = 1$.

$\sqrt{143} \approx 11.9$

Evaluate this square root on a calculator: $\sqrt{143} \approx 11.924$

Use the formula to approximate each square root. Then find each square root using your calculator. Round to the nearest thousandth.
3. $\sqrt{52}$; 5.100; 5.099
4. $\sqrt{42}$; 6.473; 6.428
5. $\sqrt{13}$; 3.606; 3.610
6. $\sqrt{10}$; 3.162; 3.162
7. $\sqrt{11}$; 3.317; 3.320
8. $\sqrt{12}$; 3.462; 3.464
9. $\sqrt{15}$; 3.873; 3.873

4-6 Reteaching
Completing the Square
You can easily graph a quadratic function if you first write it in vertex form.
Complete the square to change a function in standard form into a function in vertex form.

Problem
What is $y = x^2 - 4x + 14$ in vertex form?

Think
Write

Write an expression using the
terms that contain x.

Factor perfect square
trinomial.

Write an expression using
the terms that do not contain
x.

Complete the square.

Subtract the expression so
that the equation is unchanged.

Add the remaining constant
terms.

Exercises
Rewrite each equation in vertex form.

13. $y = x^2 + 4x + 3 \div 2 \approx 2^2 \approx 5$
14. $y = x^2 - 4x + 13 \div 2 = 2^2 = 4$
15. $y = -x^2 + 4x - 10 \div 2 = 2^2 = 5$
16. $y = x^2 - 4x + 13 \div 2 = 2^2 = 4$
17. $y = x^2 - 6x + 3 \div 2 = 3^2 = 9$
18. $y = -x^2 - 6x + 3 \div 2 = 3^2 = 9$
19. $y = x^2 + 10x - 18 \div 2 = 5^2 = 25$
20. $y = x^2 + 10x - 18 \div 2 = 5^2 = 25$
21. $y = 2x^2 + 4x - 3 \div 2 = 2 \div 2 = 1$
22. $y = 3x^2 - 12x + 8 \div 2 = 2^2 = 4$
23. $y = 3x^2 - 12x + 8 \div 2 = 2^2 = 4$
4-7 ELL Support
The Quadratic Formula

The column on the left shows the steps used to solve a problem using the quadratic formula. Use the column on the left to answer each question in the column on the right.

Problem: Solve by using the quadratic formula.

1. Read the title of the problem. What process are you going to use to solve the problem?
   Answers may vary. Sample: Solve by using the quadratic formula.

2. Write is standard form. Solve by using the quadratic formula.
   \[ ax^2 + bx + c = 0 \]
   \[ a = 4, \ b = -8, \ c = 0 \]

3. Explain how you know which value is \( a \).
   Answers may vary. Sample: It is the leading coefficient.

4. Write the quadratic formula. Explain what the symbol \( \pm \) means.
   Answers may vary. Sample: The symbol means there are two answers, one found by adding the other by subtracting.

5. Do you need to use the height of the springboard to solve the problem?
   No, you don’t need to use the height of the springboard. You only need to find the zeros of the function.

6. What are three possible methods for solving this problem?
   Graph the equation, factor the equation; use the Quadratic Formula.

7. What is the problem asking you to determine?
   The problem is asking you to determine when the rocket reaches an altitude of 1048 ft.

8. What is the problem asking you to determine?
   The problem is asking you to determine the height of the rocket 1048 ft.

9. What is the problem asking you to determine?
   The problem is asking you to determine the altitude of the rocket at 1048 ft.

10. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

11. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

12. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

13. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

14. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

15. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

16. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

17. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

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22. What is the problem asking you to determine?
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23. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

24. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

25. What is the problem asking you to determine?
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26. What is the problem asking you to determine?
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27. What is the problem asking you to determine?
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28. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

29. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

30. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

31. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

32. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

33. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

34. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

35. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.

36. What is the problem asking you to determine?
    The problem is asking you to determine the altitude of the rocket at 1048 ft.
4-7 Practice [continued] Form K

The Quadratic Formula

Evaluate the discriminant for each equation. Determine the number of real solutions.

10. $12x^2 + 8x + 7 = 0$
11. $x^2 + 5x - 4 = 0$
12. $2x - 3 = -x^2$

121, 2 real solutions
$-23, 0$ real solutions
24, $2$ real solutions

Solve each equation using any method. When necessary, round real solutions to the nearest hundredth.

13. $4x^2 + 7 = 9x$
14. $x^2 - 4x = -4$
15. $3x + 6 = -x^2$

$-31, 0$ real solutions
0, 1 real solution
$-135, 0$ real solutions

Without graphing, determine how many x-intercepts each function has.

22. $y = 2x^2 + 3x + 5$
23. $y = 2x^2 - 4x + 1$
24. $y = x^2 + 5x + 3$

$3$ x-intercepts
$2$ x-intercepts
$0$ x-intercepts

25. $y = 2x^2 - 12x + 7$
26. $y = -2x^2 + 8x - 3$
27. $y = x^2 + 10x + 64$

$0$ x-intercepts
$2$ x-intercepts
$1$ x-intercept

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A N S W E R S
4-7 Retraining
The Quadratic Formula
You can solve some quadratic equations by factoring or completing the square.
You can solve any quadratic equation ax^2 + bx + c = 0 by using the Quadratic Formula.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Notice the ± symbol in the formula. Whenever b^2 - 4ac is not zero, the Quadratic Formula will result in two solutions.

Problem
What are the solutions for \(2x^2 - 5x + 2 = 0\)?

Exercise
What are the solutions for \(2x^2 - 5x + 2 = 0\)?

Notice the Quadratic Formula will result in two solutions.

2.

Complete the vocabulary chart by filling in the missing information.

<table>
<thead>
<tr>
<th>Word or Word Phrase</th>
<th>Definition</th>
<th>Picture or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>imaginary unit</td>
<td>The imaginary unit is a complex number whose square is -1.</td>
<td>( z = \sqrt{-1} )</td>
</tr>
<tr>
<td>pure imaginary number</td>
<td>A pure imaginary number is of the form ( a + bi ) where ( a = 0 ) and ( b \neq 0 ).</td>
<td></td>
</tr>
<tr>
<td>complex number</td>
<td>2. any number of the form ( a + bi ) where ( a ) and ( b ) are real numbers</td>
<td>7 - 4i</td>
</tr>
<tr>
<td>complex number plane</td>
<td>In the complex number plane, the point ((a, b)) represents the complex number (a + bi). To graph, locate the real part on the horizontal axis and the imaginary part on the vertical axis.</td>
<td></td>
</tr>
<tr>
<td>absolute value of a complex number</td>
<td>The absolute value of a complex number is its distance from the origin in the complex number plane.</td>
<td>(</td>
</tr>
<tr>
<td>complex conjugates</td>
<td>number pairs of the form (a + bi) and (a - bi)</td>
<td></td>
</tr>
</tbody>
</table>

4-8 Retraining
Complex Numbers
There are three possible outcomes when you take the square root of a real number \(n\):

\[
\begin{align*}
\text{If } a^2 &= 0 & \Rightarrow & & \text{true real number} & \text{(one positive and one negative)} \\
\text{If } a^2 &= 1 & \Rightarrow & & \text{one real value} & \text{[0]} \\
\text{If } a^2 &= -1 & \Rightarrow & & \text{no real values} & \\
\end{align*}
\]

Now consider the quadratic formula \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\). The value under the radical symbol determines the number of real solutions that exist for the equation \(ax^2 + bx + c = 0\).

\[
\begin{align*}
\text{If } b^2 - 4ac &= 0 & \Rightarrow & & \text{one real solution} & \\
\text{If } b^2 - 4ac &= 25 & \Rightarrow & & \text{two real solutions} & \\
\text{If } b^2 - 4ac &< 0 & \Rightarrow & & \text{no real solutions} & \\
\end{align*}
\]

Problem
What is the number of real solutions of \(-3x^2 + 7x = 2\)?

Exercise
What is the value of the discriminant and what is the number of real solutions for each equation?

1.

4-8 Ell Support
Complex Numbers
A student wrote the numbers 1, 5, 1, and \(4 + 3i\) to represent the vertices of a quadrilateral in the complex number plane. What type of quadrilateral has these vertices?

Know
1. The vertices of the quadrilateral are:
   1, 5, 1 + 3i, and \(4 + 3i\)

2. You can write the vertices in the form \(a + bi\) as:
   \(1 + 0i, 5 + 0i, 1 + 3i, \) and \(4 + 3i\)

Need
3. To solve the problem I need to:
   - graph the numbers in the complex plane

Plan
4. How do you find the coordinates that represent each complex number?
   - For the complex number \(a + bi\), the real part \(a\) is the horizontal coordinate and the imaginary part \(b\) is the vertical coordinate.

5. What are the points you need to graph?
   \(1, 0, 5, 0, 1, 3, 0, 4, 3\)

6. Graph your points in the complex plane. Connect the points with straight lines to form a quadrilateral.

7. What type of quadrilateral did you draw? Explain how you know.
   - Trapezoid: the top and bottom sides are parallel, but the left and right sides are not parallel.
Simplify each expression.

12. \( \sqrt{-25} \)

15. \( \sqrt{-16} \)

18. \( \sqrt{-36} \)

21. \( \sqrt{-49} \)

Simplify each number by using the imaginary number \( i \).

5. \( 4 \sqrt{-1} \)

8. \( 10 \sqrt{-1} \)

11. \( 14 \sqrt{-1} \)

14. \( 20 \sqrt{-1} \)

Plot each complex number and find its absolute value.

7. \( -3 + 3i \)

10. \( 6 - 4i \)

13. \( -4 + 6i \)

16. \( 2i \)

Simplify each expression.

10. \( -2 \cdot 3x - (5 - 2x) \cdot 3 + i \)

13. \( -5 \cdot 3x - (8 - 2x) \cdot 3 + j \)

16. \( 3x \cdot 3x + 3 \cdot 36 \)

19. \( 4i \cdot \sqrt{15} - 4i \)

22. \( -3 + \sqrt{-1} - (3 - \sqrt{-1}) - 8 + 4i \)

25. \( (2x - 7j) - (4 + 5i) \)

28. \( (2i - \sqrt{-1}) - 8 + 4i \)

27. \( 3 + \sqrt{-1} - (3 + \sqrt{-1}) \cdot 16 - 20i \)

Simplify each expression.

9. \( (9 + 6i) - (2i) \cdot (9 - 2i) + (6i - 9) \cdot (9 + 2i) + (9 - 6i) \cdot (9 + 2i) + (9 + 6i) \)

11. \( 3(3 + 4i) \cdot 4x - 5 + (2x - 7) - (4 + 5i) \)

Write each quotient as a complex number.

12. \( \frac{4}{5i} \)

13. \( \frac{2i}{4} \)

14. \( \frac{1}{2} - i \)

16. \( \frac{2}{3} - \frac{1}{x} \)

Find all solutions to each quadratic equation.

38. \( x^2 - 2x + 5 = 0 \) 

41. \( 2x^2 - 3x + 5 = 0 \) 

43. \( 3x^2 - 2x + 7 = 0 \)

Solve each equation.

15. \( 2x^2 - 50 = 0 \)

16. \( 3x^2 + 13 = -2 \)

17. \( x^2 + 49 = 0 \)

18. \( 6x^2 + 1 = 0 \)

19. \( 5x^2 - 91 = 0 \)

20. \( x^2 + 27 = -9 \)

24. Error Analysis: Robert solved the equation \( 2x^2 + 16 = 0 \). His solution was \( x = \pm \sqrt{-8} \). What errors did Robert make? What is the correct solution? Robert made two errors. He left a negative number under the radical sign, and he did not simplify \( \sqrt{8} \). The correct solution is \( x = \pm 2\sqrt{2} \).
9. What is the simplified form of (8 - 2)\(i\)?

8. What is the simplified form of (5 - 4)\(i\)?

7. What is the simplified form of (11 - 6)\(i\)?

6. What is the simplified form of (12 - 8)\(i\)?

5. What is the simplified form of (14 - 10)\(i\)?

4. What is the simplified form of (9 - 4)\(i\)?

3. What is the simplified form of (3 - 2)\(i\)?

2. What is the simplified form of (1 - 0)\(i\)?

1. What is the simplified form of (0 - 5)\(i\)?

Multiple Choice

For Exercises 1–8, choose the correct letter.

1. What is the simplified form of (8 - 2)\(i\)?

2. What is the simplified form of (11 - 6)\(i\)?

3. What is the simplified form of (5 - 4)\(i\)?

4. What is the simplified form of (12 - 8)\(i\)?

5. What is the simplified form of (14 - 10)\(i\)?

6. What is the simplified form of (9 - 4)\(i\)?

7. What is the simplified form of (3 - 2)\(i\)?

8. What is the simplified form of (1 - 0)\(i\)?

Short Response

5. What are the solutions of \(2x^2 + 3x + 6 = 0\)? Show your work.

[1] incorrect answers and no work shown OR no answers given.

[2] quadratic formula properly used, but some computational errors OR correct solutions without work shown.

[3] quadratic formula properly used, correct solutions with work shown.

[4] incorrect answers and work shown OR no answers given.

Standardized Test Prep

Complex Numbers

When multiplying complex numbers, use the Distributive Property or FOIL.

\[ (a + bi)(c + di) = ac + adi + bci + bdi^2 \]

The magnitude of the vector is the absolute value of the complex number.

\[ |a + bi| = \sqrt{a^2 + b^2} \]

The complex conjugate of a complex number \(a + bi\) is the complex number \(a - bi\).

To divide complex numbers, use complex conjugates to simplify the denominator.

**Problem**

What is the quotient \(\frac{1}{4} + \frac{1}{i}\)?

\[ \frac{1}{4} + \frac{1}{i} = \frac{1}{4} + \frac{1}{i} \cdot \frac{i}{i} = \frac{1}{4}i \]

Exercises

Find the complex conjugate of each complex number.

10. \(1 - 2i\)

11. \(3 + 5i\)

12. \(i\)

13. \(3 + 2i\)

14. \(2 + 3i\)

15. \(-5 - 2i\)

Write each quotient as a complex number.

16. \(\frac{1}{2} + \frac{1}{2i}\)

17. \(\frac{1}{2} - \frac{1}{2i}\)

18. \(\frac{3}{2} + \frac{1}{2i}\)

19. \(\frac{1}{2} - \frac{1}{2i}\)

20. \(\frac{1}{2} + \frac{1}{2i}\)

21. \(\frac{1}{2} - \frac{1}{2i}\)

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13. Solve each system by graphing. Check your answers.

11. Solve each system by substitution. Check your answers.

9. Solve each system by elimination. Check your answers.

3. Solve each system by elimination. Check your answers.

1. Solve each system by graphing. Check your answers.

4. Solve each system by graphing. Check your answers.

2. Solve each system by graphing. Check your answers.

5. Solve this system of equations by substitution. Check your answers.

8. Solve each system by graphing. Check your answers.

6. Solve each system by substitution. Check your answers.

7. Solve each system by elimination. Check your answers.

10. Solve each system by substitution. Check your answers.

4. Solve each system by graphing. Check your answers.

25. Solve each system by graphing. Check your answers.

24. Solve each system by graphing. Check your answers.

23. Solve each system by graphing. Check your answers.

22. Solve each system by graphing. Check your answers.

21. Solve each system by graphing. Check your answers.

20. Solve each system by graphing. Check your answers.

19. Solve each system by graphing. Check your answers.

18. Solve each system by graphing. Check your answers.

17. Solve each system by graphing. Check your answers.

16. Solve each system by graphing. Check your answers.

15. Solve each system by graphing. Check your answers.

14. Solve each system by graphing. Check your answers.

13. Solve each system by graphing. Check your answers.

12. Solve each system by graphing. Check your answers.

11. Solve each system by graphing. Check your answers.

10. Solve each system by graphing. Check your answers.

9. Solve each system by graphing. Check your answers.

8. Solve each system by graphing. Check your answers.

7. Solve each system by graphing. Check your answers.

6. Solve each system by graphing. Check your answers.

5. Solve each system by graphing. Check your answers.

4. Solve each system by graphing. Check your answers.

3. Solve each system by graphing. Check your answers.

2. Solve each system by graphing. Check your answers.

1. Solve each system by graphing. Check your answers.

**Answers**
For Exercises 1−4, choose the correct letter.

Multiple Choice

For Exercises 1−4, choose the correct letter.

1. What is the solution of the system?
   a. $y = x^2 - 4x + 5$  
   b. $y = -2x + 4$  
   c. $y = -x + 2$  
   d. $y = 2x - 2$

2. What is the solution of the system?
   a. $y = x^2 - 2x + 2$  
   b. $y = x^2 + 3x - 2$  
   c. $y = -x + 2$  
   d. $y = 4x + 4$

3. What is the solution of the system?
   a. $y = x^2 - 4x + 3$  
   b. $y = -2x + 4$  
   c. $y = -x + 2$  
   d. $y = 4x + 4$

4. What is the solution of the system?
   a. $y = x^2 - 4x + 5$  
   b. $y = x^2 + 2x - 5$  
   c. $y = -x + 2$  
   d. $y = 4x + 4$

Short Response

5. What is the solution of the system? Solve by graphing.
   $y < -x^2 + 3x - 2$

6. What is the solution of the system?
   a. $y = -x^2 + 3x - 2$  
   b. $y = x^2 - 3x + 4$  
   c. $y = x^2 + 3x - 2$  
   d. $y = -x^2 + 2x - 4$

7. What is the solution of the system?
   a. $y = -x^2 + 2x + 2$  
   b. $y = x^2 - 3x + 2$  
   c. $y = x^2 + 3x - 2$  
   d. $y = -x^2 + 2x - 4$

8. What is the solution of the system?
   a. $y = -x^2 + 2x + 2$  
   b. $y = x^2 - 3x + 2$  
   c. $y = x^2 + 3x - 2$  
   d. $y = -x^2 + 2x - 4$

9. What is the solution of the system?
   a. $y = -x^2 + 2x + 2$  
   b. $y = x^2 - 3x + 2$  
   c. $y = x^2 + 3x - 2$  
   d. $y = -x^2 + 2x - 4$

10. Reasoning: What is the least number of solutions a quadratic system can have? Explain what that means...

A quadratic system can have no solutions. This means the graphs of the equations do not intersect.

11. You work at a restaurant whose weekly profit is given by the formula $P = -x^2 + 14x + 68$, where $x$ is the average price of the food, in dollars.

The manager wants to add delivery service, which will cost the restaurant $750.00 per week.

a. Use a graphing calculator to find the highest average price $c$ the restaurant can sell the food at and still make a profit if they add delivery. $\text{Profit at } c = \text{Profit for delivery}$

b. What will the weekly profit $P(x)$ be if the restaurant sells its food at this average price and doesn’t offer delivery? $\text{Profit without delivery}$

c. Reasoning: Even though these equations have two solutions, why is only one solution useful? (Hint: Remember this is a real situation.)

The other solution is negative. You cannot have negative prices.

12. Solve each system by graphing. Check your answers.

What is the solution of the system?

13. Solve each system by substitution. Check your answers.

What is the solution of the system?

14. Solve each system using your graphing calculator.

What is the solution of the system?

15. Solve each system using your graphing calculator.

What is the solution of the system?

16. Error Analysis: A classmate said that the quadratic system $y = 2x^2 - x - 3$ has no solutions. Her work is below. What mistake did she make? What is the solution of this system?

2x^2 - x - 3 = 0
2x + 3 = 0
x = -1.5
2x + 3 = 0
x = 0.5

17. In the second line of her solution, she added 3 to both sides of the equation and subtracted 1 from the right side instead of adding 3 to both sides (0, 3) and (1, 4).
4-9 Reteaching

Quadratic Systems

You used graphing and substitution to solve systems of linear equations. You can use these same methods to solve systems involving quadratic equations.

**Problem**

What is the solution of the system of equations?

\[ \begin{align*}
  y &= x^2 + 2x - 5 \\
  y &= 3x + 4
\end{align*} \]

**Solution:**

1. Write one equation.
   \[ y = x^2 + 2x - 5 \]

2. Substitute \( y = 3x + 4 \) for \( y \) in the linear equation.
   \[ x^2 + 2x - 5 = 3x + 4 \]

3. Write the quadratic equation.
   \[ x^2 - x - 9 = 0 \]

4. Factor the quadratic expression.
   \[ (x - 3)(x + 3) = 0 \]

5. Solve for \( x \).
   \[ x = 3, x = -3 \]

The solutions are \( (3, 13) \) and \( (-3, -5) \). Check these by graphing the system and identifying the points of intersection.

**Exercises**

Solve each system.

1. \( \begin{align*}
  y &= x^2 + 3x - 5 \\
  y &= 3x - 1
\end{align*} \)
2. \( \begin{align*}
  y &= x^2 + 2x - 7 \\
  y &= 3x - 1
\end{align*} \)
3. \( \begin{align*}
  y &= x^2 - 5x + 1 \\
  y &= 3x - 1
\end{align*} \)
4. \( \begin{align*}
  y &= x^2 - 5x + 3 \\
  y &= 3x - 7
\end{align*} \)
5. \( \begin{align*}
  y &= x^2 - 5x + 1 \\
  y &= 3x - 1
\end{align*} \)
6. \( \begin{align*}
  y &= x^2 - 5x + 3 \\
  y &= 3x - 7
\end{align*} \)

**Chapter 4 Quiz 1**

Form G

**Lessons 4.1 through 4.4**

**Do you know HOW?**

Graph each function.

1. \( y = -x^2 + 3x + 2 \)
2. \( y = -x + 3x^2 - 4 \)

**Chapter 4 Quiz 2**

Form G

**Lessons 4.5 through 4.9**

**Do you know HOW?**

Complete the square.

1. \( x^2 + 9x + \frac{81}{4} \)
2. \( 2x^2 - 12x + 9 \)

Evaluate the discriminant of each equation. Determine how many real solutions each equation has.

3. \( 2x^2 - 4x + 5 = 0; 0 \)
4. \( x^2 - 11x + 10 = 0; 1 \)

Simplify each expression.

5. \( \left( \frac{2}{3} + 4 \right) \left( \frac{1}{3} - 2 \right) = \left( \frac{2}{3} \right) \left( \frac{1}{3} \right) - \left( \frac{2}{3} \right) \left( 2 \right) \)
6. \( \frac{2}{3} \left( \frac{1}{3} \right) = \frac{2}{9} \)

Solve each system using the Quadratic Formula.

7. \( \begin{align*}
  y &= x^2 - 2x + 8 \\
  y &= 2x + 3
\end{align*} \)
8. \( \begin{align*}
  y &= x^2 + x + 3 \\
  y &= x^2 + 2x + 5
\end{align*} \)

Do you UNDERSTAND? **Open Ended**

Write a system of two quadratic equations that has exactly one ordered pair as the solution. Answer may vary. Sample: \( y = x^2 \) and \( y = x^2 - 1 \), while \( x^2 \)-terms cannot be further simplified.

10. **Open Ended**

Write a system of two quadratic equations that has exactly two ordered pairs as the solution. Answer may vary. Sample: \( y = x^2 + 1 \) and \( y = x^2 + 2 \)

11. **Reasoning**

Is it possible for a system of two quadratic equations to have exactly three ordered pairs for the solution? If so, give an example. no
Chapter 4 Chapter Test
Form G

Do you know HOW?

1. Write the equation of the parabola in standard form. Find the coordinates of the points on the other side of the axis of symmetry corresponding to P and Q. Label these points P’ and Q’, respectively. 
   \[ y = -x^2 - 6x - 5 \] 
   P: (-3, -14)  Q: (1, -10)

2. Graph each quadratic function. Name the axis of symmetry and the coordinates of the vertex.
   \[ y = x^2 - 4x + 4 \] 
   Vertex: (2, 0)  Axis of symmetry: x = 2

3. Graph each quadratic function. The given vertex and through the given point.
   \[ y = (x - 3)^2 + 2 \] 
   Vertex: (3, 2)  Point: (5, 0)

4. Write each quadratic equation in standard form. Find the coordinates of the points on the other side of the axis of symmetry corresponding to the vertex.
   \[ y = x^2 - 5x \] 
   Vertex: (2.5, -2.5)  Point: (1, -4)

5. Write each expression in factored form. Simplify each expression.
   \[ 32x^3 - 27x^2 = 9x^2(4x - 3) \]

6. Write each quadratic equation in standard form. Solve each quadratic equation.
   \[ x^2 - 16 = 0 \] 
   Solutions: x = 4, -4

7. Write each function in vertex form. Sketch the graph of the function and label its vertex.
   \[ y = (x - 2)^2 + 3 \] 
   Vertex: (2, 3)

8. Write each function in vertex form. Sketch the graph of the function and label its vertex.
   \[ y = 2(x - 1)^2 - 5 \] 
   Vertex: (1, -5)

9. Write each function in vertex form. Sketch the graph of the function and label its vertex.
   \[ y = -(x + 3)^2 + 2 \] 
   Vertex: (-3, 2)

10. Write each function in vertex form. Sketch the graph of the function and label its vertex.
    \[ y = (x - 4)^2 - 1 \] 
    Vertex: (4, -1)

11. Find the maximum area that can be enclosed by a rectangular picture frame possible. Anthony has 10 ft of framing and wants to make the largest rectangular picture frame possible. Find the maximum area that can be enclosed by his frame. 
    a. 25 ft²

12. Open-Ended Write a complex number with an absolute value between 3 and 8. 
    Answers may vary. Sample: 3 - 4i

Chapter 4 Part A Test
Form K

Do you know HOW?

1. Write each quadratic equation in standard form.
   \[ y = 3x^2 + 2x - 10 \]

2. Graph each function.
   \[ y = x^2 - 2x - 1 \]

3. Identify the axis of symmetry, minimum or maximum value, domain, and range of each function.
   \[ y = -x^2 - 2x - 15 \] 
   Vertex: (-1, -17)  Axis of symmetry: x = -1  Minimum: -17  Domain: all real numbers  Range: all real numbers ≤ -17

4. Write each expression in factored form.
   \[ 2x(x + 1) \]

5. Sketch a graph of the quadratic function with the vertex (0, 0) and through the point (2, 4). Then, write the equation of the parabola in vertex form and describe how the function was transformed from the parent function.
   y = x^2 + 2; The parabola shifted to the right 2 and up 6.

Chapter 4 Part A Test (continued)
Form K

Do you know HOW?

1. Solve each equation by factoring the perfect square trinomial.
   \[ x^2 - 3x - 10 = 0 \] 
   Solutions: x = 5, -2

2. Solve each system.
   \[ y = x^2 + 2x - 3 \]
   \[ y = x^2 + 2x - 3 \]
   \[ y = x^2 - x + 2 \]
   \[ y = 2x + x - 6 \]

3. Solve the following systems of inequalities by graphing.
   \[ y < x^2 - 2x - 3 \]
   \[ y < x^2 - 2x - 3 \]

4. Do you understand?
   a. Will the ball go over the net? If not, tell if it hits the net on the way up or the way down? No, it will hit the net on the way down.
   b. Reasoning The vertex of a parabola is a point in Quadrant IV and (x, y) in the equation y = ax² + bx + c. How many real solutions will the equation ax² + bx + c = 0 have? None.
   c. Open-Ended Write a complex number with an absolute value between 3 and 8. Answers may vary. Sample: 3 - 4i

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1. \( x^2 + 3x - 4 = 0 \)
   \[ x = \frac{-3 \pm \sqrt{9 + 16}}{2} \]
   \[ x = -4 \text{ or } x = 1 \]

2. \( y = x^2 + 10x - 21 \)
   \[ x = -\frac{10}{2} \pm \frac{\sqrt{100 + 84}}{2} \]
   \[ x = -5 \pm 7 \]
   \[ x = 2 \text{ or } x = -12 \]

3. \( -3x^2 - 2x + 7 = 0 \)
   \[ x = \frac{2 \pm \sqrt{4 + 84}}{-6} \]
   \[ x = -\frac{1}{3} \text{ or } x = 1 \]

Find the discriminant and determine the number of real solutions for each equation.

10. \( x^2 + 2x - 6 = 0 \)
    - 11: 0 real solutions

Solve the following systems of equations.

13. \( y = x^2 - 2x - 1 \), \( y = x - 1 \)
    - Graphing

14. \( y = x^2 + 4x - 6 \)
    \( x = -2 \pm \sqrt{4 + 24} \)
    - Completing the square

15. \( y = x^2 - 2x - 2 \)
    \( (1, -1) \) and \( (4, -6) \)
    - Writing

Graph each number on the complex plane. Then find its absolute value.

16. \( 7 - 2i \)
17. \( 6i \)
18. \( -2 + 3i \)

Give the coordinates of the vertex and the equation of the axis of symmetry.

19. Give an example of when you would use a linear model to model data.
   - A linear model is used when the relationship between the variables is linear.

20. Give an example of when you would use a quadratic model to model data.
   - A quadratic model is used when the relationship between the variables is quadratic.

21. Writing
   Explain how you would use the Quadratic Formula to solve \( 3x^2 + 5 = x + n \).
   - First, rewrite the equation in \( ax^2 + bx + c = 0 \) form. Then determine the values of \( a, b, \) and \( c \). Use these in the Quadratic Formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

22. Writing
   Explain the relationship between the \( x \)-intercepts of quadratic function and the zeros of a quadratic function.
   - The \( x \)-intercepts are the \( x \)-coordinates where the quadratic function equals zero.

23. The period of a pendulum is the time the pendulum takes to swing back and forth. The function \( l = \frac{1}{2} \pi \sqrt{\frac{L}{g}} \) relates the length of a pendulum to the period.
   - a. If a pendulum is 30 ft long, what is the period of the pendulum in seconds? \( p = 6.1 \text{ s} \)
   - b. Reasoning
      Why does only one of the solutions work here? The other solution is negative and you cannot have negative time.

The problem asks you to find the value of \( k \) that would make \( 4x^2 - 2kx + 100 \) a perfect square trinomial. You said \( k = \pm 40 \). Your friend said \( k = 40 \). Who is correct? What mistake was made?

Your friend is correct. You forgot to factor 4 and 100.
Chapter 4 Project Teacher Notes: On Target

About the Project

The Chapter 4 Project gives students an opportunity to use quadratic equations in real-life situations. Students use tables and graphs to study the paths of arrows. Students also make a three-dimensional target or a moving video target. Students display the target and their other findings.

Introducing the Project

- Encourage students to keep all project-related materials in a separate folder.
- Ask students to show the path of an arrow if it is aimed horizontally. Ask them how the paths changes if the arrow is aimed upward and to name the shape of this path.
- Have students look at Activity 4. Encourage them to start this part of the project now so they will have time to complete good targets.

Activity 1: Graphing

Students graph possible parabolic paths for arrows shot while standing or while seated in a wheelchair. Then, they identify the similarities and differences between their graphs.

Activity 2: Analyzing

Students find a parabolic model for the path of an arrow.

Activity 3: Modelling

Students graph data relating the weight of an arrow to its spine, the distance the center of the arrow bends when a constant weight is attached. Then, they decide whether a linear model or a quadratic model is a better fit for the data.

Activity 4: Researching

Students research archery styles using three-dimensional targets or moving video targets, and then create their own targets.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results. Ask students to review their project work and update their folders.

- Have students review their methods for making their graphs, for setting equations to model the graph, and for creating the targets for the project.
- Ask groups to share insights from experiments that resulted from completing the project, such as shortcuts they found for graphing, for modeling, or for making their targets.
Chapter 4 Project Manager: On Target

Getting Started
Read the project. As you work on the project, you will need a calculator, a measuring tape, materials on which you can record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist
☐ Activity 1: sketching parabolic paths
☐ Measure standing and sitting height to your shoulder.
☐ Activity 2: finding equations
☐ Use given information to write the equation of a parabola. Find the vertex of the parabola.
☐ Activity 3: modeling using regression
☐ Activity 4: researching archery
☐ Determine whether there is an archery club in your area.
☐ Presentation
☐ How might a videotaped presentation be more useful in studying parabolic paths of arrows than a real-life demonstration?

Scoring Rubric
4 Calculations are correct. Graphs are neat, accurate, and labeled correctly, and they clearly show the differences between the situations. Explanations are thorough and well thought out. The target is designed well and neatly made. The display is well organized.
3 Calculations and explanations are mostly correct, with some minor errors. Graphs are neat and mostly accurate with minor errors in scale. The target is designed adequately, but is not neatly made. The display presents clear information, but is not well organized.
2 Calculations contain both minor and major errors. Graphs are not accurate. Explanations lack detail. The target is poorly designed.
1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project
Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of Project

Activity 3: Modeling
Arrows need to use arrows that do not bend easily. The table shows how the weight of an arrow affects its spine, or the distance the center of the arrow bends when a certain constant weight is attached. Graph the data in the table to find a linear and a quadratic model for the data. Use the regression feature on your calculator to find each model. Which model is a better fit? Explain.

<table>
<thead>
<tr>
<th>Weight (in grams)</th>
<th>140</th>
<th>150</th>
<th>170</th>
<th>175</th>
<th>205</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (in inches)</td>
<td>1.4</td>
<td>1.25</td>
<td>0.93</td>
<td>0.78</td>
<td>0.43</td>
</tr>
</tbody>
</table>

linear: \( y = -0.0152x + 3.513 \)
quadratic: \( y = 0.000056x^2 - 0.0046x + 3.172 \)

Activity 4: Researching
Research the new styles of archery that use three-dimensional targets or moving video targets. Create one of these targets using readily-available materials or a computer program. Check students’ work.

Finishing the Project
The activities should help you to complete your project. Present your project for this chapter as a visual display, a demonstration or, if equipment is available, as a videotape.

Reflect and Revise
Present your information to a small group of classmates. Decide if your work is complete, clear, and convincing. If needed, make changes to improve your presentation.

Extending the Project
Interview an archer. Find techniques archers use to increase the range and accuracy of a shot.

Answers may vary. Sample: The quadratic model is a better fit. The graph of the quadratic function that models the data is closer to more of the points than is the graph of the linear function that models the data.

Weight (in grams) | Weight (in inches)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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5-1 ELL Support
Polynomial Functions

Determine the degree of the polynomial function with the given data.

29. behavior and number of turning points.
   28. end behavior of the graph of each polynomial function.

5-1 Practice Form G
Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. $4x + x^2$
   5. $3x^2 - x^2 - 1$
   9. $x^3$

2. $2x - 3x^2 + 1$
   6. $x^3 - 2x^2$
   10. $4x^3 - 2x + 1$

3. $x^2 - x^2 + 1$
   7. $3x^3 - 2x^2 - 1$
   11. $x^2 + x + 1$

4. $x^3 - x^2 + 1$
   8. $x^2 - x^2$
   12. $x^3 - 2x^2 - 1$

Determine the end behavior of the graph of each polynomial function.

22. $y = 3x^2 + 6x - x^2 + 2$
   23. $y = 3x^2 - 2x^2 + 2$
   25. $y = 3x^2 + 9 - 5x^2 - 3$

24. $y = -x^2 - x + 2$
   26. $y = 12x^2 - x + 2$
   28. $y = 3x^2 - 2x^2 + 2$

Describe the shape of the graph of each cubic function by determining the end behavior and number of turning points.

31. $y = x^3 + 4x$
   32. $y = -2x^3 + 3x - 1$
   33. $y = 5x^3 + 6x^2$

Determine the degree of the polynomial function with the given data.

34. $y = 3x^3 + 2x$ 3rd degree
   35. $y = 4x^3 + 5$ 4th degree

5-1 Think About a Plan
Polynomial Functions

Package Design. The diagram at the right shows a cologne bottle that consists of a cylindrical base and a hemispherical top.

a. Write an expression for the cylinder’s volume.
   b. Write an expression for the volume of the hemispherical top.
   c. Write a polynomial to represent the total volume.

5. What is the formula for the volume of a cylinder? Define any variables you use in your formula.
   $V = \pi r^2 h$, where $r$ is the radius of the base and $h$ is the height.

2. Write an expression for the volume of the cylinder using the information in the diagram.
   $V = \pi r^2 h$

3. What is the formula for the volume of a sphere? Define any variables you use in your formula.
   $V = \frac{4}{3}\pi r^3$, where $r$ is the radius of the sphere.

4. Write an expression for the volume of the hemisphere.
   $V = \frac{1}{2}\pi r^3$

5. How can you find the total volume of the bottle?
   Add the volume of the cylinder and the volume of the hemisphere.

6. Write a polynomial expression to represent the total volume of the bottle.
   $V = \pi r^2 h + \frac{1}{2}\pi r^3$

7. Is the polynomial expression you wrote in simplest form? Explain.
   Yes, you cannot combine the terms because they are not like terms.

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### 5-1 Practice
**Form K**

#### Polynomial Functions

Write each polynomial in standard form. Then classify it by degree and by number of terms.

1. \(4x^2 - 3 + x^3\)
2. \(2x^3 - 2x^2 - 2x + 4\)
3. \(x^5 + 2x^4 - 3x^3 + x^2\)
4. \(-2x^5 + 3\)
   - **3rd degree, 3 terms**
   - **5th degree, 4 terms**
   - **4th degree, 3 terms**
   - **3rd degree, 1 term**
   - **3rd degree, 2 terms**, 2 terms

Determine the end behavior of the graph of each polynomial function.

- **3rd degree, 3 terms**
- **4th degree, 3 terms**
- **5th degree, 4 terms**

Describe the shape of the graph of each cubic function by determining the end behavior and number of turning points.

- **3rd degree, 2 turning points**
- **2nd degree, no turning points**
- **4th degree, up and down, 2 turning points**
- **5th degree, 3 turning points**

Determine the degree of the polynomial function with the given data.

#### Standardized Test Prep

**Multiple Choice**

For Exercises 1–7, choose the correct letter.

1. Which expression is binomial?  D
   - **x**
   - **(x + 3)**
   - **(x + 3)**
   - **x - 9**

2. Which polynomial function has an end behavior of up and down?  A
   - **6x^2 - 2x^2 - 3x + 4**
   - **3x^4 - 2x^3 + x^2 + 2x - 1**
   - **5x^3 - 1x^2 + 2x - 1**

3. What is the degree of the polynomial \(5x + 4x^2 + 3x^3 - 5x^7\)?  C
   - **1**
   - **2**
   - **3**
   - **4**

4. What is the degree of the polynomial represented by the data in the table at the right?  G
   - **-3**
   - **77**
   - **0**
   - **1**

5. For the table of values at the right, if the 4th differences are constant, what is the constant value?  B
   - **-12**
   - **-5**
   - **0**
   - **2**

6. What is the standard form of the polynomial \(2x^5 + 3x^4 + 2x^3 - x^2 + 7\)?  C
   - **2x^5 + 3x^4 + 2x^3 - x^2 + 7**
   - **2x^5 + 3x^4 + 2x^3 - x^2 + 7**
   - **2x^5 + 3x^4 + 2x^3 - x^2 + 7**

7. What is the number of terms in the polynomial \((2x^3 - 1x^2 + 9)\)  B
   - **2**
   - **3**
   - **4**
   - **5**

#### Short Response

8. Simplify \((3x^2 - 4x + 2) - (2x^2 + 5x + 1)**. Then name the polynomial by degree and the number of terms.
   - **3x^2 - 4x + 2 - 2x^2 - 5x - 1**
   - **1st degree, 4 terms**
   - **1st degree, 4 terms**
   - **1st degree, 4 terms**

9. Simplified polynomial is incorrect. Or simplified polynomial is incorrect, but degree and/or number of terms is incorrect. Or simplified polynomial is incorrect, but degree and terms are correct for the polynomial given.  C
   - **3x^2 - 4x + 2 - 2x^2 - 5x - 1**
   - **1st degree, 4 terms**
   - **1st degree, 4 terms**

10. No answers given.

### 5-1 Practice (continued)
**Form K**

#### Polynomial Functions

Determine the signs of the leading coefficient and the degree of the polynomial function for each graph.

- **Positive, 3rd degree**
- **Negative, 5th degree**
- **Positive, 2nd degree**

#### Error Analysis

A student claims the function \(y = -2x^3 + 5x - 7\) is a 3rd degree polynomial with ending behavior of down and up. Describe the error the student made. What is wrong with this statement?

- **The degree is odd and the leading coefficient is negative, so the ending behavior should be up and down.**

#### Table of Values

The table to the right shows data representing a polynomial function.

- **1st degree, 2 terms**
- **2nd degree, 3 terms**
- **3rd degree, 4 terms**
- **4th degree, 5 terms**

Classify each polynomial by degree and by number of terms. Simplify first if necessary.

- **2x^5 - 6x^3 - 5x^2 + 4x - 1**
  - **5th degree, 3 terms**
  - **2nd degree, 2 terms**
- **(2x^3 - 4x^2 - 3x^2 + 4)**
  - **2nd degree, 2 terms**
  - **3rd degree, 2 terms**
- **(5x^4 - 2x^2) + (x^3 + 1)**
  - **4th degree, 4 terms**
  - **3rd degree, 2 terms**
- **(2x + 1)(x + 3)**
  - **2nd degree, 3 terms**
  - **0 degree, 1 term**

#### Open-Ended

Write a 4th degree polynomial function. Make a table of values and a graph. Check students’ work.

### 5-1 Enrichment
**Polynomial Functions**

Mathematicians use precise language to describe the relationships between sets. One important relationship is described as a function. You have graphed polynomial functions. Using this one word may not seem important, but it describes a very specific relationship between the domain and range of a polynomial. The word function tells you that every element of the domain corresponds with exactly one element of the range.

- **Another important relationship between two sets is described by the word onto.**

A function from set A to set B is onto if every element in set B is matched with an element in set A. Which of the following relations shows a function from set A to set B that is one-to-one?

- **The second relation is a function that is one-to-one because all of the elements in set A are paired with exactly one element of set A.**

Describe each polynomial function. If it is not possible, explain why.

- **Describe a polynomial function that is onto but not one-to-one.**
  - Answer may vary. Sample: A cubic with two turning points is onto because every y-value in the range is paired with an x-value, but not one-to-one because some y-values are paired with more than one x-value.

- **Is there a polynomial function that is one-to-one but not onto?**
  - Yes. Answer may vary. Sample: Odd degree polynomials are one-to-one, but they are also onto.

- **Describe a polynomial function that is both onto and one-to-one.**
  - Answer may vary. Sample: The cubic \(y = x^3\) is onto because every y-value is paired with an x-value, and it is one-to-one because every y-value is paired with exactly one x-value.
5-1 Reteaching Polynomial Functions

Problem
What is the classification of the following polynomial by its degree? by its number of terms? What is its end behavior?

Step 1 Write the polynomial in standard form. First, combine any like terms. Then, place the terms of the polynomial in descending order from greatest exponent value to least exponent value:

\[5x^4 - 3x^3 + 4x^2 + 3x^2 - 12 - x - 3x^4\]

Combine like terms.

Place terms in descending order.

Step 2 The degree of the polynomial is equal to the value of the greatest exponent. This will be the exponent of the first term when the polynomial is written in standard form.

\[\boxed{6}\]
The degree of the polynomial is 6.

Step 3 Count the number of terms in the simplified polynomial. It has 5 terms.

Step 4 To determine the end behavior of the polynomial (the directions of the graph to the far left and to the far right), look at the degree of the polynomial (6) and the coefficient of the leading term (a).

If a is positive and n is even, the end behavior is up and up.

If a is positive and n is odd, the end behavior is down and up.

If a is negative and n is even, the end behavior is down and down.

If a is negative and n is odd, the end behavior is up and down.

The leading term in this polynomial is \(a \times x^n\). If a is positive and \(n\) is positive, then the end behavior is up and up.

Exercises
What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?

1. \(8 - 4x^3 + x^3 - 2\)
   - 3rd degree; 3 terms; up and down

2. \(15x^7 - 7\)
   - 7th degree; 2 terms; down and up

3. \(2x - 6x^3 - 9\)
   - 2nd degree; 3 terms; down and down

5-2 ELL Support Polynomials, Linear Factors, and Zeros

There are two sets of note cards below that show how to write a cubic polynomial function with zeros of \(-3, 1, 4\) and 0 in standard form. The set on the left explains the thinking. The set on the right shows the steps. Write the thinking and the steps in the correct order.

Think Cards

- Multiply \((x - 3)\) and \((x - 4)\).
- Use the Distributive Property to multiply \(x + 3\) and \((x - 5x + 4)\).
- Simplify.

Write Cards

- Write a linear factor for each zero.

Think

First, you should write a linear factor for each zero.

Second, you should multiply \((x - 1)\) and \((x - 6)\).

Then, you should use the Distributive Property to multiply \(x + 3\) and \((x - 5x + 4)\).

Finally, you should simplify.

Write

\[f(x) = (x + 3)(x - 1)(x - 4)\]

Step 1 \(f(x) = (x + 3)(x - 1)(x - 4)\)

Step 2 \((x + 3)(x - 1)(x - 4)\)

Step 3 \((x - 1)(x - 4)\)

Step 4 \(x^3 - 3x^2 - 11x + 12\)

5-2 Think About a Plan Polynomials, Linear Factors, and Zeros

Measurement: The volume of a cubic foot of a CD holder can be expressed as

\[V(x) = -x^3 - x^2 + 6x\]

Assume that the height is greater than the width.

a. Factor the polynomial to find linear expressions for the height and the width.

b. Graph the function. Find the x-intercepts. What do they represent?

c. What is the volume of the CD holder?

What is the maximum volume of the CD holder? How do you know which factor is the height and which factor is the width?

Graph the function on a graphing calculator. How can you find the x-intercepts?

Set the factors equal to zero and solve or use the zero feature.

What are the x-intercepts? What do they represent?

-3, 0, 2; values of \(x\) that result in a volume of 0.

What are the limits of each of the factors? What is a realistic domain for the function? Explain.

Each dimension of the CD holder must be greater than 0, so each factor should be greater than 0. So \(2 - x > 0\), \(x > 0\), and \(x + 3 > 0\). The solution of all these inequalities is \(0 < x < 2\). To find the maximum volume, take the derivative of the function and find the critical points.

Find the maximum volume of the function on the domain \(0 < x < 2\). Use the maximum feature of the CALC menu with Left bound = 0 and Right bound = 2.

What is the maximum volume of the CD holder? About 4.00 ft³
Find the zeros of each function. Then graph the function.

19. $y = x^3 - 2x^2 + 3x - 1$
20. $y = x^3 - 4x^2 + 5x - 2$
21. $y = x^3 - 2x^2 + x - 1$
22. $y = x^3 + 2x^2 - 3x - 1$
23. $y = x^3 - 3x^2 + 2x - 1$
24. $y = x^3 + x^2 - 5x + 6$
25. $y = x^3 - x^2 - 3x + 2$
26. $y = x^3 + x^2 - 4x - 4$

Write a polynomial function in standard form with the given zeros.

0, multiplicity 2; 4, 5

11. $y = (x - 4)(x - 5)(x + 2)$
12. $y = (x - 4)^2(x + 5)$
13. $y = (x - 4)(x - 5)(x + 2)^2$
14. $y = (x - 4)^3(x + 5)$
15. $y = (x - 4)(x - 5)^2(x + 2)$
16. $y = (x - 4)^2(x - 5)(x + 2)^2$

Writing a polynomial function in standard form with given zeros.

17. $y = x^3 - 2x^2 + x - 1$
18. $y = x^3 + 2x^2 - 3x - 1$
19. $y = x^3 - 3x^2 + 2x - 1$
20. $y = x^3 + x^2 - 5x + 6$
21. $y = x^3 - x^2 - 3x + 2$
22. $y = x^3 + x^2 - 4x - 4$

Find the zeros of each function. State the multiplicity of each zero.

$y = x^3 - 2x^2 + x - 1$
$y = (x - 4)(x - 5)(x + 2)$
$y = (x - 4)^2(x + 5)$
$y = (x - 4)(x - 5)^2(x + 2)$
$y = (x - 4)^2(x - 5)(x + 2)^2$
$y = (x - 4)(x - 5)^2(x + 2)^2$

Find the zeros of each function. Then graph the function.

6. $y = x^3 - 2x^2 + x - 1$
7. $y = x^3 - 3x^2 + 2x - 1$
8. $y = x^3 - 4x^2 + 5x - 2$

Writing a polynomial function in standard form with the given zeros.

$y = x^3 - 2x^2 + x - 1$
$y = (x - 4)(x - 5)(x + 2)$
$y = (x - 4)^2(x + 5)$
$y = (x - 4)(x - 5)^2(x + 2)$
$y = (x - 4)^2(x - 5)(x + 2)^2$
$y = (x - 4)(x - 5)^2(x + 2)^2$

Find the zeros of each function. State the multiplicity of each zero.

15. $y = x^3 - 2x^2 + x - 1$
16. $y = (x - 4)(x - 5)(x + 2)$
17. $y = (x - 4)^2(x + 5)$
18. $y = (x - 4)(x - 5)^2(x + 2)$
19. $y = (x - 4)^2(x - 5)(x + 2)^2$

Find the relative maximum and relative minimum of the graph of each function.

25. $f(x) = x^3 - 7x + 12$
26. $f(x) = x^3 - x^2 - 3x + 2$
27. $f(x) = x^3 + x^2 - 4x - 4$
28. $f(x) = x^3 - 6x + 9$

30. Reasoning: A polynomial function has a zero at $x = -a$. Find one of its factors.

31. The side of a cube measures 3 units long. Express the volume of the cube as a polynomial. $27x^3 = 54x^3 + 36x^2 + 8x$
5-2 Standardized Test Prep
Polynomials, Linear Factors, and Zeros

Multiple Choice
For Exercises 1–6, choose the correct letter.

1. What are the zeros of the polynomial function \( f(x) = (x - 3)(x + 1)(x - 1) \)?
   - A. \( x = 3, x = -1, x = 1 \)
   - B. \( x = -3, x = 1, x = 1 \)
   - C. \( x = 3, x = -1, x = -1 \)
   - D. \( x = -3, x = 1, x = 1 \)

2. What is the factored form of \( 2x^2 + 5x - 3 \)?
   - A. \( (x + 3)(2x - 1) \)
   - B. \( (x - 3)(2x + 1) \)
   - C. \( (x + 3)(2x + 3) \)
   - D. \( (x - 3)(2x - 1) \)

3. Which is the cubic polynomial in standard form with roots 5, -6, and 1?
   - A. \( x^3 - 2x^2 - 18x \)
   - B. \( x^3 - 2x^2 + 18x \)
   - C. \( x^3 + 2x^2 - 18x \)
   - D. \( x^3 + 2x^2 + 18x \)

4. What is the relative minimum and relative maximum of \( f(x) = 6x^2 - 5x + 127 \)?
   - A. min 0, max 127
   - B. min 127, max 0
   - C. min -5, max 127
   - D. min 10.2, max 13.8

5. What is the multiplicity of the zero of the polynomial function \( f(x) = (x + 3)^2 \)?
   - A. 4
   - B. 5
   - C. 20
   - D. 620

6. For the polynomial function \( f(x) = (x - 2)^3 \), which behavior will the graph exhibit at the \( x \)-intercept?
   - A. linear
   - B. quadratic
   - C. cubic
   - D. quartic

Short Response
7. A rectangular box is 24 in. long, 12 in. wide, and 18 in. high. If each dimension is increased by \( x \) in, what is the polynomial function in standard form that models the volume \( V \) of the box? Show your work.

8. Each zero (you can write the polynomial.

9. Find the zeros of the function \( f(x) = x^2 - 5x + 6 \).
   - A. zero 1, 2
   - B. zero 2, 2
   - C. zero 1, 1
   - D. zero 3, 1

10. What are the zeros of the polynomial function \( f(x) = 2x^2 - 8x + 6 \)?
    - A. zero 2, 3
    - B. zero 3, 1
    - C. zero 1, 1
    - D. zero 2, 2

5-2 Enrichment
Polynomials, Linear Factors, and Zeros

Fast Factoring of Monic Quadratic Trinomials

Thus far, you have factored trinomials primarily by trial and error. This method can be quite slow, especially if the constant term of a quadratic trinomial is large, such as 72, since there are many different ways to factor this number: 1 and 72, 2 and 36, 3 and 24, and 8 and 9. With larger numbers, the trial-and-error method becomes time-consuming.

A polynomial is called monic if the coefficient of the term of highest degree is 1. Monic quadratic trinomials can be factored quite rapidly using a combination of difference of squares and equation solving.

Suppose that we wish to factor the monic quadratic trinomial \( x^2 + bx + c \). Assume there are numbers \( r \) and \( s \) such that

\[ r + s = b \]
\[ rs = c \]

Subtracting \( c \) from both sides results in

\[ r + s = b \]
\[ r - s = \sqrt{b^2 - 4c} \]

Therefore, \( b = 2r \) and \( r = x^2 - x \).

Solve for \( r \) and \( s \) in terms of \( b \) and \( c \):

\[ r = \frac{b + \sqrt{b^2 - 4c}}{2} \]
\[ s = \frac{b - \sqrt{b^2 - 4c}}{2} \]

The following example shows how this technique works.

Factor: \( x^2 - 10x + 26 \)

\[ x = \frac{10 \pm \sqrt{(10)^2 - 4(1)(26)}}{2(1)} \]
\[ x = \frac{10 \pm \sqrt{100 - 104}}{2} \]
\[ x = \frac{10 \pm \sqrt{-4}}{2} \]

So \( r = 6 \) and \( s = 4 \).

Verify that this is the correct factorization:

\[ (x - 6)(x - 4) = x^2 - 10x + 24 \]

Use this technique for each of the following.

1. \( x^2 - 4x - 21 \)
2. \( x^2 + 5x + 6 \)
3. \( x^2 - 12x + 27 \)
4. \( x^2 + 7x - 18 \)
5. \( x^2 - 5x - 6 \)
6. \( x^2 - 10x + 25 \)
7. \( x^2 + 16x + 25 \)
8. \( x^2 + 18x + 49 \)

5-2 Reteaching (continued)
Polynomials, Linear Factors, and Zeros

You can use a polynomial function to find the minimum or maximum value of a function that satisfies a given set of conditions.

Problem
Your school wants to put in a swimming pool. The school wants to maximize the volume while keeping the sum of the dimensions at 40 ft. The length must be 2 times the width, what should each dimension be?

Step 1 First, define a variable. Let \( x = \) the width of the pool.

Step 2 Determine the length and depth of the pool using the information in the problem.

The length must be 2 times the width, so length = \( 2x \).

The length plus width plus depth must equal 40 ft, so \( 2x + x + r = 40 \).

Step 3 Create a polynomial in standard form using the volume formula

\[ V = \text{length} \times \text{width} \times \text{depth} \]

\[ V = (2x)(x)(r) \]

\[ V = 2x^2 r \]

Step 4 Graph the polynomial function. Use the MAXIMUM feature. The maximum volume is 2107 ft³ as a width of 9 ft.

Step 5 Evaluate the remaining dimension: width = 9 ft

- length = 18 ft
- depth = 8 ft

Exercises
11. Find the dimensions of the swimming pool if the sum must be 50 ft and the length must be 3 times the depth.

12. Find the dimensions of the swimming pool if the sum must be 40 ft and the depth must be one tenth of the length.

13. Find the dimensions of the swimming pool if the sum must be 60 ft and the length and width are equal.

Answer: length = 20 ft, width = 20 ft, depth = 20 ft
5-3 **Solving Polynomial Equations**

### Problem
What are the real or imaginary solutions of the equation $x^2 + 3x = 0$?

Explain your work.

### Exercises
What are the real or imaginary solutions of the equation $2x^2 + 3x = 0$?

Explain your work.

### Practice
**Form G**

Find the real or imaginary solutions of each equation by factoring.

1. $x^2 - 2x + 1 = 0$
2. $x^2 + 4x + 4 = 0$
3. $x^2 + x - 2 = 0$
4. $x^2 - 5x + 6 = 0$
5. $x^2 - 10x + 24 = 0$
6. $x^2 - 6x + 8 = 0$
7. $x^2 - 4x + 4 = 0$
8. $x^2 - 9x + 18 = 0$
9. $x^2 - 5x + 6 = 0$
10. $x^2 - 3x + 2 = 0$
11. $x^2 - 4x + 4 = 0$
12. $x^2 - 5x + 4 = 0$
13. $x^2 - 6x + 5 = 0$
14. $x^2 - 7x + 10 = 0$
15. $x^2 - 8x + 16 = 0$
16. $x^2 - 9x + 8 = 0$
17. $x^2 - 10x + 25 = 0$
18. $x^2 - 12x + 36 = 0$
19. $x^2 - 14x + 49 = 0$
20. $x^2 - 16x + 64 = 0$

Find the real solutions of each equation by graphing.

17. $x^2 - 8x + 16 = 0$
18. $x^2 - 10x + 25 = 0$
19. $x^2 - 12x + 36 = 0$
20. $x^2 - 16x + 64 = 0$

For Exercises 23 and 24, write an equation to model each situation. Then solve each equation by graphing.

23. The volume of a container is 84 ft$^3$. The width, the length, and the height are $x$, $x + 1$, and $x + 2$, respectively. What are the container’s dimensions?

24. The product of three consecutive integers is $-110$. What are the integers?

### Think About a Plan
**Solving Polynomial Equations**

### Geometry
The width of a box is 2 m less than the length. The height is 1 m less than the length. The volume is 60 m$^3$. What is the length of the box?

### Know
1. The volume of the box is 60 m$^3$.
2. The formula for the volume of a rectangular prism is $V = l \times w \times h$.
3. The length of the box is equal to the width.
4. The height of the box is equal to the length.

### Need
5. To solve the problem I need to:
   - write an equation expressing the volume of the box two ways and solve the equation for the length of the box.

### Plan
6. Define a variable. Let $x =$ length.
7. What variable expressions represent the width and the height of the box?
   - $x - 2$ and $x - 1$.
8. What equation expresses the volume of the box in two ways?
   - $x(x - 2)(x - 1) = 60$
9. How can you use a graphing calculator to help you solve the equation?
10. What is the solution of the equation?
11. What are the dimensions of the box? Are the solutions reasonable?

### Practice (continued)
**Form G**

Solve each equation.

25. $x^2 - 5x + 6 = 0$
26. $x^2 - 9x + 20 = 0$
27. $x^2 - 16x + 64 = 0$
28. $x^2 - 10x + 25 = 0$
29. $x^2 - 14x + 49 = 0$
30. $x^2 - 21x + 10 = 0$
31. $x^2 - 12x + 36 = 0$
32. $x^2 - 16x + 64 = 0$

33. Over 3 years, you saved your earnings from a summer job. The polynomial $100x^3 - 1200x^2 + 4000$ represents your savings, with interest, at the end of the 3 years. The annual interest rate equals $x - 1$. Find the interest rate needed so that you will have $4000 at the end of 3 years.

34. Error Analysis: Your friend claims that the roots of $x^2 - 7x + 12 = 0$ are 4, 2, and 0. What did your friend do wrong? What are the correct factors? Your friend forgot to divide by $x$ when solving an equation to find the third factor. The correct factors are $(x - 4)(x - 2)$.

35. The container at the right consists of a cylinder on top of a hemisphere. The container holds 100 cu cm. What is the radius of the container, to the nearest hundredths of a centimeter? 3.58 cm

36. Suppose a 2-in. slice is cut from one face of the cheese block as shown. The remaining block has a volume of 224 in$^3$.

37. Reasoning: A test question asks you to find three integers whose product is 612. Do you have enough information to solve the problem? Explain. Yes, there are multiple solutions.

38. Your mother is 23 years older than you. Your father is 3 years older than your mother. The product of all three ages is 52,130. How old is your father?
5-3 Practice  
Solving Polynomial Equations

Find the real or imaginary solutions of each equation by factoring.
1. \( x^2 + 10x + 25 = 0 \)
   
2. \( 2x^2 - 3x - 2 = 0 \)
   
3. \( x^3 - 8x^2 + 16x = 0 \)

Solve each equation.
4. \( x^2 - 4 = 0 \)
5. \( x^2 + 4x + 4 = 0 \)
6. \( x^2 - 9 = 0 \)

11. \( 2x^2 + 7x + 3 = 0 \)
12. \( x^2 - 1 = 0 \)
13. \( x^2 - 1 = 0 \)

14. Writing: Show how you can rewrite \( \frac{1}{2} - a \) as a difference of two cubes.

5-3 Practice (continued)  
Solving Polynomial Equations

Find the real solutions of each equation by graphing.

15. \( x^2 - 3x - 4 = 0 \)
16. \( x^2 - 5x + 6 = 0 \)
17. \( x^2 - 6x + 10 = 0 \)

For Exercises 22–25, write an equation to model each situation. Then solve each equation by graphing.

22. The volume \( V \) of a container is 64 in. \(^3\). The width, the length, and the height are \( x \), \( x \), and \( x + 1 \) respectively. What are the container’s dimensions?

23. The product of three consecutive integers is 720. What are the numbers?

24. The length of a box is 3 cm less than the width. The length is 2 cm less than the height. The volume is 50 cm \(^3\). What is the width of the box?

25. Your sister is 8 years older than you. Your mother is 25 years older than your sister. The product of all three ages is 18,816. How old is your mother?

5-3 Standardized Test Prep  
Solving Polynomial Equations

Multiple Choice

For Exercises 1–6, choose the correct letter.

1. If you factor \( x^2 - 8 \) in the form \((x - a)(x + b) + c\), what is the value of \( a\)?
   
2. The product of three integers is \( x \), \( x \), and \( x + 5 \). What are the integers? 

3. Over 3 years, you save $500, $600, and $700 from babysitting jobs. The polynomial \( 500x^2 + 600x + 700 \) represents your total bank account balance after 3 years. The annual interest rate is \( x \). What is the interest rate needed so that you will have $2000 after 3 years? 

4. Which polynomial equation has the zeros \( -1, -3, \) and \( 1 \)?
   
5. Your brother is 3 years younger than you. Your sister is 4 years younger than you. The product of your ages is 1872. How old is your sister? 

6. What are the real roots of \( x^2 - 8 = 0 \)?

Short Response

7. You have a block of wood with a depth of \( z \) units, a length of \( 5z \) units, and a height of \( 2z \) units. You need to cut a slice off the top of the block to decrease the height by \( z \). The new block still will have a volume of 480 cubic units. 
   
a. What are the dimensions of the new block? 
   
b. What is the volume of the slice?

5-3 Enrichment  
Solving Polynomial Equations

Factoring \( x^2 + a^2 \)

There is a formula for factoring \( x^2 - a^2 \), a formula for factoring \( x^3 - a^3 \), and a formula for factoring \( x^3 + a^3 \). Why isn’t there a formula for factoring \( x^2 + a^2 \)?

Since \( x^2 + a^2 \) must be the product of two first-degree polynomials, assume that \( x^2 + a^2 = (x + bi)(x - bi) \). Use the distributive property to expand the expression on the right.

\( x^2 + a^2 = (x + bi)(x - bi) \)

Subtract \( a^2 \) from each side.

Use the quadratic formula to solve for the roots of the equation.

\( x^2 + a^2 = (x + bi)(x - bi) \)

If these two functions are the same, they must have the same graph, the same slope, and the same y-intercept.

1. Find the slope of \( y = x^2 \).
2. Find the slope of \( y = (x + c)(x - c) \).
3. What relationship must exist between \( a \) and \( c \) to get \( y = x^2 \)?

4. Find the y-intercept of \( y = x^2 \).
5. What relationship must exist among \( a \), \( b \), and \( c \) to make the original equation \( x^2 + a^2 = (x + bi)(x - bi) \)?

Solve for \( x^2 + a^2 = (x + bi)(x - bi) \) using the quadratic formula.

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13. Find the real or imaginary solutions of each polynomial equation.

What are the real or imaginary solutions of the polynomial equation

The solutions are ...

Exercises

Find the real or imaginary solutions of each polynomial equation.

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.  
10.  
11.  
12.  
13.  
14.  
15.  
16.  

Exercises

19. A slice of wood 3 in. thick is cut off a cube of wood. The remaining solid has a volume of 320 in. 3. What are the dimensions of the original block of wood? 8 in. × 8 in. × 8 in.

20. The water level in a cylindrical fish tank is 8 in. from the top. The depth of the water is the same as the width of the tank, which is half its length. The volume of the water in the tank is 4394 in. 3. What is the volume of the fish tank? 5764 in. 3
Determine whether each binomial is a factor of \( x^2 + 3x - 18 = 24 \).

12. Yes
13. Yes
14. No

Divide using synthetic division.

11. \((x^2 - 8x + 12x - 6) = (x - 5)\)
12. \((x^2 + 3x - 9x - 4) = (x + 2)\)
13. \((-2x^2 + 15x - 15x - 9) = (x + 1)\)
14. \((-x^2 + 5x + 5x + 25) = (x - 5)\)
15. \((3x^2 + 7x - 10) = (x - 2)\)
16. \((x^3 + 2x^2 + 2x + 1) = (x + 1)\)
17. \((x^3 + 5x^2 + 7x + 12) = (x - 4)\)
18. \((2x^3 + 2x^2 + 3x + 4) = (x + 1)\)

Use synthetic division and the given factor to completely factor each polynomial function.

19. \((x^3 + 2x^2 - 3x - 5) = (x - 1)\)
20. \((x^3 + 3x^2 - 4x + 2) = (x + 2)\)
21. \((x^3 + 2x^2 - 3x + 4) = (x - 2)\)
22. \((x^3 + 3x^2 - 2x + 1) = (x + 1)\)

36. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 - 3x + 11 \) divided by \( x - 3 \). Your friend says that the quotient is \( x^4 + 3x^3 + 18x^2 + 24x + 36 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.

45. What is \( P(x) = 2x^2 - 6x + 24 \)?
46. What is \( P(2) \) for \( P(x) = x^3 - 2x^2 + 4x - 8 \)?

40. The expression \( x^3 + 16x^2 + 60x + 60 \) represents the volume of a flower box in cubic inches. The expression \( x - 4 \) represents the depth of the box. Assume that the height is greater than the width.

42. If \( x^3 + 3x^2 - 2x + 1 \), what are the dimensions of the flower box? Height: \( x \) in.; length: \( 10 \) in.; width: \( x + 2 \) in.

46. If \( x^3 + 4x^2 - 6x + 4 \), what are the dimensions of the flower box? Height: \( x \) in.; length: \( 10 \) in.; width: \( x + 2 \) in.

32. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 - 2x + 1 \) divided by \( x - 1 \). Your friend says that the quotient is \( x^4 - x^3 + 2x^2 + x - 6 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.

36. Writing: What are the other dimensions of the flower box?

33. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 + 2x + 1 \) divided by \( x - 1 \). Your friend says that the quotient is \( x^4 + x^3 + x^2 + x + 1 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.

36. Writing: What does it mean if \( Q(x) \) is a factor of the polynomial function \( P(x) = x^3 + 4x^2 + 2x + 3 \)?
It means that \( x + 4 \) is a factor of the polynomial.

36. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 - 3x + 11 \) divided by \( x - 2 \). Your friend says that the quotient is \( x^4 + 3x^3 + 18x^2 + 24x + 36 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.

36. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 - 3x + 11 \) divided by \( x - 3 \). Your friend says that the quotient is \( x^4 + 3x^3 + 18x^2 + 24x + 36 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.

36. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 - 3x + 11 \) divided by \( x - 3 \). Your friend says that the quotient is \( x^4 + 3x^3 + 18x^2 + 24x + 36 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.

36. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 - 3x + 11 \) divided by \( x - 3 \). Your friend says that the quotient is \( x^4 + 3x^3 + 18x^2 + 24x + 36 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.

36. Error Analysis: Using synthetic division, you can see that the quotient is \( x^2 - 3x + 11 \) divided by \( x - 3 \). Your friend says that the quotient is \( x^4 + 3x^3 + 18x^2 + 24x + 36 \). What mistake was made?
Your friend is incorrect. You forgot to change the sign of the divisor from subtraction to addition.
5-4 Standardized Test Prep
Dividing Polynomials

Gridded Response
Solve each exercise and enter your answer in the grid provided.
1. What is P(2) given that P(x) = x^4 - 3x^3 + 5x - 10?
2. What is the missing value in the following synthetic division?
   | 4 | 0 | -1 | 2 | 12 | -4 | 1 | -4 | -10 | -12 |
   | 4 | -4 | -16 | -50 | 160 |
3. What is the remainder when x^4 - 4x^3 + 10x - 10 is divided by x + 3?
4. How many unique factors does x^4 - 4x^3 + 3x^2 - 14x + 8 have, including (x + 3)?
5. How many terms are there in the simplified form of \( \frac{x^4 - 2x^3 - 3x^2 + 12x + 36}{x - 6} \)?

Answers

| 1 | 4 | 2 | 0 | 1 | 2 | 1 | 2 | 3 | 4 | 5 |

5-4 Reteaching
Dividing Polynomials

Problem
What is the quotient and remainder? Use polynomial long division to divide
2x^3 + 6x - 7 by x + 1.

Step 1 Find the first term of the quotient, divide the highest-degree term of 2x^3 by the highest-degree term of the divisor, x + 1. Circle these terms before dividing.

Step 2 Multiply x + 1 by the term, 2x, in the quotient. 2x(x + 1) = 2x^2 + 2x. Align like terms.

Step 3 Subtract to get 4x. Bring down the next term, 7.

Step 4 Divide the highest-degree term of 4x + 7 by the highest-degree term of x + 1. Circle these terms before dividing.

Step 5 Repeat Steps 2 and 3. The remainder is 3 because its degree is less than the degree of x + 1.

Check the answer by multiplying \((x + 1)(2x^2 + 2x + 4)\) and adding 3, \((x + 1)(2x^2 + 2x + 4) + 3 = 2x^3 + 6x + 7\).

Exercises
Divide using polynomial long division.
1. \( (2x^3 + 4x^2 - 7) \div (x - 1) \)
2. \( (3x^3 - 2x^2 + 3x - 4) \div (x + 1) \)
3. \( (x^3 - 2x^2 + 5x - 3) \div (x - 3) \)
4. \( (x^2 + 3x - 4) \div (x - 2) \)

5-4 Reteaching (continued)
Dividing Polynomials

Problem
Use synthetic division to divide \( x^3 + 13x^2 + 46x + 48 \) by \( x + 3 \). What is the quotient and remainder?

Step 1 Set up your polynomial divisor.

Step 2 Reverse the sign of the constant, 13, in the divisor. Write the coefficients of the dividend: 1 13 46 48.

Step 3 Bring the first coefficient, 1, down to the bottom line.

Step 4 Multiply the coefficient, 1, by the divisor, -3. Put this product, -3, underneath the second coefficient, 13, and add those two numbers: 13 + (-3) = 10.

Step 5 Continue multiplying and adding through the last coefficient. The final sum is the remainder.

The quotient is \( x^2 + 16x + 16 \). Since the remainder is 0, \( x + 3 \) is a factor of \( x^3 + 13x^2 + 46x + 48 \).

Exercises
What is the quotient and remainder of the following polynomials?
11. \( (x^3 - 2x^2 + 8) \div (x + 2) \)
12. \( (x^3 - 7x^2 + 57x - 14) \div (x - 3) \)
13. \( (3x^3 + 4x^2 - 9x + 12) \div (x + 1) \)
14. \( (x^3 - 15x^2 + 23) \div (x - 2) \)
15. \( (x^3 + x + 10) \div (x + 2) \)
16. \( (x^3 - 12x^2 - 18x + 10) \div (x + 4) \)

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what are the rational roots of 4x^3 + 17x^2 - 4x + 3 = 0? Explain your work.

write the original equation.

identify the factors of the constant term.

identify the leading coefficient.

use the rational root theorem to identify the possible rational roots.

test the possible rational roots.

factor the polynomial using synthetic division.

write a polynomial function with rational coefficients so that

x = 5, -5, and 2 are roots.

write a polynomial function with rational coefficients so that

x = 4, -4, 3, -3 are roots.

write a polynomial function with rational coefficients so that

x = 6, -6, 3, -3 are roots.

write a polynomial function with rational coefficients so that

x = 10, 10, 5, 5 are roots.

write a polynomial function with rational coefficients so that

x = 11, 11, 7, 7 are roots.

write a polynomial function with rational coefficients so that

x = 2, 2, 1, 1 are roots.

write a polynomial function with rational coefficients so that

x = 12, 12, 4, 4 are roots.

write a polynomial function with rational coefficients so that

x = 15, 15, 5, 5 are roots.

write a polynomial function with rational coefficients so that

x = 18, 18, 3, 3 are roots.

write a polynomial function with rational coefficients so that

x = 20, 20, 5, 5 are roots.

write a polynomial function with rational coefficients so that

x = 24, 24, 6, 6 are roots.

write a polynomial function with rational coefficients so that

x = 25, 25, 5, 5 are roots.

write a polynomial function with rational coefficients so that

x = 26, 26, 6, 6 are roots.
To start, list the constant term’s factors:

\[ w^2, 2 \]

the leading coefficient of the polynomial is 3.

What is a quartic polynomial function with rational coefficients that has roots of 5, 1, 4, 1, 8, and \((-x)^4\)?

List all possible rational roots for the polynomial equation:

\[ x^4 - 2x^2 + 4x + 8 = 0 \]

Find all rational roots for \( P(x) = 0 \):

10. \( P(x) = x^4 + 5x^3 + 2x - 8 \)

11. \( P(x) = -x^3 - 4x^2 + 12x + 12 \)

12. \( P(x) = x^4 + 14x^3 + 53x + 40 \)

13. \( P(x) = x^4 - 3x^2 - 4x - 12 \)

14. \( P(x) = x^4 + 5x^3 - 3x - 45 \)

15. \( P(x) = x^4 + 2x^2 - x - 9 \)

16. \( P(x) = x^4 - 7x^3 + 14x + 8 \)

Theorem's About Roots of Polynomial Equations

Standardized Test Prep

Multiple Choice

For Exercises 1–5, choose the correct letter:

1. A fourth-degree polynomial with integer coefficients has roots 2 and 3 + \( \sqrt{5} \). Which number cannot also be a root of this polynomial?
   - C
   - E
   - F
   - G

2. A quartic polynomial \( P(x) \) has rational coefficients. If \( \sqrt{5} \) and \( 6 + i \) are roots of \( P(x) = 0 \), what is one additional root?
   - G
   - H
   - J
   - K

3. What is a quartic polynomial function with rational coefficients that has roots \(-2, 3, 0, 2\)?
   - C
   - D
   - E
   - F

4. What does Descartes’ Rule of Signs tell you about the real roots of \( x^4 - 2x^2 + 4x - 4 \)?
   - 1 positive real root and 1 or 3 negative real roots
   - 1 positive real root and 1 or 2 negative real roots
   - 2 positive real roots and 1 or 3 negative real roots
   - 2 positive real roots and 1 or 2 negative real roots

5. What is a rational root of \( x^5 + 2x^4 - 9x^3 - 23x^2 - 17x + 12 = 0 \)?
   - C
   - D
   - E
   - F

Extended Response

6. A third-degree polynomial with rational coefficients has roots -4, 1, 4. If the leading coefficient of the polynomial is \( \frac{1}{2} \), what is the polynomial? Show your work.

\[ \frac{1}{2} x^3 + 3x^2 - 5x - 6 \]

Proven Questions

Theorem's About Roots of Polynomial Equations

A polynomial function \( P(x) \) with rational coefficients has the given roots. Find two additional roots of \( P(x) = 0 \):

18. \( 1, 4 \) and \( \sqrt{7} \)

19. \( 3, -1 \) and \( -\sqrt{3} \)

20. \( -4, 4, 1, 0 \)

21. \( 6, \sqrt{2} \) and \( -\sqrt{2} \)

22. \( -4, -1 \) and \( 2, -\sqrt{5} \)

23. \( 2, 1 \) and \( -1, -2 \)

Write a polynomial function with rational coefficients so that \( P(x) = 0 \) has the given roots.

24. \( 3, 2 \) and \( 1, 0 \)

25. \( -2, -3 \) and \( -4 \)

26. \( 4, 6 \) and \( -1 \)

27. \( 1, 3 \) and \( 0 \)

28. \( 0, -1 \) and \( -2 \)

29. \( 2, 3, 4, 5 \) and \( 6 \)

Lower and Upper Bounds for Real Roots of Polynomial Equations

At times, the list of possible rational roots for a polynomial equation is rather lengthy. However, you can use patterns to shorten the list. One pattern involves finding lower and upper bounds for the polynomial equation.

Consider the polynomial equation \( x^3 - 3x^2 - 4x - 6 = 0 \).

1. List all the possible rational roots for the equation:
   - 1, 2, 3, 6, 1, 1, 1, 1, 1, 1, 1, 1

When testing the possible rational roots, if the last two numbers of the synthetic division table are all positive, then the number is in the upper bound.

To start, count and identify the number of sign changes in \( P(x) \).

You can use patterns to shorten the list. One pattern involves finding lower and upper bounds for the polynomial equation.

5. What does Descartes’ Rule of Signs say about the number of positive real roots and negative real roots for each polynomial function?

31. \( P(x) = x^4 - x^2 - 8x + 12 \)

32. \( P(x) = 2x^4 + 3x^2 - 5x - 2 \)

33. \( P(x) = x^4 - 3x^3 + 5x^2 + 4x + 1 \)

34. What does Descartes’ Rule of Signs say about the number of positive real roots and negative real roots for the polynomial equation?

35. What is a rational root of the polynomial equation:

\[ x^3 + 3x^2 - 4x - 6 = 0 \]

36. What is a rational root of the polynomial equation:

\[ x^3 + 3x^2 - 4x - 6 = 0 \]

37. What is a rational root of the polynomial equation:

\[ x^3 + 3x^2 - 4x - 6 = 0 \]

38. What is a rational root of the polynomial equation:

\[ x^3 + 3x^2 - 4x - 6 = 0 \]

39. What is a rational root of the polynomial equation:

\[ x^3 + 3x^2 - 4x - 6 = 0 \]

40. What is a rational root of the polynomial equation:

\[ x^3 + 3x^2 - 4x - 6 = 0 \]
9. Use your knowledge of the Fundamental Theorem of Algebra to answer the number of complex roots for each polynomial equation.

3.
Step 3
Substitute each possible root into the polynomial until you find one that causes the polynomial to equal zero. This is one rational root.

4.
Step 4
Determine the polynomial by synthetic division using the first rational root as the divisor.

5.
Step 5
If the divisor is a second-degree polynomial, factor to find any additional rational roots. If the divisor does not factor, there are no additional rational roots. If the divisor is greater than a second-degree polynomial, stop. Steps 1–4 found the divisor is a second-degree polynomial. 4x^2 + 12x + 8 does not factor. The rational roots of 4x^2 + 20x + 8x = 4 are -1 and -2.

Exercises
Find all rational roots for P(x) = 0.
1. P(x) = x^3 - x^2 - 4x + 2
2. P(x) = x^4 + 2x^3 + 3x^2 - 2x - 3
3. P(x) = x^4 - 2x^3 + x^2 - 3x - 1
4. P(x) = x^3 - 3x^2 + 2x + 3

6-5 Reteaching
Theorem About Roots of Polynomials Equations

Problem
What are the rational roots of 2x^4 + 3x^3 - 11x^2 + 26x - 10?

Step 1
Determine the factors of the constant term and the factors of the leading coefficient.
constant term: 1, 2, 3, 5, 10, 20
leading coefficient: 2, 1

Step 2
Find all the possible roots by dividing the factors of the constant term by the factors of the leading coefficient.

Step 3
Identify the possible roots into the polynomial until you find one that causes the polynomial to equal zero. This is one rational root.

Step 4
Determine the polynomial by synthetic division using the first rational root as the divisor.

Step 5
If the divisor is a second-degree polynomial, factor to find any additional rational roots. If the divisor does not factor, there are no additional rational roots. If the divisor is greater than a second-degree polynomial, stop. Steps 1–4 found the divisor is a second-degree polynomial. 2x^2 + 12x + 8 does not factor. The rational roots of 2x^2 + 20x + 8x = 4 are -1 and -2.

Exercises
Write a third-degree polynomial function f(x) with rational coefficients so that f(1) = 0 has roots -1, 2, and 3.

5. 1. 2
3. 5
4. 7
5. 9
6. 11
7. 13
8. 15
9. 17
10. 19
11. 21
12. 23
13. 25
14. 27
15. 29
16. 31
17. 33
18. 35
19. 37
20. 39

6-5 Reteaching
Theorem About Roots of Polynomials Equations

Problem
What is a third-degree polynomial function f(x) with rational coefficients so that f(1) = 0 has roots -1, 2, and 3?

Because 2 - 3 = 1 is a root, its complex conjugate 2 - 3 = 1 is also a root. Write the factored form of the polynomial.

Multiply the factors together.

Combine like terms.

Combine like terms.

A third-degree polynomial function with rational coefficients so that f(1) = 0 has roots 4 and 2. Solve for b.

Exercises
Write a third-degree polynomial function f(x) with rational coefficients so that f(1) = 0 has roots -1, 2, and 3.

5. 1. 2
3. 5
4. 7
5. 9
6. 11
7. 13
8. 15
9. 17
10. 19
11. 21
12. 23
13. 25
14. 27
15. 29
16. 31
17. 33
18. 35
19. 37
20. 39

5-6 ELL Support
The Fundamental Theorem of Algebra

Choose the word from the list that best completes each sentence.
complex roots degree Fundamental Theorem of Algebra Quadratic Formula synthetic division

1. The degree of a polynomial is the greatest degree among its monomials.
2. Synthetic division can be used to factor a polynomial.
3. The degree of a polynomial tells you how many complex roots the equation has.
4. The Fundamental Theorem of Algebra states that the number of complex roots of a polynomial equation is equal to the degree of the polynomial.
5. The Quadratic Formula can be used to find the complex roots of a quadratic equation.

Use your knowledge of the Fundamental Theorem of Algebra to answer the following questions.
6. Which of the following statements does not correctly state the Fundamental Theorem of Algebra?
7. Every polynomial of degree n ≥ 1 with complex coefficients has exactly n complex roots, including multiple roots.
8. Every polynomial of degree n ≥ 1 with complex coefficients has exactly n + 1 complex roots.
9. Every polynomial function of degree n ≥ 1 with complex coefficients has at least one complex root.
10. Every polynomial of degree n ≥ 1 with complex coefficients has no less than one complex root.

Use your knowledge of the Fundamental Theorem of Algebra to identify the number of complex roots for each polynomial equation.
7. x^2 + 2x^2 - 2 = 0 3 complex roots
8. x^3 - 2x^3 + 3x - 1 = 0 5 complex roots
9. x + 5 = 0 1 complex root
10. x^2 - 4x - 2 = 0 2 complex roots

5-6 Think About a Plan
The Fundamental Theorem of Algebra

Bridges
A twist in a river can be modeled by the function f(x) = \frac{2}{x} + \frac{1}{x} - \frac{3}{x^2}, -3 ≤ x ≤ 2. A city wants to build a road that goes directly along the x-axis. How many bridges would the city have to build?

Know
1. The function has exactly 1 complex roots.
2. Check students’ work.
3. Check students’ work.

Need
4. To solve the problem you need to:
   a. find the number of real roots of the function on the interval -3 ≤ x ≤ 2
5. Graph the function on a graphing calculator. What viewing window should you use?
   Answers may vary. Sample: Xmin = -5, Xmax = 5, Ymin = -5, Ymax = 5
6. What does the graph tell you?
   The function has 3 real roots on the interval -3 ≤ x ≤ 2
7. How many bridges would the city have to build?
   1
   Answers may vary. Sample: Because the function has 3 real roots, it crosses the x-axis 3 times, so, the road will cross the river 3 times.
17. Find the number of complex roots for each equation.

18. Find all the zeros of each function.

4. Without using a calculator, find all the roots of each equation.

5. To start, use a graphing calculator to find the possible rational roots.

6. Find all the zeros of each function.

7. For each equation, state the number of complex roots, the possible number of real roots, and the possible rational roots.

8. Find the number of complex roots for each equation.

9. Find all the zeros of each function.

10. Find all the zeros of each function.

11. Find all the zeros of each function.

12. Find all the zeros of each function.

13. Find all the zeros of each function.

14. Find all the zeros of each function.

15. Find all the zeros of each function.

16. Find all the zeros of each function.

17. Find all the zeros of each function.

18. Find all the zeros of each function.

19. Find all the zeros of each function.

20. Find all the zeros of each function.

21. Find all the zeros of each function.

22. Find all the zeros of each function.

23. Find all the zeros of each function.

24. Find all the zeros of each function.

25. Find all the zeros of each function.

26. Find all the zeros of each function.

27. Find all the zeros of each function.

28. Find all the zeros of each function.

29. Find all the zeros of each function.

30. Find all the zeros of each function.

31. Find all the zeros of each function.

32. Find all the zeros of each function.

33. Find all the zeros of each function.

34. Find all the zeros of each function.

35. Find all the zeros of each function.

36. Find all the zeros of each function.

37. Find all the zeros of each function.

38. Find all the zeros of each function.

39. Find all the zeros of each function.

40. Find all the zeros of each function.

41. Find all the zeros of each function.

42. Find all the zeros of each function.

43. Find all the zeros of each function.

44. Find all the zeros of each function.

45. Find all the zeros of each function.

46. Find all the zeros of each function.

47. Find all the zeros of each function.

48. Find all the zeros of each function.

49. Find all the zeros of each function.

50. Find all the zeros of each function.

51. Find all the zeros of each function.

52. Find all the zeros of each function.

53. Find all the zeros of each function.

54. Find all the zeros of each function.

55. Find all the zeros of each function.

56. Find all the zeros of each function.

57. Find all the zeros of each function.

58. Find all the zeros of each function.

59. Find all the zeros of each function.

60. Find all the zeros of each function.
5-6 Standardized Test Prep
The Fundamental Theorem of Algebra

Multiple Choice
For Exercises 1–6, choose the correct letter.

1. Which number is a zero of \( f(x) = x^3 + 4x^2 - 5x \) with multiplicity 1? 
   A. -1, 2, -1 
   B. -1, 2, 1 
   C. 2, -1, -1 
   D. 2, -1, 1

2. One root of the equation \( x^2 + x - 2 = 0 \) is 1. What are the other two roots? 
   F. -1, 2 
   G. -2, 1 
   H. 1, 2 
   J. -1, -2

3. A polynomial with real coefficients has 3, 2, and \(-1\) as three of its zeros. What is the least possible degree of the polynomial? 
   C. 3 
   D. 6 
   E. 5 
   F. 4

4. How many times does the graph of \( x^2 + 27 \) cross the x-axis? 
   G. 0 
   H. 1 
   I. 2 
   J. 4

5. Which of the following is the polynomial with zeros at \( -\frac{1}{2}, 2i, \) and \(-2\)? 
   A. \( x^3 + 2x^2 - 4x - 2 \) 
   B. \( x^3 + 2x^2 - 4x + 2 \) 
   C. \( x^3 + 2x^2 + 4x - 2 \) 
   D. \( x^3 + 2x^2 + 4x + 2 \)

6. A polynomial with real coefficients has roots of \( -2, -\sqrt{3}, \) and \( \sqrt{3} \). Which of the following could be another root of this polynomial? 
   I. -2 
   J. -1

Short Response
7. One root of the equation \( x^3 - 4x^2 - 6x + 3 = 0 \) is \(-1\). How many more complex roots does this equation have? What are all the roots? 
   Show your work.

5-6 Reteaching
The Fundamental Theorem of Algebra

Problem
What are all the complex roots of \( f(x) = x^3 + 4x^2 - x + 10 \)?

Because this is a fourth-degree polynomial, you know it will have four roots.

Step 1
Because the polynomial is already in standard form, you can use the Rational Root Theorem to determine possible rational roots. The possible rational roots are: \( \pm 1, \pm 2, \pm 5, \pm 10 \).

Step 2
Evaluate the polynomial for each possible root until you find one that causes the polynomial to equal zero. This is a rational root. In this case, one rational root is 2.

Step 3
Use synthetic division with a divisor of 2 to begin factoring the polynomial.

Step 4
Repeat Steps 1–3 until you have a polynomial of degree 2 or less.

Step 5
If the divisor is a second-degree polynomial, factor to find any additional roots.

Exercises
Find all the complex roots of each polynomial.

1. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \) 
   2. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \) 
   3. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \) 
   4. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \)

5. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \) 
   6. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \) 
   7. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \) 
   8. \( x^4 + 4x^3 + 12x^2 + 8x + 16 \)

5-6 Enrichment
The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra tells us that a polynomial, \( P(x) \), of degree \( n \geq 1 \) has exactly \( n \) complex roots. An equivalent statement is that a polynomial, \( P(x) \), of degree \( n \geq 1 \) can be factored into \( n \) linear factors. Linear factors are the building blocks for every polynomial. Let’s consider a similar theorem that applies to the building blocks of all positive integers. The Fundamental Theorem of Arithmetic states that every integer \( N > 1 \) can be uniquely written as a product of finitely many prime numbers.

Examples
5. Explain why \( N > 1 \) is a condition of the Fundamental Theorem of Arithmetic.

Answers may vary. Sample: The number 1 is not considered prime because it cannot be written as a product of 1 and another number.

2. Write the number 36 as a product of primes. 
   \( 36 = 2 \times 3 \times 3 \times 2 \times 3 \times 3 \)

3. The Fundamental Theorem of Arithmetic tells us that there is only one way to write 36 as a product of prime numbers. The prime factors of 36 can be rearranged but will always be a form of \( 2^2 \times 3^2 \). Write the number 48 as a product of primes. Express your answer using exponents. 
   \( 48 = 2^4 \times 3 \)

4. Write the number 100 as a product of primes expressed using exponents. 
   \( 100 = 2^2 \times 5^2 \)

5. Use your prime factorization to determine all of the other factors of 100.

2, 3, 4, 5, 6, 10, 12, 15, 20, 24, 25, 30, 40, 50, 60, 75, 100

6. If the prime factorization can help you determine the factors of an integer, can the factored form of a polynomial help you determine nonlinear factors of a polynomial? For example, if \( P(x) = (x + 11)(x + 9)(x + 5) \), then what are the 3 quadratic factors of \( P(x) \)?

\( (x + 11)(x + 9)(x + 5) = x^3 + 26x^2 + 252x + 945 \)

7. Describe the similarities between the Fundamental Theorem of Algebra and the Fundamental Theorem of Arithmetic.

Answers may vary. Both theorems describe the number and types of factors. One addresses factors of a number and the other address factors of a polynomial.
5-7 Think About a Plan
The Binomial Theorem

Geometers the side length of a cube is given by the expression $(2x + 8)$. Write a binomial expression for the area of a face of the cube and the volume of the cube. Then use the Binomial Theorem to expand and rewrite the expressions in standard form.

Understanding the Problem
1. What is the formula for the area of a face of a cube? $A = a^2$
2. What is the formula for the volume of a cube? $V = a^3$
3. What is the problem asking you to determine?
   expressions in standard form for the face area and volume of a cube with side length $(2x + 8)$

Planning the Solution
4. What is a binomial expression for the area of a face of this cube? $(2x + 8)^2$
5. What is a binomial expression for the volume of this cube? $(2x + 8)^3$
6. How can you use the Binomial Theorem to expand these expressions?
   Answers may vary. Sample: Use $n = 2$ for the expression for area and use $n = 3$ for the expression for volume. For both expressions, $a = 2x$ and $b = 8$

Getting the Answer
7. What is an expression for the area of a face of the cube written in standard form?
   $(2x + 8)^2$
8. What is an expression for the volume of the cube written in standard form?
   $(2x + 8)^3$
57 Practice

The Binomial Theorem

Expand each binomial.

1. $(x + 3)^2$
   - $x^2 + 6x + 9$
   - $x^2 + 6x + 9$

2. $(x - 2)^2$
   - $x^2 - 4x + 4$
   - $x^2 - 4x + 4$

3. $(x + 1)^3$
   - $x^3 + 3x^2 + 3x + 1$
   - $x^3 + 3x^2 + 3x + 1$

4. $(x - 1)^3$
   - $x^3 - 3x^2 + 3x - 1$
   - $x^3 - 3x^2 + 3x - 1$

5. $(x + 2)^3$
   - $x^3 + 6x^2 + 12x + 8$
   - $x^3 + 6x^2 + 12x + 8$

6. $(x - 3)^3$
   - $x^3 - 9x^2 + 27x - 27$
   - $x^3 - 9x^2 + 27x - 27$

Find the specified term of each binomial expansion.

6. second term of $(x - 4)^5$
   - $-20x^4$
   - $-20x^4$

7. fourth term of $(x - 3)^3$
   - $-54x^2$
   - $-54x^2$

8. fifth term of $(x - 1)^4$
   - $4x^3$
   - $4x^3$

9. third term of $(x + y)^3$
   - $3x^2y$
   - $3x^2y$

10. fifth term of $(x + y)^4$
    - $10x^3y$
    - $10x^3y$

11. second term of $(2x + y)^3$
    - $12x^2y$
    - $12x^2y$

12. second term of $(x + y)^4$
    - $2xy^3$
    - $2xy^3$

13. third term of $(x + y)^4$
    - $3x^3y$
    - $3x^3y$

57 Standardized Test Prep

Multiple Choice

For Exercises 1–7, choose the correct letter.

1. What is the expanded form of $(a - b)^2$? D
   - $a^2 - 2ab + b^2$
   - $a^2 - 2ab + b^2$

2. What is the third term in the expansion of $(x + 3)^2$? F
   - $9x$
   - $9x$

3. What is the coefficient of the third term in the expansion of $(2x - 3)^2$? D
   - $-6x^2$
   - $-6x^2$

4. Which term in the expansion of $(2x - 3)^2$ has coefficient $200$? G
   - second term
   - third term
   - fourth term
   - fifth term

5. What is $n$ if $405x^4$ appears in the expansion of $(x + y)^9$? C
   - 6
   - 7
   - 8
   - 9

6. What is the 6th term in the 12th line of Pascal’s Triangle? G
   - 252
   - 462
   - 782
   - 1327

7. What is the expanded form of $(2x - y)^3$? B
   - $8x^3 - 12x^2y + 6xy^2 - y^3$
   - $8x^3 - 12x^2y + 6xy^2 - y^3$

8. The coefficient of the fourth term in the expansion of $(x + y)^3$ is 6.
   - The value of $n$ is 3.
   - The value of $n$ is 3.

9. What is the 10th term in the 12th line of Pascal’s Triangle? G
   - 120
   - 120

10. What is the 5th term in $(x - y)^6$? A
    - $-20x^4y^2$
    - $-20x^4y^2$

11. What is the coefficient of the 12th term in the expansion of $(x - y)^6$? A
    - $20x^4y^2$
    - $20x^4y^2$

12. What is the coefficient of the 12th term in the expansion of $(x - y)^6$? A
    - $20x^4y^2$
    - $20x^4y^2$

57 Enrichment

The Binomial Theorem

Pascal’s Triangle is helpful for expanding powers of a binomial sum, but what if you wanted to expand $(a + b)^n$. You might not want to write out all of the rows of Pascal’s Triangle. In cases like this, you can use another form of the Binomial Theorem.

1. This form of the Binomial Theorem uses the operation called factorial. The expression $\frac{n!}{k!(n - k)!}$ is read “$n$ factorial” and is computed by multiplying all of the counting numbers from $n$ down to 1. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

2. An unexpected fact arises for $0!$. $0!$ is defined as 1. Fractions involving factorials can be simplified first. Rewrite each factorial in the fraction $6!$ as multiplication and then reduce. What is the simplified form?

$$\frac{6!}{2!4!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 4 \times 3 \times 2} = 15$$

Expressions involving factorials can be used to write a different form of the Binomial Theorem. This form states that the $k^{th}$ term in the expansion of $(a + b)^n$ is

$$\binom{n}{k} a^{n-k} b^k$$

where $\binom{n}{k}$ is the $k^{th}$ coefficient of Pascal’s Triangle.

3. Find the third term in the expansion of $(a + b)^5$ using the Binomial Theorem. What value will you substitute in for $a$? What value will you substitute in for $b$? $n = 5, a = 1, b = 2; 15a^4b^1 = 30a^4b$.

4. Use Pascal’s Triangle to verify that you have found the correct third term. The complete term is $\binom{5}{2} a^3 b^2 = 10a^3 b^2$. What is the coefficient of this term in the Binomial Theorem? $10$.

5. fifth term of $(x + y)^5$ 1350x^2y^3

6. ninth term of $(x + y)^5$ 209,245,425

7. twelfth term of $(x + y)^5$ 1,444,724,756x^7y^4

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3. Write the expansion of each binomial.

To create Pascal’s Triangle, start by writing a triangle of 1’s. This triangle forms the first two rows. Each row has one more element than the one above it. Each row begins with a 1, and then each element in the sum of the two lowest elements in the row above. The last element in each row is 1.

Problem
What is the expansion of \((x + y)^3\)? Use Pascal’s Triangle.

Step 1
The power of the binomial corresponds to the second number in each row of Pascal’s Triangle. Because the power of this binomial is 3, use the row of Pascal’s Triangle with 5 as the second number. The numbers of this row are the coefficients of the expansion.

Step 2
The exponents of the variables in the expansion begin with the power of the binomial and decrease to 0. The exponents of the variables in the expansion begin with 0 and increase to the power of the binomial.

Step 3
Simplify all terms to write the expansion in standard form.

Exercises
Write the expansion of each binomial.

1. \((x + 1)^3\)
2. \((x + y)^3\)
3. \((x + 1)^3\)
4. \((x - 1)^3\)

Ell Support

5. Match each word phrase in Column A with the matching term in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>interpolation</td>
<td>estimating outside the domain</td>
</tr>
<tr>
<td>((n + 1)) Point Principle</td>
<td>measures the fit between a model and a set of data</td>
</tr>
<tr>
<td>extrapolation</td>
<td>estimating within the domain</td>
</tr>
<tr>
<td>(R^2) value</td>
<td>used to find a model that fits a set of data</td>
</tr>
</tbody>
</table>

6. Match each graph in Column A with the matching term in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Linear</td>
</tr>
<tr>
<td>B</td>
<td>Quadratic</td>
</tr>
<tr>
<td>C</td>
<td>Cubic</td>
</tr>
</tbody>
</table>

7. Which form of estimation is more reliable: interpolation or extrapolation?

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Answers

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Find a polynomial function whose graph passes through each set of points.

1. \((4, -1)\) and \((-3, 13)\)
   \[ y = -3x + 7 \]

2. \((1, 2)\) and \((-8, -2)\)
   \[ y = 2x - 5 \]

3. \((7, -1)\) and \((-3, 3)\)
   \[ y = 4x - 12 \]

4. \((-2, 1)\), \((-1, 2)\), \((0, 3)\), and \((1, 4)\)
   \[ y = 2x^2 + 4x + 6 \]

5. \((3, 2)\), \((2, -3)\), \((-1, 6)\), and \((-2, -6)\)
   \[ y = 3x^2 - 2x + 5 \]

6. \((-3, 1)\), \((-2, 2)\), \((-1, 4)\), and \((0, 6)\)
   \[ y = 2x^2 - 4x + 5 \]

7. \((-1, -1)\), \((-2, 2)\), \((-3, 3)\), and \((0, 6)\)
   \[ y = -x^3 + 4x - 1 \]

Find a polynomial function that best models each set of values.

9. Let \(x\) = the number of years after 1985.
   \[ f(x) = 0.038x^4 - 0.956x^3 + 0.01x^2 + 49.3 \]

11. Let \(x\) = the number of years after 1985.
    \[ f(x) = 0.00013x^5 - 0.015x^4 + 0.3017x^3 + 74.7 \]

Find a cubic and a quartic model for each set of values. Then determine which model best represents the values.

13. \(f(x) = x^3 - 2x^2 + 5x + 3\); \(g(x) = x^4 - 3x^3 + 4x^2 + 4x + 6\); the cubic and the quartic models best represent the values.

14. \(f(x) = x^3 + 5.8x^2 + 6x - 2.11\); \(g(x) = 2x^4 - 3x^3 + 4x^2 + 6x + 5\); the quartic model best represents the values.

Find a polynomial function whose graph passes through each set of points.

1. \((-4, 3)\), \((-3, 2)\), and \((-1, 5)\)
   \[ P(x) = ax^3 + bx^2 + cx + d \]

2. \((-1, 3)\) and \((-4, -6)\)
   \[ y = 3x + 6 \]

3. \((-1, -19)\), \((0, 5)\), \((1, 8)\) and \((4, 53)\)
   \[ 3x^3 - 4x^2 + 3x - 1 \]

4. \((-4, -47)\), \((-1, 7)\), and \((1, 3)\)
   \[ y = -2x^3 - 3x^2 + 6x + 5 \]

5. \((-1, 16)\), \((0, 6)\), and \((4, 16)\)
   \[ y = -x^2 - 2x + 8 \]

6. \((-4, -3)\), \((-1, 12)\), \((0, 5)\) and \((1, 2)\)
   \[ y = x^3 - 2x^2 + 6x + 5 \]

Find a polynomial function that best models each set of values.

8. Let \(x\) = 6.

9. Let \(x\) = the number of years since 1990.

Find a polynomial function whose graph passes through each set of points.

1. \((-4, 3)\), \((-3, 2)\), and \((-1, 5)\)
   \[ P(x) = ax^3 + bx^2 + cx + d \]

2. \((-1, 3)\) and \((-4, -6)\)
   \[ y = 3x + 6 \]

3. \((-1, -19)\), \((0, 5)\), \((1, 8)\) and \((4, 53)\)
   \[ 3x^3 - 4x^2 + 3x - 1 \]

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5. \((-1, 16)\), \((0, 6)\), and \((4, 16)\)
   \[ y = -x^2 - 2x + 8 \]

6. \((-4, -3)\), \((-1, 12)\), \((0, 5)\) and \((1, 2)\)
   \[ y = x^3 - 2x^2 + 6x + 5 \]
Short Response

5. Find both a cubic and quartic model for the set of values at the right, what is the estimated Consumer Price Index in 1945? H

[Table with data points for CPI from 1910 to 1980]

6. What is the domain of this function? Is that what are the possible values for x, the size of square that can be cut out? 0 < c < 4.25

7. Explain why you cannot cut out squares that are larger than 4.25 in. on each side. (0, 0), (0.25, 0), (0.5, 0.5)

8. What are the coordinates of the x-intercepts?

9. When does the graph change from increasing to decreasing? Answers may vary. Example: The graph changes from increasing to decreasing between 2000 and 2005. This is when the polynomial changes from a higher degree to a lower degree.

5-8 Reteaching
Polynomial Models in the Real World

Problem

What polynomial function has a graph that passes through the four points (0, -1), (-1, -7), (2, 27), and (-2, 27)?

Step 1 Use the point-slope form of a linear equation to find the equation of the line that passes through the points (0, -1) and (-2, 27).

Step 2 Substitute the x and y values from the four points given in the problem. Now you have four linear equations in four unknowns (a, b, c, d).

Step 3 Subtract the coefficients of the bottom two equations.

Step 4 Multiply the first two equations by 2 and subtract the second two equations.

Step 5 Use the Cubic Regression calculator with the four original points to check your answer.

Exercises

Find the polynomial function that passes through each set of points.

1. (1, -4), (-2, 0), (0, 2), and (-1, 10) y = x^3 - 3x^2 + 2x + 2

2. (1, 0), (-3, 0), and (0, -6) y = x^4 - 3x^2 - 6

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Identify the constant of proportionality in each of the following functions.

1. A function written in the form \( y = ax \) is called a ______ power function
   - constant of proportionality

2. The constant \( a \) in the function \( y = ax \) is called the ______ constant of proportionality

Determine whether each of the following functions is a power function.

3. \( y = 3x - 4 \) ______
   - not a power function

4. \( y = 4x^5 \) ______
   - power function

5. \( y = 0.25x^6 \) ______
   - power function

6. \( y = 7x^2 + 5x \) ______
   - not a power function

7. \( y = \frac{1}{x} \) ______
   - power function

Identify the constant of proportionality in each of the following functions.

8. \( y = 5x \) ______
   - 5

9. \( y = \frac{7}{x} \) ______
   - 0.7

10. \( y = \frac{1}{x^2} \) ______
    - \( \frac{1}{x^2} \)

11. \( y = 0.5x^2 \) ______
    - 0.5

**Multiple Choice**

12. Given the function \( P = ax^2 \), what is the value of \( a \) if \( P = 225 \) when \( x = 5 \)?
   - a. 5
   - b. 9
   - c. 25
   - d. 45
   - e. 81

13. Given the function \( P = ax^4 \), what is the value of \( a \) if \( P = 9 \) when \( x = 3 \)?
   - a. \( \frac{1}{2} \)
   - b. 9
   - c. 25
   - d. 45
   - e. 6

Think About a Plan

Physics: The formula \( K = \frac{1}{2}mv^2 \) represents the kinetic energy of an object. If the kinetic energy of a ball is 10 lb-ft\(^2\), when is it thrown with a velocity of 4 ft/s?

- How much kinetic energy is generated if the ball is thrown with a velocity of 8 ft/s?

Know:
1. The kinetic energy of a ball is \( 10 \text{ lb-ft}^2 \) when the velocity of the ball is \( 4 \text{ ft/s} \)

2. Check students’ work.

Need:
1. To solve the problem I need to: solve the kinetic energy equation for \( m \).

Plan:
4. What equation can you use to find the value of \( m \) for the ball? \( 10 = \frac{1}{2}mv^2 \)

5. Solve the equation. \( m = 1.25 \)

6. What equation can you use to find the kinetic energy generated if the ball is thrown with a velocity of 8 ft/s? \( K = \frac{1}{2}mv^2 \)

7. Simplify. \( K = 40 \text{ lb-ft}^2 \)

8. Is the solution reasonable? Explain.
   - Yes; kinetic energy increases by a factor of 4 when the velocity doubles. The change in the kinetic energy is the square of the change in the velocity.
5-9 Practice  
Form K
Transforming Polynomial Functions

Short Response
5. Determine the cubic function that is obtained from the parent function \( y = x^3 \) after each sequence of transformations.
   a. a vertical stretch by a factor of 2, a vertical translation 3 units down, and a horizontal translation 5 units left

   To start, multiply by 2 to stretch.
   \( y = 2x^3 \)

   Then, subtract 3 to translate down.
   \( y = 2x^3 - 3 \)

   Finally, subtract 5 to translate left.
   \( y = 2(x + 5)^3 - 3 \)


6. Find all the real zeros of each function.
   a. \( y = 3x^3 - 13x^2 + 12x \)

   Using the Factor Theorem, we can see that \( x = 1 \) is a root.
   \( (x - 1)(3x^2 - 10x + 12) = 0 \)

   The quadratic factor can be factored as:
   \( (x - 1)(3x - 4)(x - 3) = 0 \)

   So, the real zeros are:
   \( x = 1, \frac{4}{3}, 3 \)

   b. \( y = 5x^3 - 12x^2 - x + 1 \)

   Using the Factor Theorem, we can see that \( x = 1 \) is a root.
   \( (x - 1)(5x^2 - 7x - 1) = 0 \)

   The quadratic factor can be factored as:
   \( (x - 1)(5x + 1)(x - 7) = 0 \)

   So, the real zeros are:
   \( x = 1, -\frac{1}{5}, 7 \)

   c. \( y = 2x^3 - 3x^2 - 17x + 28 \)

   Using the Factor Theorem, we can see that \( x = 2 \) is a root.
   \( (x - 2)(2x^2 + x - 14) = 0 \)

   The quadratic factor can be factored as:
   \( (x - 2)(2x - 7)(x + 2) = 0 \)

   So, the real zeros are:
   \( x = 2, \frac{7}{2}, -2 \)


5-9 Enrichment  
Form K
Transforming Polynomial Functions

Finding a quartic function with the given \( x \)-values as its only real zeros.
12. \( x = 1 \) and \( x = 4 \)

   To start, use the Factor Theorem to write the equation of a quartic with roots at 1 and 4.

   \( y = (x - 1)(x - 4) \cdot Q(x) \)

   Where \( Q(x) \) has zeros.

   Answers may vary. Sample:
   \( y = x^4 - 5x^3 + 9x^2 - 7x - 6 \)

13. \( x = -2 \) and \( x = 3 \)

   Answers may vary. Sample:
   \( y = x^4 + x^3 - 9x^2 + 3x + 6 \)

14. \( x = -3 \) and \( x = 2 \)

   Answers may vary. Sample:
   \( y = x^4 + x^3 - 9x^2 + 3x + 6 \)

15. \( x = 7 \) and \( x = 2 \)

   Answers may vary. Sample:
   \( y = x^4 + x^3 - 9x^2 + 3x + 6 \)

16. \( x = -3 \) and \( x = -4 \)

   Answers may vary. Sample:
   \( y = x^4 + x^3 - 9x^2 + 3x + 6 \)

17. \( x = 3 \) and \( x = 5 \)

   Answers may vary. Sample:
   \( y = x^4 + x^3 - 9x^2 + 3x + 6 \)

18. \( x = 3 \) and \( x = 5 \)

   Answers may vary. Sample:
   \( y = x^4 + x^3 - 9x^2 + 3x + 6 \)

19. You are driving a car at a speed of 21.1 mph. The radius of the circle you are making is 1.5 ft. The acceleration is equal to one over the radius times the velocity squared.

   a. What is the acceleration? \( \frac{21.1}{1.5} = 14.07 \) ft/sec²

   b. What is the velocity if the acceleration is 28 ft/sec²?

   \( v = \sqrt{\frac{a}{r}} \)

   \( v = \sqrt{\frac{28}{1.5}} \approx 9.38 \) ft/sec

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15. Reasoning The volume of a box is \( x^3 + 4x^2 + 4x \). Explain how you know the box is not a cube:
The factors are \((x + 1)(x + 2)(x + 4)\); these are the dimensions of the box; because they are not all the same, the box cannot be a cube.

16. Error Analysis For the polynomial function \( y = \frac{1}{4}x^6 + x + 6 \), your friend says the end behavior of the graph is down and up. What mistake did your friend make?
- The leading coefficient is positive and the degree is even, so the end behavior of the graph is up and down.
- \( y = \frac{1}{4}x^6 + x + 6 \) is not a polynomial function.

Chapter 5 Quiz 2

Do you know HOW? Expand each binomial:
1. \((2a - 1)^3\)
2. \((x + 3)^3\) Expand each polynomial:
3. \(16x^2 - 32x + 16 \)
4. \((x + 3)^2\) Use the Rational Root Theorem to list all possible rational roots for each equation:
5. \(x^3 - 10x^2 + 31x - 30 = 0\)
6. \(x^3 + 2x^2 - 5x - 6 = 0\) Use the binomial theorem to find each term of the expansion:
7. \((x + 2)^4\)
8. \((2x + 3)(2x - 3)\)

Do you UNDERSTAND? Expand each binomial:
9. \((x + y)^2\)
10. \((x + 1)(x - 1)\) Find a polynomial function with the given zeros:
11. \(x = 2, x = -1\)
12. \(x = 3, x = -2\) Find the domain of each function:
13. \(f(x) = \frac{1}{x}\)
14. \(g(x) = \sqrt{x - 1}\) Find the domain and range of each function:
15. \(f(x) = \frac{1}{x^2}\)
16. \(g(x) = \sqrt{x + 1}\) Find the domain of each function:
17. \(f(x) = \frac{1}{x^2 - 4}\)
18. \(g(x) = \sqrt{x^2 - 1}\) Graph each function:
19. \(f(x) = x^2 - 4\)
20. \(g(x) = |x|\) Graph each function:
21. \(f(x) = x^3 - 2x^2 + x - 2\)
22. \(g(x) = x^2 + 3x + 2\) Graph each function:
23. \(f(x) = |x^2 - 4|\)
24. \(g(x) = \sqrt{x^2 - 1}\) Graph each function:
25. \(f(x) = \sqrt{x^2 - 1}\)
26. \(g(x) = \sqrt{x^2 - 4}\)
Do you know HOW?
Write each polynomial in standard form. Then classify by degree and by number of terms.
1.
$$2x^4 + 3x^3 + 4x^2 + 5x + 6$$
2.
$$3x^3 - 2x^2 + 4x - 5$$
3.
$$5x^4 - 3x^3 + 2x^2 - x + 1$$
Find the zeros of each function. State the multiplicity of any multiple zeros.
4.
$$f(x) = x^3 - 2x^2 + x - 2$$
5.
$$g(x) = x^4 + 2x^3 - 3x^2 - 4x + 1$$
6.
$$h(x) = x^5 - 3x^4 + 3x^3 - x^2 + 1$$
Find the real solutions of each equation by graphing. Where necessary, round to the nearest hundredth.
7.
$$x^2 + 4x + 3 = 0$$
8.
$$x^3 - x^2 - 3x + 2 = 0$$
9.
$$x^4 - 4x^3 + 6x^2 - 4x + 1 = 0$$
10.
$$x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1 = 0$$

Do you UNDERSTAND?
11. True or false? If a function has the complex roots 2, 5, 3, then the polynomial must be a fourth-degree polynomial.
12. True or false? If a polynomial has a binomial of the form $x - a$, it is a factor of the polynomial.
13. True or false? The real solutions of a polynomial of degree k must be less than or equal to k.

Chapter 5 Part A Test
Lessons 5–1 through 5–4
Do you know HOW?
Write each polynomial function in standard form. Then classify it by degree and by number of terms.
1.
$$f(x) = x^3 + 2x^2 + 3x + 4$$
2.
$$g(x) = x^4 + 3x^3 + 2x^2 + 4x + 5$$
3.
$$h(x) = x^5 - 2x^4 + 3x^3 - x^2 + 5x + 1$$
4.
$$k(x) = x^6 + 2x^5 - 3x^4 + 4x^3 + 5x^2 - 6x + 7$$
5.
$$l(x) = x^7 - 3x^6 + 5x^5 - 7x^4 + 9x^3 - 11x^2 + 13x - 15$$
Find the zeros of each function. State the multiplicity of any multiple zeros.
6.
$$f(x) = x^3 + 2x^2 + x + 2$$
7.
$$g(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$$
8.
$$h(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$$
9.
$$k(x) = x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$$
10. Expand each binomial.
$$a. (x + 2)^2$$
$$b. (3x - 4)^2$$
$$c. (2x + 3)^2$$

Do you UNDERSTAND?
11. True or false? For a given number $a$, the binomial $(x - a)$ is a factor of the polynomial if and only if the number $a$ is a solution of the polynomial.
12. True or false? If a polynomial has a binomial of the form $x + a$, it is a factor of the polynomial.
13. True or false? The real solutions of a polynomial of degree k must be less than or equal to k.
Chapter 5 Performance Tasks

Task 1
a. Draw the related graph of \( y = -x \) by \( b = -2 \). Determine the multiplicity of each root.

b. Draw the related graph of \( y = ax \) by \( b = 2 \). Determine the multiplicity of each root.

c. Reverse the equations found in parts a and b in standard form.

d. Given the equation \( y = ax + b \), find the roots of each equation in standard form.

Task 2
a. Use division to find the remaining roots of \( y = x^4 - 4x^3 + 6x^2 - 4x + 1 \).

b. Use division to find the remaining roots of \( y = x^4 - 4x^3 + 6x^2 - 4x + 1 \).

c. Use the roots found in parts a and b to reverse the function in factored form.

Correct answers, without work shown. OR student able to complete part of the task.

[Student understood that division was involved but was unable to correctly graph the equations or solve for equation roots.]

[Student divided equations incorrectly but followed through the process correctly using the given data.]
Multiple Choice
For Exercises 1–14, choose the correct letter.

1. Which relation is not a function? D
   \( y = x^2 \)
   \( y = x + 2 \)
   \( y = 2x \)
   \( x = 2 \)

2. For which of the following sets of data is a linear model reasonable? F
   \( \{(0, 1), (1, 2), (2, 3), (3, 4), (4, 5)\} \)
   \( \{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4)\} \)
   \( \{(0, 1), (1, 2), (2, 3), (3, 4), (4, 5)\} \)
   \( \{(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)\} \)

3. Which is a solution of the system of inequalities?
   \( \{x | x > 1\} \)
   \( \{x | x < 1\} \)
   \( \{x | x = 1\} \)
   \( \{x | x = 1, 2\} \)

4. Which of the following is the equation of a parabola?
   \( y = a \)
   \( y = ax \)
   \( y = ax^2 \)
   \( y = ax^3 \)

5. For which of the following sets of data is a linear model reasonable?
   \( \{(1, 2), (2, 4), (3, 6), (4, 8)\} \)
   \( \{(1, 2), (2, 4), (3, 5), (4, 6)\} \)
   \( \{(1, 2), (2, 3), (3, 4), (4, 5)\} \)
   \( \{(1, 2), (2, 3), (3, 5), (4, 6)\} \)

6. Which of these is a direct variation? C
   \( y = x \)
   \( y = ax \)
   \( y = ax^2 \)
   \( y = ax^3 \)

7. Which polynomial is written in standard form? D
   \( 2x^3 + 5x^2 + 3x + 1 \)
   \( 2x^3 + 5x^2 + 3x - 1 \)
   \( 2x^3 + 5x^2 - 3x + 1 \)
   \( 2x^3 + 5x^2 - 3x - 1 \)

8. Solve the system.
   \( \{(x, y) | x + y = 0\} \)
   \( \{(x, y) | x - y = 0\} \)
   \( \{(x, y) | x + y = 1\} \)
   \( \{(x, y) | x - y = 1\} \)

9. Solve the system.
   \( \{(x, y) | x + 2y = 4\} \)
   \( \{(x, y) | x - 2y = 4\} \)
   \( \{(x, y) | x + 2y = 3\} \)
   \( \{(x, y) | x - 2y = 3\} \)

10. What is the axis of symmetry of \( y = 2(x - 3)^2 + 5 \)?
    \( x = 2 \)
    \( x = 3 \)
    \( x = 3 \)
    \( x = 5 \)

11. A sixth-degree polynomial function with rational coefficients has complex roots \( 4, i, 7, 2, -i \). Which of the following cannot be another complex root of this polynomial? B
    \( 2i \)
    \( -2i \)
    \( -2 \)
    \( -4 \)

12. Solve \( x + 3(x + 4) = 0 \)
    \( x + 3(4) + 4 = 0 \)
    \( x = 3 \)
    \( x = -4 \)
    \( x = 0 \)

13. Which relation is a function? B
    \( \{(1, 2), (3, 5), (6, 4), (2, 1)\} \)
    \( \{(1, 2), (3, 5), (6, 4), (2, 1), (6, 5)\} \)
    \( \{(1, 2), (3, 5), (6, 4), (2, 1), (6, 5), (5, 3)\} \)
    \( \{(1, 2), (3, 5), (6, 4), (2, 1), (6, 5), (5, 3), (5, 7)\} \)

14. Find the roots of \( x^3 = 15x + 5 - 15 = 0 \).
    \( x = -1, 1, 3 \)
    \( x = -1, 1, 5 \)
    \( x = -1, 5, 3 \)
    \( x = -1, 5, 1 \)

Short Response
15. Open-ended. Write the equation of a direct variation in slope-intercept form. Write the y- and x-intercepts. Answers may vary. Sample: \( y = 3x; \) (0, 0).

16. Writing. Explain how to solve a polynomial equation in standard form with roots \( a, b, c, d \). Check student’s work.

17. Evaluate \( 2x^2 - 5x + 4 \) for \( x = 3 \).

18. Graph the inequality \( 2x - 3y < 4 \).

19. Use Pascal’s Triangle or the Binomial Theorem to expand \( (x - 2)^5 \).

20. Determine the equation of the graph of \( y = x^3 \) under a vertical stretch by a factor of 3, a reflection across the x-axis, a horizontal translation 3 units left, and a vertical translation 3 units up. The equation of the graph can be expressed as \( y = ax^3 + bx^2 + cx + d \).

Extended Response
21. An arrow is shot upward. In height, it is given by the equation \( h = -16t^2 + 32t + 8 \), where \( h \) is the height in feet. after 2 seconds? (2) Incorrect equations OR multiple computational errors

Activity 1: Graphing
Students graph the given polynomial and fine-tune the equation to make the graph pleasing for a car hood.

Activity 2: Analyzing
Students research the design of cars or other objects that have curved parts and use inverse matrices to write equations for one of their curves.

Activity 3: Graphing
Students use their calculators to find more accurate equations to model the curves for their projects.

Finishing the Project
You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results. Ask students to review their project work and update their folders.

Have students review their methods for finding and recording curves and equations used for the project.

Ask groups to share their insights that resulted from completing the project, such as techniques they found to be most accurate and easiest to use.

Identify and label ten points on the sketch you made in Activity 2. Do you think you found the equation of a parabola? Explain your reasoning. Then find the new function using the graphing calculator and the CalcLeft feature.

Check students’ work.

Activity 3: Graphing
Students select two points on the sketch they made in Activity 2. Do you think the function that best fits these points will be more accurate than the function you found using four points? Explain your reasoning. Then find the new function using the graphing calculator and the CalcLeft feature.

Check students’ work.
Chapter 5 Project: Curves by Design

(continued)

Finishing the Project

The activities should help you to complete your project. Make a poster to display the sketch and graphs you have completed for the object you have chosen. On the poster, include your research about the object.

Reflect and Revise

Before completing your poster, check your equations for accuracy, your graph designs for neatness, and your written work for clarity. Is your poster eye-catching, exciting, and appealing, as well as accurate? Show your work to at least one adult and one classmate. Discuss improvements you could make.

Extending the Project

Interview someone who uses a computer-assisted design (CAD) program at work. If possible, arrange to have a demonstration of the program. Find out what skills, education, or experience helped the person successfully enter the field of computer-assisted design.

Chapter 5 Project Manager: Curves by Design

Getting Started

Read the project. As you work on the project, you will need a calculator, materials on which you can record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist

☐ Activity 1: modeling a curve
☐ Activity 2: finding a cubic model
☐ Activity 3: finding a better fit
☐ object model

Suggestions

☐ Make small changes in the equation at first.
☐ Label the turning points.
☐ Use the regression feature of your graphing calculator.

Is a cubic function the best model for the object you chose? Why or why not? How can you determine the curve that best models the shape of your object using a graphing calculator?

Scoring Rubric

4 Your equations and solutions are correct. Graphs are neat and accurate. All written work, including the poster, is neat, correct, and pleasing to the eye. Explanations show careful reasoning.
3 Your equations are fairly close to the graph designs, with some minor errors. Graphs, written work, and the poster are neat and mostly accurate with minor errors. Most explanations are clear.
2 Your equations and solutions contain errors. Graphs, written work, and the poster could be more accurate and neat. Explanations are not clear.
1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project

Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of Project
Find each real root.

Find all the real cube roots of each number.

Find all the real fourth roots of each number.

Find all the real square roots of each number.

Complete the vocabulary chart by filling in the missing information.

Choose the word or phrase from the list that best completes each sentence.

Select the word or phrase from the list that best completes each sentence.

6. The _____ indicates the degree of the root.

7. The _____ in the expression $\sqrt[3]{27}$ is 3.

8. Given the equation $a^n = b$, $a$ is the _____ of $b$.

9. In a radical expression, the _____ indicates the degree of the root.

10. When a number has both a positive and a negative root, the positive root is considered the _____.

6-1 Practice

Find all the real square roots of each number.

1. $\pm 8, \pm 9, \pm 100, \pm 1000$ 4. $0.0025, -0.025, 0.25$

2. $-196, 196$ 3. $10,000, -100, 1000$

Find all the real cube roots of each number.

5. $\pm 6, \pm 64$ 6. $\pm 125, \pm 125$

Find all the real fourth roots of each number.

9. $\pm 256, \pm 64, \pm 4, \pm 4, 14, 16, 256$ 7. $\pm 2, \pm 5, \pm 5, \pm 5$

Find all the real cube roots of each number.

12. $\pm 12, \pm 12, \pm 12, \pm 12$ 15. $\pm 10, \pm 10, \pm 10, \pm 10$

13. $\pm 12, \pm 12, \pm 12, \pm 12$ 16. $\pm 16, \pm 16, \pm 16, \pm 16$

14. $\pm 12, \pm 12, \pm 12, \pm 12$ 18. $\pm 20, \pm 20, \pm 20, \pm 20$

17. $\pm 12, \pm 12, \pm 12, \pm 12$ 20. $\pm 10, \pm 10, \pm 10, \pm 10$

Simplify each radical expression. Use absolute value symbols when needed.

21. $\sqrt[3]{8}$ 22. $\sqrt[3]{54}$ 23. $\sqrt[4]{16}$

24. $\sqrt[4]{256}$ 25. $\sqrt[4]{256}$ 26. $\sqrt[4]{81}$

27. $\sqrt[4]{256 + 64}$ 28. $\sqrt[4]{256 + 64}$ 29. $\sqrt[4]{256 - 64}$

30. $\sqrt[4]{25}$ 31. $\sqrt[4]{25}$ 32. $\sqrt[4]{25 - 64}$

33. A cube has volume $v = 8$, where $v$ is the length of a side. Find the side length for a cube with volume 8000 cm$^3$.

34. The temperature $T$ in degrees Celsius ($^\circ$C) of a liquid at minutes after heating is given by the formula $T = 90 - 0.16$. When is the temperature 80$^\circ$C? 30 min

6-1 Practice (continued)

Find the two real solutions of each equation.

35. $x^2 = 4, -2, 2$ 36. $x^2 = 1, -1, 1$

37. $x^2 = 0.16, -0.4, 0.4$ 38. $x^2 = \frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}$

39. $x^2 = \frac{1}{9}, -\frac{1}{9}, \frac{1}{9}, -\frac{1}{9}$ 40. $x^2 = \frac{1}{9}, -\frac{1}{9}, \frac{1}{9}, -\frac{1}{9}$

41. $x^2 = 0.000001, -0.001, 0.001$ 42. $x^2 = 0.001, -0.001, 0.001$

43. The number of new customers is twice the number of customers who already fell. As the number of customers who already fell.

44. Geometry The volume $V$ of a sphere with radius $r$ is given by the formula $V = \frac{4}{3} \pi r^3$.

45. A clothing manufacturer finds the number of defective blouses $d$ is a function of the total number of blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

46. The velocity of a falling object can be found using the formula $v^2 = 2gh$, where $v$ is the velocity (in feet per second) and $g$ is the acceleration due to gravity (32 ft/s$^2$).

47. The number of defective blouses $d$ is a function of the total number of blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

48. How many customers at the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

49. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

50. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

51. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

52. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

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56. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

57. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

58. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

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61. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

62. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

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67. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

68. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

69. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

70. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

71. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.

72. The number of defective blouses $d$ is a function of the total number of defective blouses $b$ produced at her factory. This function is $d = \frac{b}{10000}$.
Find each real root. To start, find a number whose square is equal to the given number.

Find all the real fourth roots of each number.

10. \( x^4 = 16 \)
11. \( x^4 = 81 \)
12. \( x^4 = 625 \)

Find all the real cube roots of each number.

13. \( x^3 = 64 \)
14. \( x^3 = 125 \)
15. \( x^3 = 125 \)

Find all the real square roots of each number.

6. \( x^2 = 0.0064 \)
7. \( x^2 = 0.027 \)
8. \( x^2 = 0.3 \)

For Exercises 1−6, choose the correct letter. Multiple Choice

1. What is the real square root of 0.0064? B
2. What is the real cube root of −64? D
3. What is the real fourth root of 256? C
4. What is the value of \( \sqrt[3]{125} \)? C
5. What is the simplified form of the expression \( \sqrt{4} \cdot \sqrt{27} \)? D
6. What are the real solutions of the equation \( x^2 = 16 \)? C

Short Response

7. The volume \( V \) of a cube with side length \( s \) is \( V = s^3 \). A cubic storage bin has volume 5832 cubic inches. What is the length of the side of the cube? Show your work.

Rounding Roots and Radicals

Computers treat radicals such as \( \sqrt{x} \) as if they were rounded to a preassigned number of decimal places. Most computers round numbers according to an algorithm that uses the largest integer less than or equal to a given number. This function is called the greatest integer function and is written as \( y = \lfloor x \rfloor \).

As you can see, the graph of the greatest integer function is not continuous. The open circles indicate that the endpoints are not included as part of the graph. The command INT in most popular spreadsheet programs serves the same purpose as the greatest integer function. For instance, \( \text{INT}(3.5) = 3 \), \( \text{INT}(1.05) = 1 \), \( \text{INT}(7) = 7 \).

To round a number to \( r \) decimal places, a computer performs the following procedure:

Step 1: Multiply \( x \) by \( 10^r \).
Step 2: Add 0.5 to the result.
Step 3: Find INT of the result.
Step 4: Multiply the result by \( 10^{-r} \).

Fill in the table below to see how this procedure works.

A computer that rounds numbers after each operation may introduce rounding errors into calculations. To see the effects of rounding errors, perform each of the following computations for \( x = 2 \) and different values of \( r \). First find the given root and write the answer to \( r + 1 \) digits after the decimal. Carry out the four steps to get the answer and then raise the result to the given power. Write the answer again to \( r + 1 \) digits after the decimal and carry out the four steps to get the final answer.

1.2345 6.987
2.00001 3.00000
3.99999 2.99999
2.00001
2.00000
6-1 Reteaching

Roots and Radicals

- For any real numbers a and b and any positive integer n, if a is raised to the nth power equals b, then a is an nth root of b. Use the radical sign to write a root.

- The following expressions are equivalent:

  power
  \[ b = a^n \]

  index
  \[ \sqrt[n]{b} = a \]

  radical sign
  \[ \sqrt[n]{b} = a \]

- Write as the cube of a product.

- What is the real-number roots of each radical expression?

  - a. \( \sqrt[3]{27} \)
    - Because \( (3)^3 = 27 \), 3 is a third (cube) root of 27.
    - Therefore, \( \sqrt[3]{27} = 3 \).
    - [Notice that \( (-3)^3 = -27 \), so –3 is not a cube root of 27.]

  - b. \( \sqrt[3]{-125} \)
    - Because \( (-5)^3 = -125 \) and \( (-5)^3 = -125 \), both \(-5\) and \(-5\) are real-number fourth roots of 125.

  - c. \( \sqrt[3]{-1} \)
    - Because \( (-1)^3 = -1 \) and \( (-1)^3 = -1 \), both \(-1\) and \(-1\) are real-number fourth roots of -1.

  - d. \( \sqrt[3]{-25} \)
    - Because \( (-25)^{1/3} = -1.5974 \) and \( (-5)^3 = -125 \), there are no real-number square roots of -25.

6-2 ELL Support

Multiplying and Dividing Radical Expressions

- To solve a problem I need to find:
  - the distance from the surface of the Earth to its center
  - the first satellite orbits at an altitude of 300 mi
  - the second satellite orbits at an altitude of 200 mi

6-3 Think About a Plan

Multiplying and Dividing Radical Expressions

- Solving:
  - the difference in the velocities of a satellite orbiting at an altitude of 100 mi
  - one orbiting at an altitude of 200 mi

- Plan:
  - Rewrite the formula for the circular velocity of a satellite using a for the altitude of the satellite.
  - \( v = \sqrt{\frac{gr}{a}} \)
  - Use your formula to find the velocity of a satellite orbiting at an altitude of 100 mi
  - about 17,000 m/s (17.0 km/s)
  - Use your formula to find the velocity of a satellite orbiting at an altitude of 200 mi
  - about 17,200 m/s (17.2 km/s)

- How much greater is the velocity of a satellite orbiting at an altitude of 100 mi than one orbiting at an altitude of 200 mi?
  - about 200 m/s (0.2 km/s)
6-2 Practice

Form G

Multiplying and Dividing Radical Expressions

Multiply, if possible. Then simplify.

1. $\sqrt{2} \cdot \sqrt{8}$
2. $\sqrt{3} \cdot \sqrt{7}$
3. $\sqrt{2} \cdot \sqrt{9}$
4. $\sqrt{4} \cdot \sqrt{2}$

Simplify. Assume that all variables are positive.

5. $\sqrt{8}$
6. $\sqrt{12}$
7. $\sqrt{24}$
8. $\sqrt{18}$

Multiply and simplify. Assume that all variables are positive.

9. $3\sqrt{2} \cdot 2\sqrt{3}$
10. $\sqrt{a} \cdot \sqrt{b}$
11. $\sqrt{x} \cdot \sqrt{y}$
12. $\sqrt{a} \cdot \sqrt{b} \cdot \sqrt{c}$

Divide and simplify. Assume that all variables are positive.

13. $\frac{\sqrt{18}}{\sqrt{2}}$
14. $\frac{\sqrt{8}}{\sqrt{2}}$
15. $\frac{\sqrt{32}}{\sqrt{8}}$
16. $\frac{\sqrt{48}}{\sqrt{12}}$

Simplify each expression. Assume all variables are positive.

17. $\sqrt{12} \cdot \sqrt{2}$
18. $\sqrt{16} \cdot \sqrt{4}$
19. $\sqrt{25} \cdot \sqrt{3}$
20. $\sqrt{49} \cdot \sqrt{7}$

13. Simplify. Assume that all variables are positive.

14. Multiply, if possible. Then simplify. To start, find all perfect square factors.

15. $\sqrt{18}$
16. $\sqrt{24}$
17. $\sqrt{32}$
18. $\sqrt{48}$

16. Multiply and simplify. Assume all variables are positive.

19. $\sqrt{16} \cdot \sqrt{2}$
20. $\sqrt{25} \cdot \sqrt{3}$
21. $\sqrt{32} \cdot \sqrt{2}$
22. $\sqrt{48} \cdot \sqrt{3}$

17. Simplify each expression. Assume all variables are positive.

23. $\sqrt{18} \cdot \sqrt{2}$
24. $\sqrt{25} \cdot \sqrt{3}$
25. $\sqrt{32} \cdot \sqrt{2}$
26. $\sqrt{48} \cdot \sqrt{3}$

18. Error Analysis: Your classmate simplified $\sqrt{18} \cdot \sqrt{25}$ to $5\sqrt{6}$. What mistake did she make? What is the correct answer? She thought the indexes were the same. They are different, so you cannot multiply the radicals.

19. A square rug has sides measuring $\sqrt{6}$ ft by $\sqrt{6}$ ft. What is the area of the rug? $6 \text{ ft}^2$
6-2 Standardized Test Prep

Multiplying and Dividing Radical Expressions

Multiple Choice
For Exercises 1−5, choose the correct letter. Assume that all variables are positive.

1. What is the simplest form of \( \sqrt[5]{x^2} \cdot \sqrt[5]{x^3} \)?
   - (a) \( \sqrt[5]{x^5} \)
   - (b) \( \sqrt[5]{x^7} \)
   - (c) \( \sqrt[5]{x} \)
   - (d) \( \sqrt[5]{x^4} \)

2. What is the simplest form of \( \sqrt[3]{8x^3} \)?
   - (a) \( 2x \)
   - (b) \( 2x^2 \)
   - (c) \( 2x^3 \)
   - (d) \( 2x^4 \)

3. What is the simplest form of \( \sqrt{25y^4} \)?
   - (a) \( 5y^2 \)
   - (b) \( 5y^3 \)
   - (c) \( 5y^4 \)
   - (d) \( 5y^5 \)

4. What is the simplest form of \( \sqrt{4y^2} \)?
   - (a) \( 2y \)
   - (b) \( 2y^2 \)
   - (c) \( 2y^3 \)
   - (d) \( 2y^4 \)

5. What is the simplest form of \( \sqrt{16z^5} \)?
   - (a) \( 4z^2 \)
   - (b) \( 4z^3 \)
   - (c) \( 4z^4 \)
   - (d) \( 4z^5 \)

Short Response
6. The volume \( V \) of a wooden beam is \( V = \pi l^2 \), where \( l \) is the length of the beam and \( w \) is the length of one side of its square cross section. If the volume of the beam is 1200 in.\(^3 \) and its length is 90 in., what is the side length? Show your work.

\[ V = \pi l^2 \]
\[ 1200 = \pi l^2 \]
\[ l^2 = \frac{1200}{\pi} \]
\[ l = \sqrt{\frac{1200}{\pi}} \]

7. Choose three odd numbers larger than 50 and write them as the sum of three primes.

\[ 53 = 3 + 5 + 5 \]
\[ 63 = 7 + 7 + 9 \]
\[ 73 = 11 + 5 + 7 \]

ANSWERS

6-2 Enrichment

Multiplying and Dividing Radical Expressions

To simplify the radical \( \sqrt{n} \), you look for a perfect nth power among the factors of the radicand \( a \). When this factor is not obvious, it is helpful to factor the number into primes. Prime numbers are important in many aspects of mathematics. Several mathematicians throughout history have unsuccessfully tried to find a pattern that would generate the nth prime number. Other mathematicians have offered conjectures about primes that remain unresolved.

1. Goldbach’s Conjecture states that every even number \( n \geq 2 \) can be written as the sum of two primes. For example, \( 4 = 2 + 2 \) and \( 10 = 3 + 7 \). Choose three even numbers larger than 50 and write them as a sum of two primes.
   \[ \text{Answers may vary. Sample: } 52 = 17 + 35 \]

2. The Odd Goldbach’s Conjecture states that every odd number \( n \geq 3 \) can be written as the sum of three primes. For example, \( 7 = 2 + 2 + 3 \). Choose three odd numbers larger than 50 and write them as a sum of three primes.
   \[ \text{Answers may vary. Sample: } 51 = 13 + 13 + 15 \]

3. Another interesting pattern emerges when you examine a subset of the prime numbers. Make a list of the primes less than 50.

\[ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 \]

4. Make this list smaller by eliminating 2 and all primes that are 1 less than a multiple of 4.

\[ 3, 5, 13, 17, 23, 31, 43 \]

5. The remaining primes in the list above are related in an interesting way. You can write each prime as the sum of two squares. Express each of these primes as a sum of two squares.

\[ 3 = 1 + 2 \]
\[ 5 = 1 + 4 \]
\[ 13 = 2 + 11 \]
\[ 17 = 8 + 9 \]
\[ 19 = 2 + 17 \]
\[ 23 = 4 + 19 \]
\[ 29 = 5 + 24 \]
\[ 31 = 10 + 21 \]
\[ 41 = 5 + 36 \]
\[ 43 = 13 + 30 \]

6. A Cullen number, named after an Irish mathematician James Cullen, is a natural number of the form \( n \cdot 2^n + 1 \). Determine the first four Cullen numbers. That is, let \( n = 1, 2, 3, 4 \).

\[ 3, 9, 33, 125 \]

7. What is the smallest Cullen number that is a prime number? (The next Cullen number that is a prime occurs when \( n = 143 \))

\[ n = 1 \]

8. A palindrome is a number that reads the same forward and backward. For example, 121 is a palindrome number. List the seven palindrome primes that are less than 149.

\[ 3, 5, 11, 101, 131 \]

6-2 Reteaching

Multiplying and Dividing Radical Expressions

You can simplify a radical if the radicand has a factor that is a perfect nth power and \( n \) is the index of the radical. For example, \( \sqrt[4]{x^6} = x^{6/4} \).

Problem
What is the simplest form of each product?

a. \( \sqrt[4]{x^2} \cdot \sqrt[4]{x^3} \)
   - Use \( \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} \).
   - \( \sqrt[4]{x^2} \cdot \sqrt[4]{x^3} = \sqrt[4]{x^5} \).
   - Use perfect first powers.
   - \( \sqrt[4]{x^4} \cdot \sqrt[4]{x} = x^{4/4} \cdot \sqrt[4]{x} \).
   - Simplify.

b. \( \sqrt[6]{y^2} \cdot \sqrt[6]{y^4} \)
   - Use \( \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} \).
   - \( \sqrt[6]{y^2} \cdot \sqrt[6]{y^4} = \sqrt[6]{y^6} \).
   - Use perfect second powers.
   - \( \sqrt[6]{y^6} = y^{6/6} \).

Exercises
Simplify each product.

1. \( \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} \)
2. \( \sqrt[4]{y^4} \cdot \sqrt[4]{y^3} \)
3. \( \sqrt[2]{a^2} \cdot \sqrt[2]{a^4} \)
4. \( \sqrt[3]{b^3} \cdot \sqrt[3]{b^2} \)
5. \( \sqrt[3]{c^3} \cdot \sqrt[3]{c^2} \cdot \sqrt[3]{c^3} \)
6. \( \sqrt[3]{d^3} \cdot \sqrt[3]{d^2} \cdot \sqrt[3]{d^3} \)
7. \( \sqrt[4]{e^4} \cdot \sqrt[4]{e^3} \)
8. \( \sqrt[5]{f^5} \cdot \sqrt[5]{f^4} \)
9. \( \sqrt[6]{g^6} \cdot \sqrt[6]{g^5} \)
10. \( \sqrt[7]{h^7} \cdot \sqrt[7]{h^6} \)

11. \( \sqrt[3]{i^3} \cdot \sqrt[3]{i^2} \cdot \sqrt[3]{i^3} \)
12. \( \sqrt[4]{j^4} \cdot \sqrt[4]{j^3} \cdot \sqrt[4]{j^4} \)
13. \( \sqrt[5]{k^5} \cdot \sqrt[5]{k^4} \cdot \sqrt[5]{k^5} \)
14. \( \sqrt[6]{l^6} \cdot \sqrt[6]{l^5} \cdot \sqrt[6]{l^6} \)

6-2 Reteaching 2

Multiplying and Dividing Radical Expressions

Rationalize the denominator means that you are rewriting the expression so that no radicals appear in the denominator and there are no fractions inside the radical.

Problem
What is the simplest form of \( \sqrt[4]{2} \)?

Rationalize the denominator and simplify. Assume that all variables are positive.

\[ \sqrt[4]{2} = \sqrt[4]{2} \]

1. Write as a square root of a fraction.
2. Make the denominator a perfect square.
4. Write the denominator as a product of perfect squares.
5. Simplify the denominator.
6. \( \sqrt[4]{2} \cdot \sqrt[4]{2} \cdot \sqrt[4]{2} \cdot \sqrt[4]{2} \)
7. Simplify the numerators.
8. Use \( \sqrt[n]{a} = a \) to simplify.

Exercises
Rationalize the denominator of each expression. Assume that all variables are positive.

7. \( \sqrt[5]{5} \)
8. \( \sqrt[6]{6} \)
9. \( \sqrt[7]{7} \)
10. \( \sqrt[8]{8} \)
11. \( \sqrt[9]{9} \)
12. \( \sqrt[10]{10} \)
13. \( \sqrt[11]{11} \)
14. \( \sqrt[12]{12} \)
6-3 ELL Support

Rationalizing the Denominator

The column on the left shows the steps used to rationalize a denominator. Use the column on the left to answer each question in the column on the right.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Rationalizing the Denominator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>What does it mean to rationalize a denominator?</td>
</tr>
<tr>
<td>2.</td>
<td>Multiply the numerator and the denominator by the conjugate of the denominator.</td>
</tr>
<tr>
<td>3.</td>
<td>Write a new equation to show why the radicals in the denominator cancel out.</td>
</tr>
<tr>
<td>4.</td>
<td>Divide ( \sqrt{7} ) in the numerator.</td>
</tr>
<tr>
<td>5.</td>
<td>Simplify.</td>
</tr>
</tbody>
</table>

1. Why do you have a product of 21? |
2. What property allows you to distribute the \( \sqrt{7} \)? |
3. What is \( \sqrt{7} \)? |
4. What number multiplied by \( \sqrt{7} \) would produce a product of 21? |

6-3 Practice

Binomial Radical Expressions

Add or subtract, if possible.

1. \( \sqrt{2} + 3 + \sqrt{2} + \sqrt{2} \) 
2. \( \sqrt{2} + 3 + \sqrt{2} - \sqrt{2} \) 
3. \( \sqrt{2} + 3 - \sqrt{2} \) 
4. \( \sqrt{2} + 3 - \sqrt{2} \) 
5. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
6. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
7. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
8. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 

Simplify.

9. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
10. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
11. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
12. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
13. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
14. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
15. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
16. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 

Multiply.

17. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
18. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
19. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
20. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
21. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
22. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
23. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
24. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 

Multiply each pair of conjugates.

25. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
26. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
27. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
28. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
29. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
30. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 

6-3 Practice (continued)

Rationalize each denominator. Simplify the answer.

31. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
32. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
33. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
34. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
35. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
36. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
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51. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
52. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
53. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
54. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
55. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
56. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
57. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
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81. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
82. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
83. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
84. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
85. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
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87. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
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92. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
93. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
94. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
95. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
96. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
97. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
98. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
99. \( \sqrt{2} + 3 - 2 \sqrt{2} \) 
100. \( \sqrt{2} + 3 - 2 \sqrt{2} \)
6-3 Practice

Form K

Binomial Radical Expressions

Simplify if possible. To start, determine if the expressions contain like radicals.

1. $\sqrt[3]{2} + \sqrt[3]{8}$
   - radicands: $2$ and $8$
   - neither radical
   - no; cannot simplify

2. $2\sqrt{2} \div \sqrt[3]{2}$
   - radicands: $2$ and $2$
   - $2$th radical
   - $2\sqrt[3]{2}$

3. $2\sqrt{2} + 2\sqrt{2}$
   - radicands: $2$ and $2$
   - both radicals: $2$
   - no; cannot simplify

4. A tile is made up of smaller squares. Each square measures 3 in. on each side. Find the perimeter of the tile. $24 \times 7$ in.

Simplify. To start, factor each radical.

5. $\sqrt{7} + \sqrt{7}$
   - $\sqrt{2}$
   - $2\sqrt{7}$

6. $\sqrt[3]{7} - \sqrt[3]{7}$
   - $\sqrt[3]{2}$
   - $0$

7. $\sqrt{2} \div \sqrt[3]{2}$
   - $\sqrt[3]{2}$
   - $\sqrt[3]{2}$

Multiply.

8. $(\sqrt{7} - \sqrt{7})(\sqrt{2} + \sqrt{2})$
   - $4\sqrt{2}$

9. $(\sqrt{7} - \sqrt{7})(\sqrt{2} - \sqrt{2})$
   - $-4\sqrt{2}$

10. $(\sqrt{7} + \sqrt{7})(\sqrt{2} + \sqrt{2})$
    - $14$

Multiply each pair of conjugates.

11. $(\sqrt{7} - \sqrt{7})(\sqrt{2} + \sqrt{2})$
    - $14$

12. $(\sqrt{7} + \sqrt{7})(\sqrt{2} - \sqrt{2})$
    - $-14$

13. $(\sqrt{7} + \sqrt{7})(\sqrt{2} + \sqrt{2})$
    - $14$

14. $(\sqrt{7} - \sqrt{7})(\sqrt{2} - \sqrt{2})$
    - $-14$

Short Response

15. A rock drops a rock from the rim of the Grand Canyon. The distance it falls in 100 feet is given by the function $d = \frac{16t^2}{2}$. How far has the rock fallen after 3.4 seconds? Show your work.
   - $d = \frac{16 \times (3.4)^2}{2} = \frac{16 \times 11.56}{2} = 92.48\text{ ft}$

2. Find the distance traveled by the rock from the rim of the Grand Canyon if it takes 9.2 seconds to reach the bottom.
   - $d = \frac{16 \times (9.2)^2}{2} = \frac{16 \times 84.64}{2} = 676.96\text{ ft}$

3. a. Find the distance the rock travels after 10 seconds.
   - $d = \frac{16 \times (10)^2}{2} = \frac{16 \times 100}{2} = 800\text{ ft}$

b. Find the distance traveled by the rock from the rim of the Grand Canyon if it takes 9.6 seconds to reach the bottom.
   - $d = \frac{16 \times (9.6)^2}{2} = \frac{16 \times 92.16}{2} = 737.28\text{ ft}$

4. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

5. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

6. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

7. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

8. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

9. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

10. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

11. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

12. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

13. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

14. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

15. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

16. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

17. A section of tile wall has the design shown at the right. The design is made up of equilateral triangles. Each side of the large triangle is 4 in. and each side of a small triangle is 2 in. Find the total area of the design to the nearest tenth of an inch.
   - $A = 17.3\text{ in.}^2$

18. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

19. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

20. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

21. Error Analysis: A classmate simplified the expression using the steps shown. What mistake did your classmate make?

22. Writing: Explain the first step in simplifying $\sqrt{10} + \sqrt{10} - \sqrt{10}$.

23. Consider how you might use a calculator to find the square of negative three. If you enter the expression $-3^2$, your calculator produces an answer of $-9$. However, the square of negative three is $(-3)^2 = 9$. Calculators follow the order of operations. Therefore, a calculator will compute $-3\times -3$ as the opposite of $9^2$. The correct input is $(-3)^2$, which is correctly evaluated as $9$. Be sure to follow the order of operations when expanding binomial radical expressions.

24. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

25. Error Analysis: A classmate simplified the expression using the steps shown. What mistake did your classmate make?

26. Writing: Explain the first step in simplifying $\sqrt{10} + \sqrt{10} - \sqrt{10}$.

27. Consider how you might use a calculator to find the square of negative three. If you enter the expression $-3^2$, your calculator produces an answer of $-9$. However, the square of negative three is $(-3)^2 = 9$. Calculators follow the order of operations. Therefore, a calculator will compute $-3\times -3$ as the opposite of $9^2$. The correct input is $(-3)^2$, which is correctly evaluated as $9$. Be sure to follow the order of operations when expanding binomial radical expressions.

28. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

29. Error Analysis: A classmate simplified the expression using the steps shown. What mistake did your classmate make?

30. Writing: Explain the first step in simplifying $\sqrt{10} + \sqrt{10} - \sqrt{10}$.

31. Consider how you might use a calculator to find the square of negative three. If you enter the expression $-3^2$, your calculator produces an answer of $-9$. However, the square of negative three is $(-3)^2 = 9$. Calculators follow the order of operations. Therefore, a calculator will compute $-3\times -3$ as the opposite of $9^2$. The correct input is $(-3)^2$, which is correctly evaluated as $9$. Be sure to follow the order of operations when expanding binomial radical expressions.

32. Graph the function $y = \sqrt{x}$ for $x \geq 0$.

33. Error Analysis: A classmate simplified the expression using the steps shown. What mistake did your classmate make?

34. Writing: Explain the first step in simplifying $\sqrt{10} + \sqrt{10} - \sqrt{10}$.
**6-3 Reteaching**
Biomedical Radical Expressions

Two radical expressions are like radicals if they have the same index and the same radicand.

Compare radical expressions to the terms in a polynomial expression.

Like terms: $\sqrt{a^2}$, $\sqrt{b^2}$  The power and the variable are the same.

Unlike terms: $\sqrt{a^2}$, $\sqrt{b^3}$  Either the power or the variable are not the same.

Like radicals: $\sqrt{a^2}$, $\sqrt{b^2}$  The index and the radicand are the same.

Unlike radicals: $\sqrt{a^2}$, $\sqrt{b^2}$  Either the index or the radicand are not the same.

When adding or subtracting radical expressions, simplify each radical so that you can find like radicals.

**Problem**

What is the sum? $\sqrt{27} + \sqrt{75}$

Find each radical.

$= \sqrt{3^3} + \sqrt{3^2 \cdot 5}$

$= 3\sqrt{3} + \sqrt{3} \cdot \sqrt{5}$

$= 3\sqrt{3} + 5\sqrt{3}$

Add like radicals.

The sum is $5\sqrt{3}$.

**Exercises**

Simplify.

1. $\sqrt{27} - \sqrt{48}$
2. $\sqrt{25} + \sqrt{8}$
3. $\sqrt{18} - \sqrt{75}$
4. $\sqrt{8} - \sqrt{2}$
5. $\sqrt{3} + \sqrt{2} - \sqrt{18}$
6. $\sqrt{18} + \sqrt{8} - \sqrt{2}$
7. $\sqrt{2} + \sqrt{3}$

Write each expression in exponential form.

8. $\sqrt{3} = 3^{\frac{1}{2}}$
9. $(\sqrt{3})^2 = 3$
10. $\sqrt{2} + \sqrt{3}$

Write each expression in radical form.

11. $\sqrt{3}$
12. $\sqrt{2} + \sqrt{3}$

Multiple Choice

12. What is the simplest term? $B$
   - $\sqrt{3}$
   - $\sqrt{2}$
   - $\sqrt{3}$
   - $\sqrt{2}$
   - $\sqrt{3}$

---

**6-4 ELL Support**

Rational Exponents

Choose the word or phrase from the list that best matches each sentence.

<table>
<thead>
<tr>
<th>rational exponent</th>
<th>radical form</th>
<th>exponential form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The expression $\sqrt{3}$ is written in ___</td>
<td>radical form</td>
<td>exponential form</td>
</tr>
<tr>
<td>2. $a^{\frac{2}{3}}$ is an exponent written in fractional form</td>
<td>rational exponent</td>
<td>exponential form</td>
</tr>
<tr>
<td>3. The expression $\sqrt[3]{a}$ is written in ___</td>
<td>exponential form</td>
<td>radical form</td>
</tr>
</tbody>
</table>

Write each expression in exponential form.

4. $\sqrt{3} = 3^{\frac{1}{2}}$
5. $(\sqrt{3})^2 = 3$
6. $(\sqrt{3})^3 = \sqrt{3}$
7. $\sqrt{3} = 3^{\frac{1}{2}}$

Write each expression in radical form.

8. $a^{\frac{2}{3}} = \sqrt[3]{a^2}$
9. $a^{\frac{1}{2}} = \sqrt{a}$
10. $a^{\frac{3}{2}} = \sqrt[2]{a^3}$

Multiple Choice

12. What is the simplest term? $B$
   - $\sqrt{3}$
   - $\sqrt{2}$
   - $\sqrt{3}$
   - $\sqrt{2}$
   - $\sqrt{3}$

---

**6-4 Think About a Plan**

Rational Exponents

**Science**
A desktop world globe has a volume of about 1300 cubic inches. The radius of the Earth is approximately equal to the radius of the globe raised to the 10th power. Find the radius of the Earth. (Hint: Use the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere.)

**Know**

1. The volume of the globe is 1300 in.$^3$.

2. The radius of the Earth is equal to ___ the radius of the globe raised to the 10th power ___.

**Need**

1. To solve the problem I need to find ___ the radius of the Earth ___.

**Plan**

4. Write an equation relating the radius of the globe $r_g$ to the radius of the Earth $r_e$.
   
5. How can you represent the radius of the globe in terms of the radius of the Earth?
   
6. Write an equation to represent the volume of the globe.
   
7. Use your previous equation and your equation from Exercise 5 to write an equation to find the radius of the Earth.
   
8. Solve your equation to find the radius of the Earth.
   
   About 3950 mi

---

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6-4 Practice Form G
Rational Exponents

Simplify each expression.

1. \( \sqrt[3]{5} \)
2. \( \sqrt{8} \)
3. \( \sqrt[4]{2} \)

Write each expression in radical form.

4. \( \sqrt[3]{5} \)
5. \( \sqrt{8} \)
6. \( \sqrt[4]{2} \)

Write each expression in exponential form.

7. \( \sqrt[3]{5} \)
8. \( \sqrt{8} \)
9. \( \sqrt[4]{2} \)

Simplify each expression.

10. \( \sqrt[3]{5} \)
11. \( \sqrt{8} \)
12. \( \sqrt[4]{2} \)

Write each expression in radical form.

13. \( \sqrt[3]{5} \)
14. \( \sqrt{8} \)
15. \( \sqrt[4]{2} \)

Write each expression in exponential form.

16. \( \sqrt[3]{5} \)
17. \( \sqrt{8} \)
18. \( \sqrt[4]{2} \)

19. \( \sqrt[3]{5} \)
20. \( \sqrt{8} \)
21. \( \sqrt[4]{2} \)

22. \( \sqrt[3]{5} \)
23. \( \sqrt{8} \)
24. \( \sqrt[4]{2} \)

25. \( \sqrt[3]{5} \)
26. \( \sqrt{8} \)
27. \( \sqrt[4]{2} \)

28. \( \sqrt[3]{5} \)
29. \( \sqrt{8} \)
30. \( \sqrt[4]{2} \)

31. The rate of inflation \( r \) that raises the cost of an item from the present value \( P \) to the future value \( F \) over 10 years is found using the formula \( i = \left( \frac{F}{P} \right)^{\frac{1}{n}} - 1 \). Round your answers to the nearest tenth of a percent.

a. What is the rate of inflation for which a television set costing \$1000 today will become one costing \$1500 in 10 years? 14.3%

b. What is the rate of inflation that will result in the price \( P \) doubling after 20 years? 7.2%

d. What is the rate of inflation that will result in the price \( P \) doubling after 20 years? 7.2%

10. Bone loss for astronauts may be prevented with an apparatus that rotates to simulate gravity. In the formula \( N = \frac{g}{r^2} \), \( N \) is the rate of rotation in revolutions per second, \( r \) is the simulated acceleration in m/s², and \( r \) is the radius of the apparatus in meters. How fast would an apparatus with the following radii have to rotate to simulate the acceleration of 9.8 m/s² that is due to Earth’s gravity?

a. \( r = 1.7 \) m; 368 rev/s
b. \( r = 1.6 \) m; 252 rev/s
c. \( r = 2.1 \) m; 218 rev/s

4. Reasoning Would an apparatus with radius 0.8 m need to spin faster or slower than the one in part (a)? Faster

22. Error Analysis Explain why the following simplification is incorrect. What is the correct simplification? You cannot multiply \( 5 \) and \( 3 \) together by multiplying bases. You have to write \( 6 \) in \( \sqrt[3]{6} \) and combine the exponents: 20 - 5 = 5.

\[
\left( \frac{6}{5} \right)^{\frac{2}{3}} = \frac{6^{\frac{2}{3}}}{5^{\frac{2}{3}}} = \frac{36}{25} = 1.44
\]
**6-4 Standardized Test Prep**

**Multiple Choice**

For Exercises 1–5, choose the correct letter.

1. What is $16^2 \cdot 8^2 \cdot 4^2$ in simplest form? D
   - $\text{A} \quad 32^6$
   - $\text{B} \quad 10^6$
   - $\text{C} \quad 10^7$
   - $\text{D} \quad 27,000$

2. What is $x^2 \cdot x^3$ in simplest form? C
   - $\text{A} \quad x^5$
   - $\text{B} \quad x^6$
   - $\text{C} \quad x^5$
   - $\text{D} \quad x^6$

3. What is $x^{\frac{2}{3}} \cdot x^{\frac{1}{2}}$ in simplest form? A
   - $\text{A} \quad x^{\frac{5}{6}}$
   - $\text{B} \quad x^{\frac{3}{2}}$
   - $\text{C} \quad x^{\frac{1}{6}}$
   - $\text{D} \quad x^{\frac{2}{3}}$

4. What is $(x^2)^3$ in simplest form? B
   - $\text{A} \quad x^5$
   - $\text{B} \quad x^6$
   - $\text{C} \quad x^{\frac{1}{3}}$
   - $\text{D} \quad x^{\frac{2}{3}}$

5. What is $(-2x^2y^3)^2$ in simplest form? C
   - $\text{A} \quad 4x^2y^6$
   - $\text{B} \quad 2x^4y^6$
   - $\text{C} \quad 4x^4y^6$
   - $\text{D} \quad 2x^2y^6$

**Short Response**

6. The surface area $S$, in square units, of a sphere with volume $V$, in cubic units, is given by the formula $S = \frac{2}{3}\pi V^{\frac{3}{2}}$. What is the surface area of a sphere with volume 243? Write your answer in terms of $\pi$. Show your work.

   \[ S = \frac{2}{3}\pi V^{\frac{3}{2}} \]
   \[ V = 243 \]
   \[ S = \frac{2}{3}\pi (243)^{\frac{3}{2}} \]
   \[ S = \frac{2}{3}\pi (27^2) \]
   \[ S = \frac{2}{3}\pi (729) \]
   \[ S = 486\pi \]

**Answers**

**6-4 Enrichment**

**Rational Exponents**

Each problem below involves rational exponents. Some of the problems are tricky. Good luck!

1. Begin with any positive number. Call it $x$. Divide by 2. Call the result $y$. Now follow these directions carefully. You may use a calculator. Final $r = \sqrt[3]{x}$.
   - a. Divide $x$ by 2. Call the result $y$.
   - b. Add $y$ and $x$. Call the result $z$.
   - c. Divide $z$ by 2. Call the result $r$.
   - d. Go back to step a. Repeat steps a–d until no longer changes. What is the relationship between the original $x$ and the final result?

2. If we take the square root of a number 6 times, it would look like this:

   \[ \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{x}}}}}} \]

Rewrite the expression above using rational exponents. \([\frac{3}{2}]^{\frac{3}{2}]^{\frac{3}{2}]^{\frac{3}{2}]^{\frac{3}{2}]^{\frac{3}{2}]}}\]

Simplify the expression above. Express the denominator of the exponent as a power of 2.

If you were to take the square root of a number 10 times, what would the denominator of the exponent be equal to if you use rational exponents? 12 times $2^{10}$.

Choose any number and repeatedly take the square root. What number is the answer approaching? \(1\).

Does the answer appear to approach the same number if you change the number you choose? \(1\).

In Exercises 3–6, assume that the square roots and the operations inside them repeat forever.

3. How much is $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \ldots$? (Hint: Let $y = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \ldots$. Then use substitution and solve the equation $y = \sqrt{2} \times y$.)

4. How much is $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \ldots$? $\sqrt{2}$

5. How much is $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \ldots$? $\sqrt{2}$

6. How much is $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \ldots$? $\sqrt{2}$

**6-4 Re-teaching**

**Rational Exponents**

You can simplify a number with a rational exponent by converting the expression to a radical expression:

\[ a^{\frac{m}{n}} = \sqrt[n]{a^m} \]

You can simplify the product of numbers with rational exponents $x$ and $y$ by raising the number to the sum of the exponents using the rule:

\[ a^m \cdot a^n = a^{m+n} \]

**Problem**

What is the simplified form of each expression?

a. \(36^2 \cdot 36^3\)
   - $36^2 \cdot 36^3 = 36^5$
   - $36^5 = 6^2 \cdot 6^3$
   - $6^2 \cdot 6^3 = 6^5$
   - $6^5$

b. Write \((x^2)^2 \cdot (x^3)^2\) in simplest form.
   - $(x^2)^2 \cdot (x^3)^2 = x^4 \cdot x^6$
   - $(x^4 \cdot x^6) = x^{10}$
   - $(x^4 \cdot x^6) = x^{10}$
   - $(x^4 \cdot x^6) = x^{10}$

**Exercises**

Simplify each expression. Assume that all variables are positive.

1. \(\sqrt{\sqrt{2}}\)
2. \(\sqrt[3]{(\sqrt{2})^3}\)
3. \(-\sqrt{15} \cdot \sqrt{15} = -15\)
4. \(-\sqrt[4]{16}\)
5. \(\sqrt{16} \cdot \sqrt{4} = 4 \cdot 2 = 8\)
6. \(\sqrt[3]{(-5)^3} = -5\)
7. \(\sqrt[3]{125} = 5\)
8. \(\sqrt[3]{-125} = -5\)
9. \(\sqrt{100} = 10\)
10. \(\sqrt{0.04} = 0.2\)
11. \(\sqrt{\frac{1}{16}} = \frac{1}{4}\)
12. \(\sqrt{\frac{9}{16}} = \frac{3}{4}\)
13. \(\sqrt{25} = 5\)
14. \(\sqrt{81} = 9\)
15. \(\sqrt{100} = 10\)
16. \(\sqrt{0.001} = 0.01\)
17. \(\sqrt{\frac{1}{4}} = \frac{1}{2}\)
18. \(\sqrt{\frac{9}{16}} = \frac{3}{4}\)

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6-5 ELL Support
Solving Square Root and Other Radical Equations

Problem

Solve the equation \( 4\sqrt{2x + 27} + 3 = 19 \). Justify your steps. Then check your solution.

1. \( 4y + 2x^2 + 3 = 19 \)
   - Rewrite the radical using a rational exponent.
   - Subtract 3 from each side.
   - Divide each side by 4.
   - Raise each side to the \( \frac{1}{2} \) power.
   - Simplify the exponents.
   - Solve for \( y \).

Check \( 4\sqrt{2(x + 3)} + 3 = 19 \) Substituting \( x \) for \( y \).

Exercise

Solve the equation \( 3\sqrt{2x - 3} + 2 = 38 \). Justify your steps. Then check your solution.

1. \( 4x - 4 \sqrt{x} + 2 = 38 \)
   - Rewrite the radical using a rational exponent.
   - Subtract 2 from each side and simplify the exponent.
   - Divide each side by 4.
   - Raise each side to the \( \frac{1}{2} \) power.
   - Simplify the exponents.
   - Solve for \( x \).

Check \( 3\sqrt{2(x - 3)} + 2 = 38 \) Substitute \( x \) for \( a \).

6-5 Practice
Form G

Solving Square Root and Other Radical Equations

Solve.

1. \( \sqrt{\frac{x}{2}} + 2 = 12 \).
2. \( 3\sqrt{x} - 8 = 25 \).
3. \( \sqrt{x + 2} = 8 \).
4. \( \sqrt{x} - 1 = 3 \).
5. \( \sqrt{x + 5} = 12 \).
6. \( \sqrt{x} + 3 = 18 \).
7. \( \sqrt{x - 2} = 8 \).
8. \( \sqrt{x + 2} = 5 \).
9. \( \sqrt{x} = 3 \).
10. \( \sqrt{x} + 3 = 0 \).

11. \( (x - 2)^2 = 1 \).
12. \( (x + 2)^2 = 4 \).
13. \( x^2 - 4x + 4 = 0 \).
14. \( x^2 - 9 = 0 \).
15. \( x^2 - 2x - 3 = 0 \).
16. \( x^2 - 5x + 6 = 0 \).
17. \( x^2 + 4x + 4 = 0 \).
18. \( x^2 - 6x + 9 = 0 \).
19. \( x^2 + 2x + 1 = 0 \).
20. \( x^2 - 2x - 3 = 0 \).
21. \( x^2 + 3x + 2 = 0 \).
22. \( x^2 - 5x + 6 = 0 \).
23. \( x^2 + 3x + 2 = 0 \).
24. \( x^2 - 6x + 9 = 0 \).

25. The area \( A \) of the window is 106 in.\(^2\). What are the width and height of the window? \( 14 \) in., \( 8 \) in.

26. The formula \( V = \frac{4}{3}\pi r^3 \) defines the volume \( V \) of a sphere with radius \( r \) in cubic meters. What is the volume of a cube with surface area \( 486 \) m\(^2\)? \( 270 \) m\(^3\).

27. A mound of sand at a rock-crushing plant is growing at the rate of \( V = 0.027x \) cubic meters per hour. When is the volume equal to 549 cubic meters? \( 20 \) hours.

6-5 Practice [continued]
Form G

Solving Square Root and Other Radical Equations

28. City officials conclude they should budget \$1 million dollars for a new library building if the population increases by \( \frac{1}{2} \) thousand people in ten years. The formula \( P = \frac{1}{2}t + 17 \) expresses the relationship between population \( P \) and time \( t \) in years. How much can the population increase without the city going over budget if they have \$5 million for a new library building?

Solve. Check for extraneous solutions.

29. \( x = 3 \).
30. \( x = -3 \).
31. \( x = 7 \).
32. \( x = -9 \).
33. \( x = 10 \).
34. \( x = 3 \).
35. \( x = 1 \).
36. \( x = 1 \).
37. \( x = 0 \).
38. \( x = 0 \).
39. \( x = -2 \).
40. \( x = -2 \).
41. \( x = 1 \).
42. \( x = 1 \).
43. \( x = 2 \).
44. \( x = 2 \).
45. \( x = 3 \).
46. \( x = 3 \).
47. \( x = 4 \).
48. \( x = 4 \).
49. \( x = 5 \).
50. \( x = 5 \).

A clothing manufacturer uses the model \( s = \frac{a}{2} \) to estimate the amount of fabric to order from a mill. In the formula, \( a \) is the number of apparel items (in hundreds) and \( s \) is the number of yards of fabric needed. If there are 500 apparel items to be manufactured, how many yards of fabric should be ordered?

52. What are the lengths of the sides of the trapezoid shown at the right if the perimeter of the trapezoid is 17 cm?

ANSWERS
Solving Square Root and Other Radical Equations

1. Solve. To start, rewrite the equation to isolate the radical.
   \[ \sqrt{x + 2} = -2 \]
   \[ x + 2 = 4 \]
   \[ x = 2 \]

2. \[ \sqrt{x - 3} = 7 \]
   \[ x - 3 = 49 \]
   \[ x = 52 \]

3. \[ 2 \sqrt{x - 2} = 6 \]
   \[ \sqrt{x - 2} = 3 \]
   \[ x - 2 = 9 \]
   \[ x = 11 \]

4. Solve.
   \[ 2(x - 2)^2 = 50 \]
   \[ x = 12 \text{ and } x = -12 \]

5. \[ 2(x + 3)^2 = 54 \]
   \[ x = 6 \]

6. \[ \sqrt{x} - 5 = x - 2 \]
   \[ x = 8 \text{ and } x = -2 \]

7. A cylindrical can holds 21.5 in³ of soup. If the can is 4 in. tall, what is the radius of the can to the nearest tenth of an inch? (Hint: \( V = \pi r^2 h \)) 1.5 in.

8. Writing Explain the difference between a radical equation and a polynomial equation. A radical equation has a variable in a radical or a variable with a fractional exponent, while a polynomial equation has a variable with whole number exponents.

9. Reasoning If you are solving \( 4(x + 3)^2 = 7 \), do you need to use the absolute value to solve for \( x \)? Why or why not? No, the numerator of the exponent \( 2 \) is not even.

10. Solve each equation. Graph each equation as a system to determine if there are any extraneous solutions.
    \[ \sqrt{3x + 2} = x \]
    \[ 3x + 2 = x^2 \]
    \[ x = -1, x = 2 \]

11. \[ \sqrt{x - 3} = x - 2 \]
    \[ x - 3 = x^2 - 4x + 4 \]
    \[ 0 = x^2 - 5x + 7 \]

12. \[ \sqrt{x + 2} = -2 \]
    \[ x + 2 = 4 \]
    \[ x = 2 \]

13. \[ \sqrt{x - 3} = x - 2 \]
    \[ x - 3 = x^2 - 4x + 4 \]
    \[ 0 = x^2 - 5x + 7 \]

14. \[ \sqrt{x + 2} = x - 2 \]
    \[ x + 2 = x^2 - 4x + 4 \]
    \[ 0 = x^2 - 5x + 2 \]

15. \[ \sqrt{x - 3} = x - 2 \]
    \[ x - 3 = x^2 - 4x + 4 \]
    \[ 0 = x^2 - 5x + 7 \]

16. Find the solutions of \( \sqrt{x - 3} = x - 2 \).
    a. Is there any extraneous solution? \( x = 1 \)
    b. Reasoning How do you know the answer in part (a)? Substitute the solutions into the original equation. If a solution does not make the equation true, then the solution is extraneous.

17. A floor is made up of hexagon-shaped tiles. Each hexagon tile has an area of 147 cm². What is the length of each side of the hexagon? (Hint: 6 equilateral triangles make one hexagon.) about 24 cm.

18. Enrichment
    When solving radical equations you will often get an extraneous solution. You can use a graph to explain why an algebraic answer is not a solution.
    1. Solve the equation \( \sqrt{x + 2} = -4 \). Is there an extraneous solution?
    2. \( x = -6 \) is an extraneous solution.
    3. Graph the two equations.
    4. Explain how you find the solution to this system of equations on your graph. What is the solution? Answers may vary. Sample: On a graph the solution to a system of equations is the point of intersection; the solution for this system is \( (7, 2) \).
    5. How can you use the solution from the graph of the system of equations to help you solve the original equation \( \sqrt{x + 2} = -4 \)? Answers may vary. Sample: The x-coordinate of the solution to the system is the solution to the original equation.
    6. How can you tell from your graph that one of your algebraic answers is an extraneous solution? Answers may vary. Sample: Because there is only one point of intersection, there can only be one solution to the equation.

Solve each equation. Graph each equation as a system to determine if there are any extraneous solutions.

19. \[ \sqrt{x + 2} = x \]
   \[ x = -2 \] 20. \[ \sqrt{x - 3} = x - 1 \]
   \[ x = 2 \] 21. No extraneous solutions
   \[ x = 3 \] 22. \[ x = 2 \] is an extraneous solution.
2. Let cards got mixed up. Darnell wrote the steps to compose the following functions on index cards, but the cards got mixed up.

Let \( f(x) = x + 7 \) and \( g(x) = x^2 \). What is \( [g \cdot f](-4) \)?

Subtract 4 from 7. Raise 3 to the 3rd power. Substitute -4 for \( x \) in \( f(x) \). Substitute 3 into \( g(x) \).

Use the note cards to write the steps in order.

1. First, substitute -4 for \( x \) in \( f(x) \).
2. Second, subtract 4 from 7.
3. Then, substitute 3 into \( g(x) \).
4. Finally, raise 3 to the 3rd power.

Exercises
Solve. Check your solutions.
1. \( \sqrt{5} = 11 - 10 \)  2. \( 2 \sqrt{2} \cdot 2 = 12 \)  3. \( \sqrt{2} \cdot 5 = 11 - 12 \)
6. \( (x + 9)^2 - 3 = 7 \)  5. \( (60 - 9)^2 - 3 = 5 - 3 \)  6. \( \sqrt{2} = 4 - 1 \)
3. \( (x + 2)^2 - 3 - 1 = 23 \)  4. \( \sqrt{11} = 3 - 1 = 11 - 2 \)  5. \( \frac{\sqrt{11}}{3} - 3 = 1 - 5 \)

Check the solution in the original equation.

The solution is 12.

Exercises
Solve. Check for extraneous solutions.
10. \( \sqrt{x+7} = \sqrt{v+2} + 2 \)  11. \( \sqrt{x} + 3 = x + 1 \) no solution 12. \( \sqrt{x} - \sqrt{v+2} = 3 \)
13. \( x - \sqrt{2} = 4 \)  14. \( x - 4 = 5 \)  15. \( \sqrt{x+5} = x - 4 \)  16. \( \sqrt{x+8} = x - 4 \)  17. \( \sqrt{v+3} = 2 - x = 2 \)
18. \( \sqrt{x+5} = x - 5 \)  19. \( \sqrt{x+3} = x + 1 \)  20. \( \sqrt{x+7} = x + 7 - 5 \)  21. \( x - \sqrt{v+4} = 40 - 47 \)

ANuirans

6-5 Reteaching Solving Square Root and Other Radical Equations

An extraneous solution may satisfy equations in your work, but it does not make the original equation true. Always check possible solutions in the original equation.

Problem
What is the solution? Check your results. \( \sqrt{x+3} = x + 3 \)
\( \sqrt{x+3} = x + 3 \) Add 3 to each side to get the radical alone on one side of the equation.
\( \sqrt{x+3} - x = 3 \) Subtract 3 from each side of the equation.
\( x - x + 3 = 3 \) Divide each side by 3.
\( 0 = 0 \) Check the solution in the original equation.

The only solution is 1.

Exercises
Solve.
10. \( \sqrt{x+7} = \sqrt{v+2} + 2 \)  11. \( \sqrt{x} + 3 = x + 1 \) no solution 12. \( \sqrt{x} - \sqrt{v+2} = 3 \)
13. \( x - \sqrt{2} = 4 \)  14. \( x - 4 = 5 \)  15. \( \sqrt{x+5} = x - 4 \)  16. \( \sqrt{x+8} = x - 4 \)  17. \( \sqrt{v+3} = 2 - x = 2 \)
18. \( \sqrt{x+5} = x - 5 \)  19. \( \sqrt{x+3} = x + 1 \)  20. \( \sqrt{x+7} = x + 7 - 5 \)  21. \( x - \sqrt{v+4} = 40 - 47 \)

6-6 Think About a Plan Function Operations
Sales A salesperson earns a 3% bonus on weekly sales over $5000. Consider the following functions.

\( g(x) = 0.03x \) \( h(x) = x - 5000 \)

a. Explain what each function above represents.
b. Which composition, \( (g \circ h)(x) \), represents the weekly bonus? Explain.

5. What does \( z \) represent in the function \( g(z) \)?
the sales amount used to calculate a 3% bonus

2. What does the function \( g(x) \) represent?
the bonus earned by the salesperson on sales

3. What does \( x \) represent in the function \( h(x) \)?
the total weekly sales made by the salesperson

4. What does the function \( h(x) \) represent?
the weekly sales over $5000 made by the salesperson

5. What is the meaning of \( h \circ g(x) \)?
First multiply the value of \( x \) by 0.03, then subtract $5000 from the result

6. Assume that \( z = 7000 \). What is \( h(g(z)) \)?
First multiply the value of \( x \) by 0.03, then subtract $5000 from the result

7. What is the meaning of \( g \circ h(x) \)?
First subtract $5000 from the value of \( x \), then multiply the result by 0.03

8. Assume that \( z = 7000 \). What is \( g(h(z)) \)?

9. Which composition represents the weekly bonus? Explain
\( g \circ h(x) \) represents the weekly bonus because you must first find the sales amount over $5000 by subtracting $5000 from the weekly sales, and then you multiply the result by the bonus percent as a decimal, or 0.03.
2. Let $f(x) = 3x + 4$ and $g(x) = x^2$. Perform each function operation and then find the domain.
   1. $(f + g)(x)$
   2. $(f - g)(x)$
   3. $(f \cdot g)(x)$
   4. $\frac{f}{g}(x)$

3. Let $f(x) = 4x - 2$ and $g(x) = x - 2$. Perform each function operation and then find the domain of the result.
   1. $(f \circ g)(x)$
   2. $(g \circ f)(x)$
   3. $(f \circ g)(x) = \frac{f}{g}$
   4. $(g \circ f)(x) = \frac{g}{f}$

4. Let $f(x) = 2x^2 - 3x - 1$ and $g(x) = 4x^2 + 2x + 1$. Perform each function operation and then find the domain of the result.
   1. $(f + g)(x)$
   2. $(f - g)(x)$
   3. $(f \cdot g)(x)$
   4. $\frac{f}{g}(x)$

5. Let $f(x) = \sqrt{x} + 2$ and $g(x) = x^2 - 4$. Find each value. To start, use the definition of composing functions to find a function rule.
   9. $f(g(4))$
   10. $(f \circ g)(-1)$
   11. $(f \circ g)(-3)$

6. Let $f(x) = \sqrt{x} + 2$ and $g(x) = (x + 2)^2$. Find each value.
   12. $(f \circ g)(5)$
   13. $(f \circ g)(0)$
   14. $(f \circ g)(4)$

7. Let $f(x) = 2x^2 - 3x - 1$ and $g(x) = 4x^2 + 2x + 1$. Find each value.
   15. $(f \circ g)(3)$
   16. $(f \circ g)(0)$

8. Find each value or expression.
   28. $(f \circ g)(3)$
   29. $(f \circ g)(0)$

9. Write a function to find the price at the store if the markup is applied before the shipping charge is added.

10. A video game store adds a 25% markup on each of the games that it sells. A boutique prices merchandise by adding 80% to its cost. It later decreases by 25% the price of items that do not sell quickly.

11. A video game store adds a 25% markup on each of the games that it sells. A boutique prices merchandise by adding 80% to its cost. It later decreases by 25% the price of items that do not sell quickly.

12. A video game store adds a 25% markup on each of the games that it sells. A boutique prices merchandise by adding 80% to its cost. It later decreases by 25% the price of items that do not sell quickly.

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14. A video game store adds a 25% markup on each of the games that it sells. A boutique prices merchandise by adding 80% to its cost. It later decreases by 25% the price of items that do not sell quickly.

15. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

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20. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

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24. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

25. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

26. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

27. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

28. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

29. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.

30. A car dealer offers a 10% discount off the list price $x$ of any car on the lot. At the same time, the manufacturer offers a $1000 rebate for each purchase of a car.
**6-6** Standardized Test Prep

**Function Operations**

**Multiple Choice**

For Exercises 1–5, choose the correct letter.

1. Let \( f(x) = 2x + 5 \) and \( g(x) = x^2 \). What is \((f \cdot g)(x)\)?
   - A. \( x^2 + 5 \)
   - B. \( 2x^2 + 10 \)
   - C. \( x^2 + 10 \)
   - D. \( 2x^2 + 5 \)

2. Let \( f(x) = 3x + 2 \) and \( g(x) = x^3 \). What is \((f \cdot g)(x)\)?
   - A. \( 3x^4 + 2x \)
   - B. \( 3x^4 + 2x^3 \)
   - C. \( 3x^4 + 2 \)
   - D. None of the above

3. Let \( f(x) = x^2 - 2x - 15 \) and \( g(x) = x + 3 \). What is the domain of \( \frac{f(x)}{g(x)} \)?
   - A. \( x \neq -3 \)
   - B. \( x \neq 0 \)
   - C. \( x \neq 5 \)
   - D. \( x \neq 5, x \neq -3 \)

4. Let \( f(x) = \sqrt[3]{x} + 1 \) and \( g(x) = 2x + 1 \). What is \((f \cdot g)(x)\)?
   - A. \( \sqrt[3]{2x^4 + 9x^2 + 2} \)
   - B. \( \sqrt[3]{2x^4 + 9} \)
   - C. \( \sqrt[3]{2x^4 + 9x^2 + 2} \)
   - D. \( \sqrt[3]{2x^4 + 9x^2 + 2x + 1} \)

5. Let \( f(x) = \frac{1}{x} \) and \( g(x) = x^2 - 2 \). What is \((f \cdot g)(x)\)?
   - A. \( \frac{x^2 - 2}{x} \)
   - B. \( \frac{1}{x(x^2 - 2)} \)
   - C. \( \frac{x^2 - 2}{x^2} \)
   - D. \( \frac{1}{x^3} \)

**Short Response**

6. Suppose the function \( f(x) = 0.03x \) represents the number of U.S. dollars equivalent to \( x \) Russian rubles. What is the number of yen equivalent to 100 Russian rubles?

7. Suppose the function \( g(x) = 100(x - 5) \) represents the number of yen equivalent to 100 Russian rubles. Show your work.

\[ (g \circ f)(x) = 100(0.03x - 5) = 3.15x \]

8. Which is the correct answer? A) \( A = C \)  
   B) \( g(f(x)) = g(x)f(x) \)  
   C) \( g(f(x)) = g(x)f(x) \)  
   D) \( f(g(x)) = f(x)g(x) \)

9. What is the domain of the resulting function? A) \( x > 0 \)  
   B) \( x 
eq -3 \)  
   C) \( x 
eq 0 \)  
   D) \( x 
eq 5 \)

10. Two functions \( f(x) \) and \( g(x) \) are equal if they have the same domains and the same value for each point in their domain. Suppose that \( f(x) = 2x + 5 \) and \( g(x) = 3x + 2 \). Are these two linear functions both true, or are the set of real numbers?

11. Which equation results from the equation \( \frac{f(x)}{g(x)} = f(x) \)?
   - A. \( f(x) = g(x) \)
   - B. \( f(x) \cdot g(x) = 1 \)
   - C. \( f(x) = x^2 + 2x + 1 \)
   - D. \( f(x) = x^2 + 2 \)

12. What is the coefficient of \( x \) in the expression for \( (g \cdot f)(x) \)?

**Enrichment**

**Function Operations**

**Composition and Linear Functions**

Two functions \( f(x) \) and \( g(x) \) are equal if they have the same domains and the same value for each point in their domain. Suppose that \( f(x) = 2x + 5 \) and \( g(x) = 3x + 2 \). Are these two linear functions both true, or are the set of real numbers?

1. If \( f(x) = g(x) \), what can you conclude by examining the values of \( f(x) \) and \( g(x) \) at \( x = 0 \)?

2. Use your conclusion to eliminate \( D \) from the definition of \( g(x) \).

3. What equation results from the equation \( \frac{f(x)}{g(x)} = f(x) \)?

4. What can you conclude about \( A \) and \( C \)?

5. What is the domain of \( f(x) \)?

6. What is the domain of \( g(x) \)?

7. What type of function is the composite of two linear functions?...

8. What is the coefficient of \( x \) in the expression for \( (g \cdot f)(x) \)?

9. What is the constant term? \( AC = D \)

10. Compute \( (g \cdot f)(x) \).

11. What equation must be satisfied if \( f \cdot g = g \cdot f \)?

12. What equations must be satisfied if \( f \cdot g = f \)?

13. What equations must be satisfied if \( f \cdot g = g \)?

14. What must occur if \( f \cdot g = 0 \)?

15. Constant functions are a subset of linear functions in which the coefficient of \( x \) is zero. What type of function is the composite of two constant functions? constant

16. a. If \( f(x) \) and \( g(x) \) are two constant functions, under what circumstances does \( h \cdot k = k \cdot h \)?

17. Under what circumstances does \( h \cdot k = k \cdot h \)?

18. Under what circumstances does \( h \cdot k = k \cdot h \)?

19. Under what circumstances does \( h \cdot k = 0 \)?

**Reteaching**

**Function Operations**

**Where you combine functions using addition, subtraction, multiplication, or division, the domain of the resulting function has to include the domains of both of the original functions.**

**Problem**

Let \( f(x) = x^2 - 4 \) and \( g(x) = \sqrt[3]{x} \). What is the solution of each function operation? What is the domain of the result?

a. \( (f \cdot g)(x) \)

b. \( (f - g)(x) \)

...c. \( (g \cdot f)(x) \)

The domain of \( f(x) \) is all real numbers. The domain of \( g(x) \) is all \( x > 0 \) for parts a–d, there are no additional restrictions on the values for \( x \), or the domain for each of these \( x > 0 \).

...d. \( (g \cdot f)(x) \)

As before, the domain is \( x > 0 \). But, because the denominator cannot be zero, eliminate any values of \( x \) for which \( g(x) = 0 \). The only value for which \( \sqrt[3]{x} = 0 \) is \( x = 0 \). Therefore, the domain of \( g \cdot f \) is \( x > 0 \).

Similarly, begin with \( x > 0 \) and eliminate any values of \( x \) that make the denominator \( f(x) \) zero: \( x^2 - 4 = 0 \) when \( x = -2 \) and \( x = 2 \). Therefore, the domain of \( f \cdot g \) is \( x > 0 \) combined with \( x = -2 \) and \( x = 2 \). In other words, the domain is \( x > 0 \) or \( x > 2 \), or all nonnegative numbers except 2.

**Exercises**

Let \( f(x) = 4x + 3 \) and \( g(x) = x^2 + 2 \). Perform each function operation and then find the domain of the result.

1. \( (f \cdot g)(x) \)

2. \( (g \cdot f)(x) \)

3. \( (g \cdot f)(x) \)

4. \( (f \cdot g)(x) \)

5. \( (g \cdot f)(x) \)

...6. \( (g \cdot f)(x) \)

Given \( f(x) = x^2 + 4x + 5 \), all real numbers

...7. \( f(x) = x^2 + 4x + 5 \), all real numbers

...8. \( f(x) = x^2 + 4x + 5 \), all real numbers

...9. \( f(x) = x^2 + 4x + 5 \), all real numbers

...10. \( f(x) = x^2 + 4x + 5 \), all real numbers

...11. \( f(x) = x^2 + 4x + 5 \), all real numbers

...12. \( f(x) = x^2 + 4x + 5 \), all real numbers

...13. \( f(x) = x^2 + 4x + 5 \), all real numbers

...14. \( f(x) = x^2 + 4x + 5 \), all real numbers

...15. \( f(x) = x^2 + 4x + 5 \), all real numbers

...16. \( f(x) = x^2 + 4x + 5 \), all real numbers

...17. \( f(x) = x^2 + 4x + 5 \), all real numbers

...18. \( f(x) = x^2 + 4x + 5 \), all real numbers

19. \( f(x) = x^2 + 4x + 5 \), all real numbers
Choose the word or phrase from the list that best matches each sentence.

17. A relation pairs element a of its domain to element b of its range. The inverse relation, 
   which pairs element b of its range to element a of its domain, is represented by ______. 

14. In a relation, each ______ is represented by ______ such that one value in the range 
   corresponds to each value in the domain.

11. The inverse of a function f is represented by ______.

8. If a relation and its inverse are functions, then they are ______ functions.

1. A relation pairs element a of its domain to element b of its range. The inverse relation, which pairs element b of its range to element a of its domain, is represented by ______.

6-7 pairs

Circle the inverse of

Circle the inverse of

13. The inverse of a relation is a relation that contains all the ordered pairs ______.

12. In a relation, each value in the ______ corresponds to each value in the domain.

10. A relation is a function if each value in the domain is ______.

9. In a relation, each value in the domain is ______.

6. Every element in the domain of a function f is represented by ______.

5. The inverse of a function f is represented by ______.

3. The inverse of a relation is a relation that contains all the ordered pairs ______.

2. A relation pairs element a of its domain to element b of its range. The inverse relation of this relation pairs element b of its range to element a of its domain.

1. A relation pairs element a of its domain to element b of its range. The inverse relation, which pairs element b of its range to element a of its domain, is represented by ______.

6-7 pairs

Circle the inverse of

Circle the inverse of

13. The inverse of a relation is a relation that contains all the ordered pairs ______.

12. In a relation, each value in the domain of a function is represented by ______.

10. A relation is a function if each value in the domain of a function is ______.

9. In a relation, each value in the domain of a function is ______.

6. Every element in the domain of a function f is represented by ______.

5. The inverse of a relation is a relation that contains all the ordered pairs ______.

3. The inverse of a relation is a relation that contains all the ordered pairs ______.

2. A relation pairs element a of its domain to element b of its range. The inverse relation of this relation pairs element b of its range to element a of its domain.

1. A relation pairs element a of its domain to element b of its range. The inverse relation, which pairs element b of its range to element a of its domain, is represented by ______.

6-7 pairs

Circle the inverse of

Circle the inverse of

13. The inverse of a relation is a relation that contains all the ordered pairs ______.

12. In a relation, each value in the domain of a function is represented by ______.

10. A relation is a function if each value in the domain of a function is ______.

9. In a relation, each value in the domain of a function is ______.

6. Every element in the domain of a function f is represented by ______.

5. The inverse of a relation is a relation that contains all the ordered pairs ______.

3. The inverse of a relation is a relation that contains all the ordered pairs ______.

2. A relation pairs element a of its domain to element b of its range. The inverse relation of this relation pairs element b of its range to element a of its domain.

1. A relation pairs element a of its domain to element b of its range. The inverse relation, which pairs element b of its range to element a of its domain, is represented by ______.
Multiple Choice

What is the domain and range of the inverse of the function?
1. \( f(x) = x^2 \) domain: all real numbers; range: \( x \geq 0 \) no, range is the set of all real numbers
2. \( f(x) = \sqrt{x} \) domain: \( x \geq 0 \); range is the set of all real numbers
3. \( f(x) = \frac{1}{x} \) domain: all real numbers; range: \( x \neq 0 \) yes, the inverse is not a function
4. \( f(x) = x^3 \) domain: all real numbers; range: all real numbers; the inverse is a function
5. \( f(x) = \frac{1}{x^2} \) domain: \( x \neq 0 \); range: all real numbers; the inverse is a function

For each function, find its inverse and the domain and range of the function and its inverse.
6. \( f(x) = x^2 \) domain: all real numbers; range: \( x \geq 0 \)
   - Inverse: \( f^{-1}(x) = \sqrt{x} \) domain: \( x \geq 0 \); range is the set of all real numbers

Find the inverse of each function. Is the inverse a function? To start, switch \( x \) and \( y \).
7. \( y = \frac{2}{x} \) yes, \( y = \frac{2}{x} \)
8. \( y = 2x + 3 \) no, \( y = \frac{x - 3}{2} \)
9. \( y = \sqrt{x} \) yes, \( y = x^2 \)
10. \( y = \frac{1}{x} \) yes, \( y = \frac{1}{x} \)

Graph each relation and its inverse.
11. \( y = x^2 \) \( y = -x^2 \)
12. \( y = \frac{1}{x} \) \( y = -\frac{1}{x} \)
13. \( y = x + 2 \) \( y = -x - 2 \)
14. \( y = 2x - 1 \) \( y = \frac{1}{2}x + \frac{1}{2} \)

Composition, Inverses, and Linear Functions

Solving an equation for one variable in terms of another is an important step in finding inverses. This step is also used in conversion formulas.

Consider the following linear functions: Let \( f \) denote the temperature in degrees Fahrenheit, \( T \) the temperature in degrees Celsius, and \( K \) the temperature in degrees Kelvin. The formula for converting degrees Fahrenheit to degrees Celsius is \( C = \frac{5}{9}(F - 32) \), and the formula for converting degrees Celsius to degrees Kelvin is \( K = C + 273 \).

1. Use composition to determine the formula for converting degrees Fahrenheit to degrees Kelvin. \( K = \frac{5}{9}(F - 32) + 273 \)
2. Solve this function for \( F \): \( F = \frac{9}{5}K - 273 \)
3. This new equation converts degrees Kelvin to degrees Fahrenheit. \( F = \frac{9}{5}K - 273 \)
4. Derive a formula in degrees Celsius to degrees Fahrenheit. \( F = \frac{9}{5}C + 32 \)
5. Derive a formula in degrees Kelvin to degrees Celsius. \( C = K - 273 \)
6. Compose these two functions to find a formula for converting degrees Kelvin to degrees Fahrenheit: \( F = \frac{9}{5}K - 273 \)

Solve each of the following problems involving functions.
7. In 1940, the cost of a new house was $10,000. By 1980, this cost had risen to $500,000. Assuming that the increase is linear, find a function expressing the cost \( c \) of a new house in terms of the year \( y \). Solve this function for \( y \). What does this new function enable you to do?
   \( c = 10,000 + 300,000 \cdot y \) \( y = \frac{c - 10,000}{300,000} \)
8. Between the ages of 5 and 15, a typical child grows at a linear rate. If Mary was 42 in 2013 and grew at a rate of 2 in. a year, find a formula that expresses Mary’s height \( h \) in inches when her age is \( a \) years. Solve this function for \( a \). What does this new function enable you to do?
   \( h = 2a + 22 \) \( a = \frac{h - 22}{2} \)
9. The air temperature, in degrees Fahrenheit, surrounding an airplane on one particular day was modeled by \( T = \frac{5}{9}S + 115 \), where \( S \) is the altitude, in feet, of the airplane. Solve this function for \( S \). What does this new function enable you to do?
   \( S = \frac{9}{5}(T - 115) \)
10. The formula \( L = 0.234S^0.5 \) models the length of a certain spring, in inches, when a weight of \( W \) ounces is attached to it. Solve this function for \( W \). What does this new function enable you to do?
   \( W = \frac{L^2}{0.234} \)
6.7 Reteaching
Inverse Relations and Functions

- Inverse operations "undo" each other. Addition and subtraction are inverse operations. So are multiplication and division. The inverse of raising a number to a power is taking the root.
- If two functions are inverses, they consist of inverse operations performed in the opposite order.

**Problem**
What is the inverse of the relation described by \( f(x) = x + 1 \)?

\[
\begin{align*}
&x = y + 1 & \text{Interchange } x \text{ and } y. \\
&x - 1 = y & \text{Solve for } y. \\
y = x - 1 & \text{The resulting function is the inverse of the original function.}
\end{align*}
\]
So, \( f^{-1}(x) = x - 1 \).

**Exercises**
Find the inverse of each function.

1. \( y = 4x - 3 \)  
   \( f^{-1}(x) = \frac{x + 3}{4} \)
2. \( y = 3x^2 + 2 \)  
   \( f^{-1}(x) = \sqrt[3]{\frac{x - 2}{3}} \)
3. \( y = (x + 1)^3 \)  
   \( f^{-1}(x) = \sqrt[3]{x} - 1 \)
4. \( y = \frac{5x}{2} + 2 \)  
   \( f^{-1}(x) = \frac{2x - 4}{5} \)
5. \( y = x - 4 \)  
   \( f^{-1}(x) = x + 4 \)
6. \( f(x) = 2(x - 1) \)  
   \( f^{-1}(x) = \frac{x}{2} + 1 \)
7. \( f(x) = \sqrt{x} \)  
   \( f^{-1}(x) = x^2 \)
8. \( f(x) = x \)  
   \( f^{-1}(x) = x \)
9. \( y = x^2 \)  
   \( f^{-1}(x) = \sqrt{x} \)
10. \( y = x - 3 \)  
    \( f^{-1}(x) = x + 3 \)
11. \( y = \sqrt{x} + 5 \)  
    \( f^{-1}(x) = x^2 - 5 \)
12. \( y = x^3 \)  
    \( f^{-1}(x) = \sqrt[3]{x} \)
13. \( f(x) = \frac{1}{x} \)  
    \( f^{-1}(x) = \frac{1}{x} \)
14. \( y = \frac{1}{x} \)  
    \( f^{-1}(x) = \frac{1}{x} \)
15. \( f(x) = \sqrt[3]{x} \)  
    \( f^{-1}(x) = x^3 \)
16. \( f(x) = 2(x - 3)^2 \)  
    \( f^{-1}(x) = \sqrt[3]{x} + 3 \)
17. \( y = \sqrt{x} + 3 \)  
    \( f^{-1}(x) = x^2 - 3 \)
18. \( y = 2x + 5 \)  
    \( f^{-1}(x) = \frac{x - 5}{2} \)
19. \( f(x) = \frac{1}{x - 1} \)  
    \( f^{-1}(x) = \frac{1}{x} + 1 \)

**ANSWERS**

6.8 Ell Support
Graphing Radical Functions

1. Circle the radical functions in the group below:
   \( \sqrt{x} = x - 4 \)
   \( \sqrt[3]{x} + 6 \)

2. Circle the square root functions in the group below:
   \( \sqrt{x} = x + 6 \)
   \( \sqrt[3]{x} + 6 \)

For Exercises 3–6, draw a line from each word or phrase in Column A to its matching term in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. parent function</td>
<td>A. ( y = \sqrt{x} )</td>
</tr>
<tr>
<td>4. translate 6 units downward</td>
<td>E. ( y = -\sqrt{x} )</td>
</tr>
<tr>
<td>5. stretch vertically by the factor ( 2 )</td>
<td>C. ( y = \sqrt{x} )</td>
</tr>
<tr>
<td>6. translate 6 units upward</td>
<td>B. ( y = \sqrt{x} )</td>
</tr>
<tr>
<td>7. reflection in x-axis</td>
<td>D. ( y = \sqrt{-x} )</td>
</tr>
<tr>
<td>8. translate 6 units to the right</td>
<td>F. ( y = \sqrt{x} + 6 )</td>
</tr>
</tbody>
</table>

Identify the meaning of the following terms in the function \( y = 2\sqrt{x} + 5 \).

- \( y = 2\sqrt{x} \)  
  - Stretch vertically by a factor of 2.
- 9.  
  - Translate 4 units to the right.
- 10.  
  - Translate 5 units upward.
Graph each function. Find the domain and range.

1. \( y = \sqrt{x} + 3 \)
2. \( y = -\sqrt{x} - 1 \)
3. \( y = \sqrt{x} - 7 \)
4. \( y = \sqrt{x} \)
5. \( y = 3\sqrt{x} - 4 \)
6. \( y = \frac{1}{2}\sqrt{x} - 5 \)

Solve each square root equation by graphing. Round the answer to the nearest hundredth, if necessary. If there is no solution, explain why.

7. \( \sqrt{2x} = 5.7 \)
8. \( \sqrt{x - 2} = 5.7 \)
9. \( \sqrt{x^2} - 1 \)

10. A submarine is at a depth \( h \) feet, below the surface of the water. The greatest distance \( d \), in miles, that can be seen from the submarine on a clear day is given by \( d = \frac{h}{2} \).
   a. If a ship is 3 miles from the submarine, at what height above the water would the submarine have to raise its periscope in order to see the ship? 6 ft
   b. If a ship is 5 miles from the submarine, what height would it have to be raised? 15 ft

11. \( y = \sqrt{x} + 4 \)
12. \( y = \sqrt{x} - 5 \)
13. \( y = \sqrt{x} - 1 \)

14. A center pivot irrigation system can water 1.5 acres of crop land. The length \( L \) in feet of rotating pipe needed to irrigate \( A \) acres is given by the function \( L = 117.75\sqrt{A} \).
   a. Graph the equation on your calculator. Make a sketch of the graph.
   b. What length of pipe is needed to irrigate 40, 80, and 130 acres? 346.7 ft, 502.4 ft, 694.5 ft

15. \( y = \sqrt{x} + 3 \)
16. \( y = -\sqrt{x} + 3 \)
17. \( y = -\sqrt{x} - 3 \)

Graph each function. To start, graph the parent function, \( y = \sqrt{x} \).

11. \( y = \sqrt{x} + 4 \)
12. \( y = \sqrt{x} - 5 \)
13. \( y = \sqrt{x} - 1 \)

14. A center pivot irrigation system can water 1.5 acres of crop land. The length \( L \) in feet of rotating pipe needed to irrigate \( A \) acres is given by the function \( L = 117.75\sqrt{A} \).
   a. Graph the equation on your calculator. Make a sketch of the graph.
   b. What length of pipe is needed to irrigate 40, 80, and 130 acres? 346.7 ft, 502.4 ft, 694.5 ft
Multiple Choice

For Exercises 1–4, choose the correct letter.

1. What is the graph of \( y = \sqrt{x} + 3 \)?
   - [ ] A
   - [ ] B
   - [ ] C
   - [ ] D

2. What is the graph of \( y = \sqrt{x} - 2 \)?
   - [ ] A
   - [ ] B
   - [ ] C
   - [ ] D

3. What is the graph of \( y = 1 - \sqrt{x} \)?
   - [ ] A
   - [ ] B
   - [ ] C
   - [ ] D

4. What is the description of \( y = \sqrt{x} + 3 \)?
   - [ ] A
   - [ ] B
   - [ ] C
   - [ ] D

Short Response

5. What is the graph of \( y = 2\sqrt{x} + 3 \)?
   - [ ] A
   - [ ] B
   - [ ] C
   - [ ] D

Exercises

Graph each function.

1. \( y = \sqrt{x} + 2 \)
2. \( y = \sqrt{x} - 4 \)
3. \( y = -\sqrt{x} + 3 \)
4. \( y = -\sqrt{x} - 3 \)
5. \( y = 2\sqrt{x} + 4 \)
6. \( y = -2\sqrt{x} + 4 \)
7. \( y = -\sqrt{x} + 1 \)
8. \( y = \sqrt{x} - 3 \)
9. \( y = 2\sqrt{x} + 2 \)
10. \( y = -\sqrt{x} - 3 \)
**Chapter 6 Quiz 1**  
**Form G**  
**Lessons 6-1 through 6-4**

**Do you know HOW?**  
Find all the real roots.

1. \( \sqrt[3]{5} \) = 5  
2. \( \sqrt[3]{-8} \) = -2  
3. \( \sqrt[3]{-8} \) = -2  
4. \( \sqrt[3]{-5} \) = \(-\sqrt[3]{5}\)

Simplify each radical expression. Use absolute value symbols when needed.

5. \( \sqrt[3]{81} \) = 3  
6. \( \sqrt[3]{32} \) = 2 \( \sqrt{2} \)  
7. \( \sqrt[3]{-64} \) = -4  
8. \( \sqrt[3]{-125} \) = -5

Find the two real solutions of each equation.

9. \( y^2 + 4 = 0 \)  
10. \( x^3 = 1 \)

Multiply or divide and simplify. Assume that all variables are positive.

11. \( 3\sqrt[3]{x} \times 3\sqrt[3]{y} \) = 3\( \sqrt[3]{xy} \)

Simplify. Rationalize all denominators. Assume that all variables are positive.

12. \( \sqrt{3} \times \frac{\sqrt{8}}{\sqrt[4]{2}} \) = \( \sqrt{12} \)

Find the inverse of each function. Is the inverse a function?

13. \( f(x) = 3x - 2 \)  
14. \( g(x) = x^2 + 1 \)  
15. \( h(x) = \frac{1}{x} \)

16. \( f(x) = 2x + 1 \)  
17. \( g(x) = \sqrt{x} \)  
18. \( h(x) = \frac{1}{x^2} \)

**Do you UNDERSTAND?**

19. **Geometry**  
What is the perimeter of the triangle at the right?

20. **Reasoning**  
Solve. \( \sqrt{3} \times \sqrt{2} = \sqrt{6} \) \( \sqrt{3} \times \sqrt{2} = \sqrt{6} \)

---

**Chapter 6 Quiz 2**  
**Form G**  
**Lessons 6-5 through 6-8**

**Do you know HOW?**  
Solve. Check for extraneous solutions.

1. \( \sqrt[3]{x^3} = 8 \)  
2. \( \sqrt[3]{x^3} = -8 \)  
3. \( \sqrt{x^2} = 16 \)

Let \( f(x) = x^2 \) and \( g(x) = x - 4 \). Perform each function operation and then find the domain.

4. \( f(x) \cdot g(x) \)  
5. \( f(x) + g(x) \)  
6. \( f(x) - g(x) \)  
7. \( f(x) \div g(x) \)

Find the inverse of each function. Is the inverse a function?

8. \( f(x) = x^3 \)  
9. \( g(x) = \sqrt{x} \)  
10. \( h(x) = \frac{1}{x} \)

11. \( f(x) = \sqrt{x} \)  
12. \( g(x) = 3x + 2 \)  
13. \( h(x) = \frac{1}{x^2} \)

**Do you UNDERSTAND?**

14. **Writing**  
Explain why it is not true that \( \sqrt[3]{x^3} = -x \), even though \( (-2)^3 = -8 \).

15. **Reasoning**  
Suppose the cost of an item is \$20 in the United States. Is it marked up \$5 and then reduced by \$5? Is the final cost equal to \$20? Use a composition of functions to justify your answer.

---

**Chapter 6 Chapter Test**  
**Form G**

Find the inverse of each function. Is the inverse a function?

16. \( f(x) = 3x + 2 \)  
17. \( g(x) = x^3 \)  
18. \( h(x) = \frac{1}{x^2} \)

**Graph**  
Find the domain and range of each function.

19. \( f(x) = x^3 \)  
20. \( g(x) = x^2 \)  
21. \( h(x) = \frac{1}{x} \)

22. **Graph**  
Find the domain and range of each function.  

23. **Graph**  
Find the domain and range of each function.

24. **Graph**  
Find the domain and range of each function.

---

**Prentice Hall Algebra 2 • Teaching Resources**

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1. Do you know HOW?
Simplify each radical expression. Use absolute value symbols when needed.

1. \( \sqrt[3]{27x^6} \)  
2. \( \sqrt[3]{-64y^9} \)  
3. \( \sqrt[3]{243x^3} \)

Simplify. Assume that all variables are positive.

4. \( \sqrt[3]{54} \)  
5. \( \sqrt[3]{277} \)  
6. \( \sqrt[3]{327} \)

Multiply or divide and simplify. Assume that all variables are positive.

7. \( \sqrt[3]{7} \cdot \sqrt[3]{5} \)  
8. \( \sqrt[3]{162} / \sqrt[3]{9} \)  
9. \( \sqrt[3]{54} / 3 \)

Simplify. Rationalize all denominators. Assume that all variables are positive.

10. \( \sqrt[3]{2} = \sqrt[3]{2} \)  
11. \( \sqrt[3]{3} - \sqrt[3]{2} \)  
12. \( \sqrt[3]{4} / \sqrt[3]{2} \)

Write each expression in simplest form.

13. \( \sqrt[3]{x^3} \)  
14. \( \sqrt[3]{x^3y^3} \)  
15. \( \sqrt[3]{80x^7} \)

ANSWERS

Do you understand?

16. Write each expression in radical form.
17. \( \sqrt[3]{27x^6} \)  
18. \( \sqrt[3]{-64y^9} \)

Write each expression in simplest form. Assume that all variables are positive.

19. \( \sqrt[3]{54} \)  
20. \( \sqrt[3]{277} \)  
21. \( \sqrt[3]{327} \)

Do you understand?

22. Writing Explain when absolute value symbols are needed when you are simplifying radical expressions. For radicals in the form \( \sqrt[n]{a} \), if the index is even, use absolute value symbols. If the index is odd, you don’t need absolute value symbols for any terms.

23. Error Analysis Explain the error in the simplification of radical expressions.

What is the correct simplification of \( \sqrt[3]{8} - \sqrt[3]{2} = \sqrt[3]{2} \)?

The product property does not apply to different indices; \( \sqrt[3]{2} \)

24. An object is moving at a speed of \( 10 - \sqrt[3]{2} \) m/s. How long will it take the object to travel 35 m?

25. Reasoning Show that \( \sqrt[3]{2} = \sqrt[3]{2} \) by rewriting \( \sqrt[3]{2} \) in exponential form.

For each pair of functions, find \( g(f(x)) \) and \( f(g(x)) \).

1. \( f(x) = 2x + 3 \) and \( g(x) = x^2 - x \)
2. \( f(x) = x^2 + 1 \) and \( g(x) = x^2 + 2 \)
3. \( f(x) = 2x + 3 \) and \( g(x) = x^2 + 2 \)
4. \( f(x) = x^2 + 1 \) and \( g(x) = x^2 + 2 \)

For each pair of functions, find the inverse of each function. Is the inverse a function?

1. \( f(x) = x^2 \)  
2. \( f(x) = \sqrt{x} \)  
3. \( f(x) = (x - 3)^2 + 1 \)

Graph. Find the domain and range of each function.

12. \( y = -\sqrt[3]{x} \)  
13. \( y = -\sqrt[3]{2x} \)  
14. \( y = -\sqrt[3]{2} \)

Do you understand?

15. Multiple choice The graph of \( y = -\sqrt[3]{x} \) is shifted 4 units up and 3 units right. Which equation represents the new graph?

16. Writing Explain the relationship between the domain of a function and the range of the function’s inverse. They are equal.

17. A store is having a sale with a 10% discount on all items. In addition, employees get a $20 discount on purchases of $100 or greater. Will an employee get a better deal if the $20 discount is applied first or if the 10% discount is applied first to their purchase of $110? The employee will pay less if the 15% discount is applied first.

18. A spherical water tank holds 6600 m³ of water. What is the diameter of the tank? (Hint: \( \pi = 3.14 \) or \( \sqrt[3]{1000} = 5 \))
Chapter 6 Performance Tasks

Give complete answers.

Task 1

a. Write a product of two square roots so that the answer, when simplified, is 12x^2. Show how your product simplifies to give the correct answer.

b. Write a quotient of two cube roots so that the answer, when simplified, is 6x. Show how your product simplifies to give the correct answer.

c. Write a product of the form \(a + \sqrt{b} (a - \sqrt{b})\) so that the answer, when simplified, is 15. Show how your product simplifies to give the correct answer.

[4] Check students’ work. All parts of Task completed correctly with work shown OR used correct process with minor computational errors.

[3] Student found products and quotient correctly but followed through the process incorrectly using incorrect simplification.

[2] Correct answer with no work shown OR student only able to complete part of Task.

[1] Student understood that squaring, cubing or multiplying was involved but was unable to correctly find the products or quotients, or to simplify them.

[0] No attempt was made to solve this problem OR answer is incorrect with no work shown.

Task 2

a. Find a radical equation of the form \(\sqrt{x + 3} = x + 2\) and its domain. Explain how you determined the domain. \(x = 0\) or \(x = -1\).

b. Find \((f - g)(x)\) and \((g - f)(x)\). Are they equal? Explain.

[4] All parts of Task completed correctly with work shown OR used correct process with minor computational errors.

[3] Student found all parts correctly but could not explain (a) OR could not explain (a) OR could not correctly find the domain for (b).

[2] Correct answer with no work shown OR student only able to complete part of Task.

[1] Student found part (a) but could not compose functions for parts (b) and (c).

[0] No attempt was made to solve this problem OR answer is incorrect with no work shown.

Give complete answers.

Task 3

a. Find the inverse of \(f(x) = \sqrt{x + 3} + 5\). Show all steps in the process. What is the domain of \(f^{-1}\)? (\(x = -3\) or \(x = 2\) domain: \(x \geq 3\)).

b. Choose a value for \(x\) and use the inverse to find \((f - f^{-1})(x)\) and \((f^{-1} - f)(x)\). For the value you chose: What can you conclude about \((f - f^{-1})(x)\) and \((f^{-1} - f)(x)\)? (\(g(x) = x^2 - ax + b\) and \(g^{-1}(x) = \frac{x - b}{a}\)).

c. Graph \(f(x)\) and \(f^{-1}(x)\) on the same axes. What relationship do you see between the two graphs? (The graphs are reflections of each other across the line \(y = x\)).

[4] All parts of Task completed correctly with work shown OR used correct process with minor computational errors.

[3] Student found all parts (a) and (b) correctly but could not explain (c) OR could not show steps in (a) OR could not explain (b).

[2] Correct answer with no work shown OR student only able to complete part of Task.

[1] Student knew to square each side of the equation, but could not complete the solution.

[0] No attempt was made to solve this problem OR answer is incorrect with no work shown.

Chapter 6 Cumulative Review

Multiple Choice

For Exercises 1–12, choose the correct letter.

1. What is an equation for this graph? \(B\)
   \[ y = \frac{1}{2}x^2 + 1 \]

2. Simplify \(f\).
   \[ f(2) = 4 \]

3. Solve the system:
   \[ \begin{align*}
   x + y &= 3 \quad \text{A}
   2x - y &= 4 \quad \text{C}
   \end{align*} \]

4. Which of the following polynomials has roots 0, 1, and 2? \(B\)
   \[ P(x) = x^3 - 3x^2 + 2x \]

5. Complete the square: \((x^2 - 2x)\).
   \[ \frac{1}{4} \]

6. Let \(f(x) = x^2 + 5x + 2\) and \(h(x) = x^2 - 2x - 6\). What is the function for \((f + h)(x)\)? \(A\)
   \[ 2x^2 + 3x - 4 \]

7. Simplify \((5 - 2)(3 - 6)\).
   \[ C\]

8. What is the solution to \(x^2 - 5x + 1 = 0\)? \(D\)
   \[ 5 \pm \sqrt{17} \]

9. If \(h(x) = 2x^2 + x - 5\) and \(g(x) = -4x - 1\), what is the value of \(h(1) - g(3)\)? \(C\)
   \[ 11 \]

Short Response

For Exercises 13 and 14, let \(f(x) = x^2 + 2x - 3\) and \(g(x) = 4x - 1\).

10. Simplify each expression:
   a. \(f(x) + 2\)
   b. \(fx(2)\)

11. Which of the following relations defines \(y\) as a function of \(x\)? \(C\)
   \[ x = y \]

12. Simplify \(\frac{2x}{x - 2}\).
   \[ \frac{2}{x - 2} \]

13. Solve by graphing:
   \[ f(x) = x^2 - 2x = 3 \quad \text{D} \]

14. To find the time \(t\) in seconds, it takes an object to fall 
   across the equation \(i = \frac{h}{g}\), a croissant dropped from the top of the Eiffel Tower takes 
   \(12.84\) to reach the ground. How tall is the Eiffel Tower? \(1244\) meters.

15. Simplify each expression. Assume that all variables are positive.
   a. \(\frac{3x^2y}{9x^3y^2}\)
   b. \(\frac{x^2}{y^3}\)

Extended Response

16. Writing: Explain how you can obtain the solutions to a system of equations by graphing.

17. Find the area of a parallelogram with vertices \((2, 5), (4, 3), (6, 2), (8, 0)\).

18. Complete the square: \(x^2 + 4x\).

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Finishing the Project
why their experimental results might differ from their theoretical results.

Students solve a formula for a given variable, then use the formula to find the period of a pendulum. For the pendulum, substitute the string length into the new equation.

Activity 1: Constructing

Students use string, coins, and binder clips to construct simple pendulums.

Activity 2: Investigating

Students perform experiments to time the swings of their pendulums and record their observations in charts.

Activity 3: Analyzing

Students solve a formula for a given variable, then use the formula to find the theoretical periods of their pendulums. They analyze their data and determine whether their experimental results might differ from their theoretical results.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results.

• Have students review their data and their calculations of the periods.
• Ask students to share their insights that resulted from completing the project.

Reflect and Revise

• Ask students if they have ever used a pendulum, or a swing-like motion.
• Have students speculate as to whether a homemade pendulum could be used to time events.

Activity 2: Investigating

To construct a simple pendulum, tie a medium binder clip to the end of a piece of string. The string is swung. Insert one coin in the binder clip. Measure the length of the string (in centimeters) and record the data in the second column.

Experiment 1

Cut a second string that is half the length of the original string. Repeat Experiment 1. Record the time it takes to make a complete swing. Then divide each of these times by two to determine the period of the pendulum. Repeat the procedure using two coins, then using three coins, recording the data in the second and third columns, respectively.

Do your experimental results give the same period as the theoretical models? What factors other than the weight might affect the period of the pendulum?

Chapter 6 Project Manager: Swing Time

Getting Started

Read the project. As you work on the project, you will need a calculator and materials on which you can record your results and make calculations. Keep all of your work for the project in a folder, along with this Project Manager.

Checklist

☐ Activity 1: constructing a pendulum
☐ Activity 2: determining the period
☐ Activity 3: comparing experimental and theoretical periods
☐ pendulum experiment

Suggestions

• Use the lightest thread or string possible.
• Have one student swing the pendulum while another student keeps track of the time.
• Isolate the pendulum by first recording the data, then take the square root of each side of the equation. Subtract the string lengths into the new equation.
• How would your results change if you pulled the pendulum back to an angle of 60°? What other changes would affect your results?

Scoring Rubric

1. Your experimental results are reasonable. Calculations are correct. Explanations are thorough and well thought out. Data, calculations, and conclusions are neatly presented.
2. Your experimental results are reasonable. Calculations are mostly correct with some minor errors. Explanations lack detail and accuracy. Data, calculations, and conclusions are not well organized.
3. Your experimental results are not reasonable. Calculations and explanations contain errors. Data, calculations, and conclusions are unorganized and lack detail.
4. Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
5. Major elements of the project are incomplete or missing.

Your Evaluation of Project

Evaluate your work, based on the Scoring Rubric:

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Choose the word or phrase from the list that best completes each sentence.

- exponential function
- exponential growth
- exponential decay
- asymptote
- growth factor
- decay factor

1. In the function \( y = 12(2)^x \), the value 2 is the \( \text{growth factor} \).
2. An \( \text{asymptote} \) is a line that a graph approaches as \( x \) or \( y \) increases in absolute value.
3. For \( \text{exponential decay} \), as the value of \( x \) increases, the value of \( y \) decreases.
4. A function in the general form \( y = ab^x \) is called an \( \text{exponential function} \).
5. For \( \text{exponential growth} \), as the value of \( x \) increases, the value of \( y \) increases.
6. In the function \( y = 4(0.3)^x \), the value 0.3 is the \( \text{decay factor} \).

Identify whether each function represents exponential growth or exponential decay.

7. \( y = 0.75(2)^x \) \( \text{exponential growth} \)
8. \( y = 0.022(0.9)^x \) \( \text{exponential decay} \)
9. \( y = 50(0.85)^x \) \( \text{exponential decay} \)
10. \( y = 12(2)^x \) \( \text{exponential growth} \)

Identify the \( y \)-intercept for each function.

11. \( y = 4(0.75)^x \) \( (0, 4) \)
12. \( y = 0.3(2)^x \) \( (0, 0.3) \)

Write a function that models the change in the animal population.

20. \( 0.7 \) bears \( \text{growth; 185} \) years
21. \( 0.6 \) bears \( \text{growth; 23} \) years
22. \( 0.1 \) bears \( \text{growth; 45} \) years
23. \( 0.2 \) bears \( \text{growth; 90} \) years

Graph each function.

1. \( y = (2.3)^x \)
2. \( y = 3^x \)
3. \( y = (0.2)^x \)
4. \( y = \frac{3}{2}x \)
5. \( a(t) = 2.5t \)
6. \( f(x) = \frac{1}{2}(0.7)^x \)

Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the \( y \)-intercept.

7. \( y = 0.9e^{0.12x} \) \( \text{decay; 0.99} \)
8. \( y = 20(3.75)^x \) \( \text{growth; 20} \)
9. \( y = 10(0.5)^x \) \( \text{growth; 10} \)
10. \( f(x) = \frac{3}{2} \) \( \text{decay;} \)
11. \( f(x) = 0.25(1.05)^x \) \( \text{growth; 0.25} \)
12. \( f(x) = \frac{1}{2}(0.7)^x \) \( \text{growth;} \)

Suppose you deposit $1500 in a savings account that pays interest at an annual rate of 6%. No money is added or withdrawn from the account.

a. How much will be in the account after 5 years? \( \approx 2000.93 \)
b. How much will be in the account after 20 years? \( \approx 4849.70 \)
c. How many years will it take for the account to contain $2000? \( \approx 10 \)
d. How many years will it take for the account to contain $4000? \( \approx 17 \)

Write an exponential function to model each situation. Find each amount after the specified time.

13. Population of 2,736,000 increases 1.4% per year for 10 years. \( y = 2736000(1.014)^x \) about 3,600,410
14. Population of 12,286,000 grows 1.5% per year for 15 years. \( y = 12286000(1.015)^x \) about 1,940,413
15. Population of 7,532,000 decreases 1.4% per year for 10 years. \( y = 7532000(0.986)^x \) about 5,814,960
16. A new car that sells for $22,000 depreciates 25% each year for 8 years. \( y = 22000(0.25)^x \) \( \approx 5295.31 \)

For each annual rate of change, find the corresponding growth or decay factor.

17. \( +41 \%) \text{ growth; 1.45} \)
18. \( -10 \%) \text{ growth; 0.9} \)
19. \( -41 \%) \text{ growth; 0.6} \)
20. \( +20 \%) \text{ growth; 1.2} \)
21. \( +10 \%) \text{ growth; 2} \)
22. \( -5 \%) \text{ growth; 0.95} \)
23. \( +3 \%) \text{ growth; 1.03} \)

In 2009, there were 1570 bears in a wildlife refuge. In 2010, the population had increased to approximately 1884 bears. If this trend continues and the bear population is increasing exponentially, how many bears will there be in 2017? \( 4580.67 \)

Write a function that models the change in the animal population.

24. \( 0.7 \) bears \( \text{growth; 185} \) years
25. \( 0.6 \) bears \( \text{growth; 23} \) years
26. \( 0.1 \) bears \( \text{growth; 45} \) years
27. \( 0.2 \) bears \( \text{growth; 90} \) years

Graph the function. Estimate the number of years until the population first drops below 15 animals. 47 years

Graph your function on a graphing calculator. Sketch your graph.

Write a function that models the change in the animal population.

28. \( 0.7 \) bears \( \text{growth; 185} \) years
29. \( 0.6 \) bears \( \text{growth; 23} \) years
30. \( 0.1 \) bears \( \text{growth; 45} \) years
31. \( 0.2 \) bears \( \text{growth; 90} \) years

An investment of $75,000 increases at a rate of 12.5% per year. What is the value of the investment after 30 years? \( $2,508,247.87 \)

Write a function that models the change in the animal population.

32. \( 0.7 \) bears \( \text{growth; 185} \) years
33. \( 0.6 \) bears \( \text{growth; 23} \) years
34. \( 0.1 \) bears \( \text{growth; 45} \) years
35. \( 0.2 \) bears \( \text{growth; 90} \) years

A new truck that sells for $39,000 depreciates 12.5% each year. What is the value of the truck after 7 years? \( $11,685.50 \)

The price of a new home is $350,000. The value of the home appreciates 2% each year. How much will the home be worth in 10 years? \( $4810.70 \)

The value of a piece of equipment has a decay factor of 0.80 per year. After 5 years, the equipment is worth $98,304. What was the original value of the equipment? \( $200,000 \)

Your friend drops a rubber ball from 8 ft. You notice that its rebound is 32.5 in. on the first bounce and 22 in. on the second bounce.

a. What exponential function would be a good model for the height of the ball? \( y = 0.50 \cdot 0.96^x \)
b. How high will the ball bounce on the fourth bounce? \( 4.70 \) in.

The population of an endangered bird is decreasing at a rate of 0.75% per year.

How many birds will there be in 100 years? \( 2,250,000 \)
Finding the exponential function that models the cran population is $b$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t$. The exponential function that models the crane population is $y = 270(1.05)^t. The exponential function that models the crane population is $y = 270(1.05)^t. 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3. decay. Then find the
Determine whether the function represents exponential growth or exponential
Exercises
Step 1 Find a and b.
\[ a = 82 \]
\[ b = 1 + 0.16 \]
\[ b = 1.16 \]
Step 2 Write the exponential function.
\[ y = ab^x \]
\[ y = 82(1.16)^x \]
Step 3 Calculate.
\[ y = 82(1.16)^5 \]
\[ y = 172.22 \]
Carl will weigh about 172.22 lb in 5 years.

Exercises
Determine whether the function represents exponential growth or exponential
decay. Then find the y-intercept.
1. \[ y = 8000(1.15)^x \] growth; \[ 8000 \]
2. \[ y = 200(0.75)^x \] decay; \[ 200 \]
3. \[ y = 10(0.56)^x \] decay; \[ 10 \]
4. \[ f(x) = 6(1.1)^x \] growth; \[ 6 \]

7-2 ELL Support
Properties of Exponential Functions

Concept List

- compression
- horizontal translation 3 units to the right
- parent function
- stretch
- continuously compounded interest
- natural base exponential function
- reflection in x-axis
- vertical translation 3 units upward

Choose the concept from the list above that best represents the item in each box.

1. \[ y = 3 \cdot 5^x \] stretch
2. \[ y = 0.045 \] parent function

3. \[ y = 7\cdot 3 \] vertical translation 3 units upward
4. \[ 4(0.5)P \cdot e^{-t} \] continuously compounded interest

5. \[ y = -5\cdot 3 \] reflection in x-axis
6. \[ y = 0.567(2)^x \] compression

7. \[ y = 8 \] natural base exponential function
8. \[ y = 1\cdot e^{2x} \] horizontal translation 3 units to the right

7-2 Think about a Plan
Properties of Exponential Functions

Investment: How long would it take to double your principal in an account that pays 3% annual interest compounded continuously?

Know
1. The equation for continuously compounded interest is \[ A = Pe^{rt} \]
2. The principal is \[ P \]
3. The interest rate is \[ 0.03 \]

Need
4. To solve the problem I need to:
   Find the time \( t \) when the amount in the account is twice the original principal

Plan
5. If the principal is \( P \), then twice the principal is \[ 2P \]
6. What equation can you use to find the time \( t \) it takes to double your principal? \[ 2P = Pe^{rt} \]
7. Solve your equation for \( t \).
   \[ \ln(2) = \ln(e^{rt}) \]
   \[ \ln(2) = rt \]
   \[ 10.7 \text{ years} \]
8. Is your solution reasonable? Explain.
   Yes, it seems reasonable that it would take almost 11 years to double the principal.
Graph each function.
1. \( y = x^3 \)
2. \( y = \frac{1}{2}x^2 \)
3. \( y = x + 2 \)
4. \( y = \frac{1}{2}(x + 2)^2 \)
5. \( y = \sqrt{x} \)
6. \( y = \sqrt{x} - 2 \)
7. \( y = -x^3 + 1 \)
8. \( y = -2(x - 1)^2 \)
9. \( y = -x + 2 \)
10. \( y = -\sqrt{x} + 1 \)

Graph each function as a transformation of its parent function.
11. \( y = 2x \)
12. \( y = \frac{1}{2}x \)
13. \( y = \sqrt{2x} \)
14. \( y = -\sqrt{x} \)
15. \( y = \sqrt{-x} \)

A cake is 190°F when you remove it from the oven. You must let it cool to 72°F before you can frost it. The table at the right shows the temperature readings for the cake.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temp (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>190</td>
</tr>
<tr>
<td>5</td>
<td>169</td>
</tr>
<tr>
<td>10</td>
<td>142</td>
</tr>
<tr>
<td>15</td>
<td>114</td>
</tr>
<tr>
<td>20</td>
<td>92</td>
</tr>
</tbody>
</table>

Use the graph of \( y = x^3 \) to evaluate each expression to four decimal places.

14. \( x^2 \) 15. \( x^{1.2} \) 16. \( x^{0.5} \)

Write the parent function of each function.
1. \( y = 3x^2 - 5 \)
2. \( y = 2^{x - 1} \)
3. \( y = 2^{x+2} \)

Graph each of the following functions.
4. \( y = 2^x \)
5. \( y = 0.5 \cdot 2^x \)

Identify each function as a compression, a reflection, or a translation of the parent function.
6. \( y = 0.5 \cdot 2^x \)
7. \( y = -2^x \)
8. \( y = 2^x - 3 \)

Write a function for the indicated transformation.
9. The function \( y = 3x^2 \) vertically stretched by the factor 3 \( y = 3 \cdot 3^{x^2} \)
10. The function \( y = 7 - 2x \) translated up 8 units \( y = 7 - 2x + 8 \)
2.7.2 Standardized Test Prep
Properties of Exponential Functions

Gridded Response
Solve each exercise and enter your answer in the grid provided.

1. Suppose you deposit $1000 in a savings account that pays interest at an annual rate of 4% compounded continuously. How many years will it take for the balance in your savings account to reach $1800? Round your answer up to the nearest number of years.

2. Suppose you make $1500 at your summer job and you decide to invest this money in a savings account that pays interest at an annual rate of 5.5% compounded continuously. How many dollars will be in the account after 5 years? Express the answer to the nearest whole dollar.

3. The half-life of a radioactive substance is the time it takes for half of the material to decay. Phosphorus-32 is used to study a plant’s use of fertilizer. It has a half-life of 14.3 days. How many milligrams of phosphorus-32 remain after 52 days from a 100-mg sample? Express the answer to the nearest whole milligram.

4. A scientist notes the bacteria count in a petri dish is 40. Three hours later, she notes the count has increased to 75. Using an exponential model, how many hours will it take for the bacteria count to grow from 75 to 120? Express the answer to the nearest tenth of an hour.

Answers

1. $5.5$
2. $3579$
3. $6.23$
4. $1.5$

7.2 Reteaching
Properties of Exponential Functions
There are four types of transformations that can change the graph of an exponential function.

- **Stretcher**: The factor in $y = a^x$ can stretch the graph of an exponential function when $|a| > 1$.
- **Compress**: The factor in $y = a^x$ can compress the graph of an exponential function when $|a| < 1$.
- **Reflection**: The factor in $y = a^x$ can reflect the graph of an exponential function in the $x$-axis when $a < 0$.
- **Translation**: The graph of an exponential function translates horizontally by $t$ units, vertically by $k$ units.

**Problem**

How does the graph of $y = \left(\frac{1}{2}\right)^{x-1} + 4$ compare to the parent function $y = \left(\frac{1}{2}\right)^x$?

**Step 1** Determine the base of the function $y = \left(\frac{1}{2}\right)^{x-1} + 4$. Because $b < 1$, the graph will represent exponential decay.

**Step 2** Make a table. Find more values if necessary to get a good picture of the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$\left(\frac{1}{2}\right)^{-1} + 4 = \frac{1}{2} + 4 = \frac{9}{2}$</td>
</tr>
<tr>
<td>0</td>
<td>$\left(\frac{1}{2}\right)^0 + 4 = 1 + 4 = 5$</td>
</tr>
<tr>
<td>1</td>
<td>$\left(\frac{1}{2}\right)^1 + 4 = \frac{1}{2} + 4 = \frac{9}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>$\left(\frac{1}{2}\right)^2 + 4 = \frac{1}{4} + 4 = \frac{17}{4}$</td>
</tr>
</tbody>
</table>

**Step 3** Use the values for $x$ and $y$ from the table to graph the function.

**Step 4** For $y = \left(\frac{1}{2}\right)^{x-1} + k$, let $A = 1$ and $k = -4$. Shift the graph of the parent function above 1 unit left and 4 units down. The horizontal asymptote shifts down as well, from $y = 0$ to $y = -4$.

**Step 5** Use a graphing calculator to check your graph.

7.2 Enrichment
Properties of Exponential Functions

A Closer Look at Compounding

The formula for finding the amount of money accumulated in an account is $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

The variable $P$ represents the principal, or amount initially invested.

The variable $r$ represents the interest rate as a decimal.

The variable $n$ represents the number of times per year the interest is compounded.

The variable $t$ represents the time, or number of years for which the money is invested.

1. $750$ is invested at 3% compounded quarterly. How much is in the account after 10 yr? $\text{2331.80}$

2. Write the new formula for $P = 500, r = 0.1, n = 1$, and $t = 3$ yr. $A = 500\left(1 + \frac{0.1}{1}\right)^{1 \cdot 3}$

3. Remind that $1$ is the number of times the interest is compounded. What happens as $a$ grows? In other words, what is the effect of compounding more often? Fill in the following table. Round answers to eight decimal places.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.10000000</td>
</tr>
<tr>
<td>10</td>
<td>5.12000000</td>
</tr>
<tr>
<td>100</td>
<td>5.12460938</td>
</tr>
<tr>
<td>1000</td>
<td>5.12493293</td>
</tr>
<tr>
<td>10000</td>
<td>5.12493293</td>
</tr>
</tbody>
</table>

4. The table suggests that as $n$ increases, the value of $1 + \frac{1}{n}$ grows closer to $e$. If the value of $n$ is increased further, the decimal approximation on the table will get very close to the value of $e$. This number is used in many growth and decay applications. $2.71828183$

5. As $n$ grows, you get closer to compounding continuously. This is why the formula used for compounding continuously is $A = Pe^t$. Rework Exercise 1 assuming that compounding is continuous. $\text{2222.50}$

For problems involving continuously compounded interest, use the following formula:

- **Continuously Compounded Interest**
  \[ A(t) = Pe^{rt} \]
  - $P$ is the principal.
  - $r$ is the annual interest rate (as a decimal).
  - $t$ is the time in years.

**Problem**

Suppose you invest $1000 at an annual interest rate of 5% compounded continuously. How much will you have in the account in 10 years? $\text{2158.93}$

**What do you know?**
- The principal, $P = 1000$
- The interest rate, $r = 0.05$
- The time, $t = 10$

**Use the formulas.**

\[ A(t) = 1000e^{0.05 \cdot 10} \]

**In ten years, you will have $2158.93.**

7.2 Reteaching continued
Properties of Exponential Functions

For problems involving continuously compounded interest, use the following formulas:

- **Continuously Compounded Interest** $A(t) = Pe^{rt}$

**Problem**

Suppose you invest $500 at an annual interest rate of 7% compounded continuously. How much will you have in the account in 10 years? $\text{1013.64}$

**Exercises**

Graph each exponential function.

1. $y = 2^x$
2. $y = 3^x + 1$
3. $y = 5^x$
4. $y = (\frac{1}{2})^x$
5. $y = 2^{x+4}$
6. $y = 2^{x-4}$
7. $y = (\frac{1}{2})^{x+4}$
8. $y = (\frac{1}{2})^{x-4}$

10. Suppose you invest $750 at an annual interest rate of 7% compounded continuously.
   a. How much will you have in the account in 10 years? $\text{1303.56}$
   b. How long will it take for the account to reach $2000? $\text{15 years}$

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7-3 ELL Support
Logarithmic Functions as Inverses

For Exercises 1–3, draw a line from each word or phrase in Column A to the matching item in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. logarithmic function</td>
<td>A. a logarithm with base 10</td>
</tr>
<tr>
<td>2. common logarithm</td>
<td>B. the inverse of an exponential function</td>
</tr>
<tr>
<td>3. logarithmic scale</td>
<td>C. uses the logarithm of a quantity instead of the quantity itself</td>
</tr>
</tbody>
</table>

For Exercises 4–8, draw a line from each word or phrase in Column A to the matching item in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. parent function</td>
<td>A. ( y = 0.75 \log x )</td>
</tr>
<tr>
<td>5. stretch</td>
<td>B. ( y = 5 \log_{10} x )</td>
</tr>
<tr>
<td>6. compression</td>
<td>C. ( y = \log x - 3 )</td>
</tr>
<tr>
<td>7. reflection in ( x )-axis</td>
<td>D. ( y = \log_{10} x )</td>
</tr>
<tr>
<td>8. translation 3 units to the right</td>
<td>E. ( y = - \log x )</td>
</tr>
<tr>
<td>9. translation 3 units downward</td>
<td>F. ( y = \log (x - 3) )</td>
</tr>
</tbody>
</table>

7-3 Think About a Plan

Logarithmic Functions as Inverses

Chemistry Find the concentration of hydrogen ions in seawater, if the pH level of seawater is 8.5.

Understanding the Problem
1. What is the pH of seawater? \( 8.5 \)

2. How do you represent the concentration of hydrogen ions? \( [H^+] \)

3. What is the problem asking you to determine? the concentration of hydrogen ions in seawater

Planning the Solution
4. Write the formula for the pH of a substance: \( \text{pH} = - \log([H^+]) \)

5. Write an equation relating the pH of seawater to the concentration of hydrogen ions in seawater. \( 8.5 = - \log([H^+]) \)

Getting an Answer
6. Solve your equation to find the concentration of hydrogen ions in seawater.

- \( 8.5 = - \log([H^+]) \)
- \( -8.5 = \log([H^+]) \)
- \( 10^{-8.5} = [H^+] \)
- \( [H^+] = 10^{-8.5} \text{ or } 1.56 \times 10^{-9} \)

7-3 Practice

Logarithmic Functions as Inverses

Form G

Write each equation in logarithmic form.

1. \( 2^3 = 8 \) \( \log_2 8 = 3 \)
2. \( 5^2 = 25 \) \( \log_5 25 = 2 \)
3. \( 10^3 = 1000 \) \( \log_{10} 1000 = 3 \)
4. \( (\log 5)^2 = 9 \) \( \log_5 9 = 2 \)
5. \( 8^3 = 512 \) \( \log_8 512 = 3 \)
6. \( 2^4 = 16 \) \( \log_2 16 = 4 \)
7. \( 5^2 = 25 \) \( \log_5 25 = 2 \)
8. \( 10^2 = 100 \) \( \log_{10} 100 = 2 \)
9. \( \log_9 1 = 0 \) \( \log_9 1 = 0 \)
10. \( \log_8 1 = 0 \) \( \log_8 1 = 0 \)
11. \( \log_5 125 = 3 \) \( \log_5 125 = 3 \)
12. \( \log_10 100,000 = 5 \) \( \log_{10} 100,000 = 5 \)
13. \( \frac{1}{\log_2 4} = \frac{1}{2} \) \( \log_4 2 = \frac{1}{2} \)
14. \( \log_{10} 100,000 = 5 \) \( \log_{10} 100,000 = 5 \)
15. \( \log_5 75 = 2 \) \( \log_5 75 = 2 \)
16. \( \log_2 8 \) \( \log_2 8 = 3 \)

In 2004, an earthquake of magnitude 7.0 shook Papua, Indonesia. Compare the intensity level of that earthquake to the intensity level of each earthquake below.

17. magnitude 8.1 in Costa Rica, in 2009
18. magnitude 7.5 in Greece, in 2008
19. magnitude 8.0 in the Fiji Islands, in 2007
20. magnitude 7.9 in the Kuril Islands, in 2005
21. magnitude 8.7 in the Kuril Islands, in 2005
22. magnitude 9.0 in the Kuril Islands, in 2005
23. magnitude 8.9 in the Kuril Islands, in 2005

Graph each logarithmic function.

24. \( y = \log x \)
25. \( y = \log_2 x \)
26. \( y = \log_3 x \)
27. \( y = \log_5 x \)

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7.3 Practice

Form K

Logarithmic Functions as Inverses

Write each equation in logarithmic form.
1. \(2^2 \log 32 = 5\)
2. \(2^{41} = 3^7 \log 243 = 5\)
3. \(625 = 5^3 \log 525 = 4\)

Write each equation in exponential form.
4. \(\log_b 2 = 3 \implies b^3 = 2\)
5. \(\log_{12} 125 = 3 \implies 125 = 12^3\)
6. \(\log_{512} 3 = 3 \implies 512 = 3^3\)

Evaluate each logarithm.
7. \(\log_2 27 = x\)
8. \(\log_{12} 256 = x\)
9. \(\log_3 5 = x\)

The formula \(M_s = M_o - M_d\) is used to compare the intensity levels of earthquakes.
The variable \(I\) is the intensity measured by a seismograph. The variable \(M\) is the measurement on the Richter scale. Use the formula to answer the following problem.
10. In 1938, an earthquake of magnitude 8.5 hit San Francisco, California. Indonesia was hit by an earthquake of magnitude 8.0 in 1938. Compare the intensity of the two earthquakes. The earthquake in Indonesia was approximately 1.79 times more intense than the earthquake in San Francisco.

7.3 Practice (continued)

Form K

Logarithmic Functions as Inverses

11. Error Analysis A student drew the graph below to represent the function \(y = \log x\). What mistake did the student make when she drew her graph?

Graph each logarithmic function.
12. \(y = \log x\)
13. \(y = \log x + 2\)
14. \(y = 0.5 \log x\)

Identify each function as a compression, a stretch, or a translation of the parent function.
15. \(y = \log x + 2\) Translation 16. \(y = 0.5 \log x\) Compression

Transform the function \(y = \log x\) as indicated below.
17. Stretch by a factor of 3 and translate 6 units up \(y = 3 \log x + 6\)
18. Compress by a factor of 0.4 and reflect in the x-axis \(y = -0.4 \log x\)

7.3 Standardized Test Prep

Logarithmic Functions as Inverses

Multiple Choice
For Exercises 1−4, choose the correct letter.
1. Which of the following is the logarithmic form of the equation \(4^3 = 64\)?
   a. \(3 \log_4 64 = 4\)
   b. \(\log_2 64 = 3\)
   c. \(\log_4 4 = 3\)
   d. \(\log_2 4 = 3\)

2. What is the value of \(\log_2 87\)?
   a. \(\sqrt{2}\)
   b. 8
   c. \(\frac{\sqrt{2}}{8}\)
   d. 16

3. How does the graph of \(y = \log (x + 3)\) compare with the graph of the parent function, \(y = \log x\)?
   a. Translated 3 units to the left
   b. Translated 3 units to the right
   c. Translated 3 units up
   d. Translated 3 units down

4. In 2009, an earthquake of magnitude 6.7 shook the Kermadec Islands off the coast of New Zealand. Also in 2009, an earthquake of magnitude 5.3 occurred in the Alaska Peninsula. How many times stronger was the Kermadec earthquake than the Alaska earthquake?
   a. \(\sqrt{\frac{6.7}{5.3}}\)
   b. \(6.7\)
   c. \(\frac{6.7}{5.3}\)
   d. \(\frac{5.3}{6.7}\)

Short Response
5. A single-celled bacterium divides every hour. The number \(N\) of bacteria after \(t\) hours is given by the formula \(N = 2^t\).
   a. After how many hours will there be 64 bacteria?
   b. Explain in words or show work for how you determined the number of hours.
   c. 6 hours
   d. \(\log_2 64 = 6\) can be written in the exponential form \(2^6 = 64\). Substituting 64 for \(N\), the equation becomes \(2^t = 64\). Rewriting 64 with base 2, the equation becomes \(2^t = 2^6\). Since the bases are equal, \(t = 6\). 
   e. Incorrect exponential form OR incorrect explanation
   f. Incorrect answers and no explanation OR no answers given

Rewrite each equation in exponential form to solve the equation.
1. \(\log_{12} 48 = 4\)
2. \(\log_{17} 2 = 2\)
3. \(\log_{17} 3 > \log_{17} 2\)
4. \(\log_{18} x = \log_{18} 3\)
5. Which is greater, \(\log_{10} 2\) or \(\log_{10} 5\)?
6. \(\log_{10} 2 = 0.3010\)
7. Which is greater, \(\frac{1}{2} \log_{10} 8\) or \(\frac{1}{3} \log_{10} 8\)?
8. \(\log_{10} 2 = 0.3010\)
9. \(\log_{10} 5 = 0.6990\)
10. \(\log_{10} 8 = 0.9031\)
11. Which is greater, \(\frac{1}{2} \log_{10} 8\) or \(\frac{1}{3} \log_{10} 8\)?
12. 8.69
13. \(\log_{10} 2 = 0.3010\)
14. \(\log_{10} 5 = 0.6990\)
15. \(\log_{10} 8 = 0.9031\)
16. \(\log_{10} 10 = 1\)

Rewrite in exponential form and solve for \(x\).
9. \(\log_{10} x = 0\)
10. \(\log_{10} (2x - 7) = 0\)
11. \(\log_{10} 7 = 1\)
12. \(\log_{10} x = 0.3\)
13. \(\log_{10} x = 0\)
14. \(\log_{10} 17 = x\)
15. \(\log_{10} 0.1 = -1\)
16. \(\log_{10} x = 1\)
17. \(\log_{10} 2 = 0.3010\)
18. \(\log_{10} (x + 1) = 0\)
19. \(0 = x + 1\)
20. \(-1 = \log_{10} x = -1\)

7.3 Enrichment

Log Jams

The logarithm is a tool originally developed and used to aid in calculations, yet this viewpoint of logarithms is not the only one of interest. Logarithms are also useful when thought of as real-valued functions, or as inverse functions of the corresponding exponential functions. The idea of a logarithm as an inverse function of an exponential function means that \(\log_{10} x\) is a question to be answered. For example, you can read the expression \(\log_{10} 1000 = 3\) as “what exponent on base 10 gives 1000?” The answer is 3, because \(10^3 = 1000\).

Thinking of a logarithm as an exponent helps to order some logarithms without evaluating them. For example, the logarithms \(\log_{10} 3, \log_{10} 7,\) and \(\log_{10} 8\) are in descending order since the exponent needed on base 10 that gives 8 would be greater than 1, and 1 is in turn greater than the exponent needed on base 10 that gives 7.

You can also compose logarithms as you would compose other functions, where their domains and ranges agree. Thus, you evaluate \(\log_{10} (\log_{10} 3)\) by evaluating \(\log_{10} 3\), then evaluating \(\log_{10} 2\).

Rewrite each equation in exponential form to solve the equation.
1. \(\log_{10} x = 4\)
2. \(\log_{10} x = 3\)
3. \(\log_{10} x = \log_{10} 3\)
4. \(\log_{10} x = \log_{10} 2\)
5. \(\log_{10} x = \log_{10} 2\)
6. \(\log_{10} x = \log_{10} 3\)
7. Which is greater, \(\log_{10} 2\) or \(\log_{10} 3\)?
8. Which of the following are equal? \(\log_{10} 3\) and \(\log_{10} 2\) are equal
   a. \(\log_{10} 3\)
   b. \(\log_{10} 2\)
   c. \(\log_{10} 3\)
   d. \(\log_{10} 2\)

Rewrite in exponential form and solve for \(x\).
9. \(\log_{10} 1 = x\)
10. \(\log_{10} (2x - 7) = 0\)
11. \(\log_{10} 7 = 1\)
12. \(\log_{10} x = 0.3\)
13. \(\log_{10} x = 0\)
14. \(\log_{10} 17 = x\)
15. \(\log_{10} 0.1 = -1\)
16. \(\log_{10} x = 1\)
17. \(\log_{10} 2 = 0.3010\)
18. \(\log_{10} (x + 1) = 0\)
19. \(0 = x + 1\)
20. \(-1 = \log_{10} x = -1\)
7.3 Reteaching
Logarithmic Functions as Inverses

A logarithmic function is the inverse of an exponential function.
To evaluate logarithmic expressions, use the fact that \( x = \log_b y \) is the same as \( y = b^x \). Keep in mind that \( x = \log y \) is another way of writing \( x = \log_b y \).

**Problem**
What is the logarithmic form of \( 6^3 = 216 \)?

**Step 1** Determine which equation to use.
The equation is in the form \( b^x = y \).

**Step 2** Find \( x \), \( y \), and \( b \).
\( b = 6 \), \( x = 3 \), and \( y = 216 \)

**Step 3** Because \( y = b^x \) is the same as \( x = \log_b y \), rewrite the equation in logarithmic form by substituting for \( x \), \( y \), and \( b \).
\( x = \log_b y \)

**Exercises**
Write each equation in logarithmic form.
1. \( 4^\frac{1}{2} \)
2. \( 5^2 \)
3. \( 3^2 = 9 \)
4. \( 2^3 = 8 \)
5. \( 2 = \log_4 16 \)
6. \( 3 = \log_4 27 \)

**Problem**
What is the value of \( \log_5 125 \)?

**Step 1** Determine which equation to use.
The equation is in the form \( x = \log_b y \).

**Step 2** Find \( x \), \( y \), and \( b \).
\( b = 5 \), \( x = 3 \), and \( y = 125 \)

**Step 3** Because \( y = b^x \) is the same as \( x = \log_b y \), rewrite the equation in exponential form by substituting for \( x \), \( y \), and \( b \).
\( y = b^x \)

7.3 Reteaching (continued)
Logarithmic Functions as Inverses

**Exercises**
Write each equation in exponential form.
9. \( 3 = \log_2 8 \)
10. \( 2 = \log_{10} 100 \)
11. \( \log_2 8 = \frac{3}{2} \)
12. \( \log_4 16 = 2 \)
13. \( \log_5 125 = 3 \)
14. \( \log_6 36 = 2 \)

**Problem**
What is the value of \( \log_4 12 \)?

**Step 1** Write the equation in logarithmic form \( x = \log_b y \).
\( x = \log_b y \)

**Step 2** Rewrite in exponential form \( y = b^x \).
\( y = b^x \)

**Step 3** Solve for \( x \).
\( x = \frac{1}{2} \)

**Exercises**
Evaluate the logarithm.
21. \( \log_2 8 \)
22. \( \log_3 9 \)
23. \( \log_5 25 \)
24. \( \log 10 \)
25. \( \log 0.1 \)
26. \( \log 1 \)
27. \( \log 100 \)
28. \( \log 0.01 \)
29. \( \log 1000 \)
30. \( \log 0.001 \)

7.4 ELL Support
Properties of Logarithms

<table>
<thead>
<tr>
<th>Product Property</th>
<th>Quotient Property</th>
<th>Power Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log_a mn = \log_a m + \log_a n )</td>
<td>( \log_a \frac{m}{n} = \log_a m - \log_a n )</td>
<td>( \log_a m^n = n \log_a m )</td>
</tr>
</tbody>
</table>

Identify the property that is demonstrated by each equation.
1. \( \log_a (ab) = \log_a b + \log_a a \) - product property
2. \( \log_a b^3 = 3 \log_a b \) - power property
3. \( \log_a \frac{b}{c} = \log_a b - \log_a c \) - quotient property
4. \( \log_a (b + c) = \log_a b + \log_a c \) - product property
5. \( \log_a (bc) = \log_a b + \log_a c \) - power property

Identify the values of the symbols in the equations below.
6. \( \log_3 9 = \log_3 3^2 \) - \( a = 2 \), \( b = 3 \), \( c = 3 \)
7. \( \log_2 8 = \log_2 2^3 \) - \( a = 3 \), \( b = 2 \), \( c = 2 \)
8. \( \log_5 125 = \log_5 5^3 \) - \( a = 3 \), \( b = 5 \), \( c = 5 \)

Write each expression as a single logarithm.
9. \( \log_b a + \log_b c = \log_b (ac) \)
10. \( \log_b a - \log_b c = \log_b \left(\frac{a}{c}\right) \)
11. \( \log_b a^2 = 2 \log_b a \)
12. \( \log_b \frac{a}{c} = \log_b a - \log_b c \)

7.4 Think About a Plan
Properties of Logarithms

**Construction**
The foreman of a construction team puts up a sound barrier that reduces the intensity of the noise by 50%. By how many decibels is the noise reduced? Use the formula \( I = 10 \log \left(\frac{E}{E_0}\right) \) to measure loudness. (Hint: Find the difference between the expression for loudness for intensity \( I \) and the expression for loudness for intensity \( I_0 \) without.)

**Know**
1. You can represent the intensity of the original noise by \( I_0 \).
2. You can represent the intensity of the reduced noise by \( I \).

**Need**
4. To solve the problem I need to find the difference between the expression for the original noise intensity and the expression for the reduced noise intensity.

**Plan**
5. What is an expression for the loudness of the original construction noise? \( 10 \log \left(\frac{E}{E_0}\right) \)
6. What is an expression for the loudness of the reduced construction noise? \( 10 \log \left(\frac{E}{E_0}\right) \)
7. Use your expressions to find the difference between the loudness of the original construction noise and the loudness of the reduced construction noise.
8. The sound barrier reduced the loudness by \( \frac{3}{2} \).
7.4 Practice Form G
Properties of Logarithms

Write each expression as a single logarithm.

1. \( \log_5 4 + \log_5 3 \)
2. \( \log_7 (9 \cdot 3) \)
3. \( \log_2 4 - \log_2 2 - \log_2 8 \)
4. \( \log_3 x - 3 \log_3 y \)
5. \( \log_9 3 - \log_9 4 + \log_9 x \)
6. \( \log_7 7 - 3 \log_7 3 \)
7. \( \log_3 y - 3 \log_3 x \)
8. \( \frac{1}{2} \log_5 x + \frac{1}{3} \log_5 y - \log_5 z \)
9. \( \log_6 4 + 2 \log_6 2 \)
10. \( \log_8 2 + 2 \log_8 2 \)
11. \( \log_2 x + \log_2 y - \log_2 z \)
12. \( \log_3 x + \log_3 y - 3 \log_3 z \)
13. \( \log_3 90 - 2 \log_3 5 \)

Expand each logarithm. Simplify if possible.

14. \( \log_5 (5^2 \cdot 5^3) \)
15. \( \log_2 (2^3 \cdot 2^4) \)
16. \( \log_7 (7^2 \cdot 7^3) \)
17. \( \log_3 (3^2 \cdot 3^5) \)
18. \( \log_4 (4^2 \cdot 4^3) \)
19. \( \log_6 (6^2 \cdot 6^3) \)
20. \( \log_5 (5^2 \cdot 5^4) \)
21. \( \log_2 (2^2 \cdot 2^3) \)
22. \( \log_7 (7^2 \cdot 7^4) \)
23. \( \log_3 (3^2 \cdot 3^5) \)
24. \( \log_4 (4^2 \cdot 4^3) \)

25. \( \log_2 \left( \frac{1}{8} \right) \)
26. \( \log_3 \left( \frac{1}{9} \right) \)
27. \( \log_5 \left( \frac{1}{125} \right) \)
28. \( \log_4 \left( \frac{1}{64} \right) \)
29. \( \log_6 \left( \frac{1}{36} \right) \)
30. \( \log_7 \left( \frac{1}{122 \, 504} \right) \)

31. \( \log_5 3.5 \)
32. \( \log_3 5.655 \)
33. \( \log_5 8.268 \)
34. \( \log_5 7.109 \)
35. \( \log_3 1.785 \)
36. \( \log_2 1.113 \)
37. \( \log_2 10 \)
38. \( \log_7 11 \)
39. \( \log_5 10 \)
40. \( \log_2 5 \)

41. \( \log_7 (25 \cdot 3) \)
42. \( \log_6 (9 \cdot 4) \)
43. \( \log_5 (25 \cdot 5) \)
44. \( \log_3 (75 \cdot 3) \)
45. \( \log_4 (16 \cdot 4) \)
46. \( \log_8 (81 \cdot 8) \)
47. \( \log_5 (125 \cdot 5) \)
48. \( \log_2 (32 \cdot 2) \)
49. \( \log_7 (7 \cdot 7) \)
50. \( \log_3 (9 \cdot 3) \)
51. \( \log_4 (16 \cdot 4) \)
52. \( \log_8 (64 \cdot 8) \)
53. \( \log_9 (81 \cdot 9) \)
54. \( \log_5 (75 \cdot 75) \)
55. \( \log_5 (25 \cdot 25) \)
56. \( \log_2 (32 \cdot 32) \)

57. \( \log_8 (64 \cdot 64) \)
58. \( \log_9 (81 \cdot 81) \)
59. \( \log_4 (16 \cdot 16) \)

The concentration of hydrogen ions in a batch of homemade ketchup is \( 1 \times 10^{-4} \). What is the pH level of the ketchup? 4
Multiple Choice

1. Which statement correctly demonstrates the Power Property of Logarithms? D
   - \( \log_b a^m = m \log_b a \)
   - \( \log_b a + \log_b c = \log_b (ac) \)
   - \( \log_b a - \log_b c = \log_b \left(\frac{a}{c}\right) \)

2. Which expression is the correct expansion of \( \log_b (3^3 \cdot 2^2) \)? G
   - \( \log_b 3^3 + \log_b 2^2 \)
   - \( 3 \log_b 3 + 2 \log_b 2 \)
   - \( \log_b 3 + \log_b 2 \)

3. Which expression is equivalent to \( \log_5 125x \)? C
   - \( \log_5 125 + \log_5 x \)
   - \( 3 \log_5 5 + \log_5 x \)
   - \( \log_5 5 \cdot 3 \log_5 x \)

4. Which statement correctly expresses \( 4 \log_3 x \) as a single logarithm? F
   - \( \log_3 x^4 \)
   - \( \log_3 (x^4 + y^4) \)
   - \( \log_3 (4x + 7y) \)

Short Response

5. The pH of pure water is 7 less than the pH of the sodium hydroxide solution. Which statement correctly expresses 4 \( \log_3 x \) as a single logarithm? F
   - \( \log_3 x^4 \)
   - \( \log_3 (x^4 + y^4) \)
   - \( \log_3 (4x + 7y) \)

7-4

Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are exponents.

| Property | Formula | Why?
|----------|---------|--------
| Product Property | \( \log_c (mn) = \log_c m + \log_c n \) | When you multiply two powers, you add the exponents. Example: \( 2^3 \cdot 2^4 = 2^{3+4} \)
| Quotient Property | \( \log_c \left(\frac{m}{n}\right) = \log_c m - \log_c n \) | When you divide two powers, you subtract the exponents. Example: \( \frac{x^5}{x^2} = x^{5-2} \)
| Power Property | \( \log_c m^n = n \log_c m \) | When you take a power to a power, you multiply the exponents. Example: \( (x^2)^3 = x^{2\cdot3} \)

| Problem | What is \( 2 \log_6 6 - \log_2 27 \) written as a single logarithm? | To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.
|----------|--------|--------
| Solution | \( 2 \log_6 6 - \log_2 27 = \log_6 6^2 - \log_2 27 \) | Use the Power Property twice.
| Solution | \( = \log_6 36 - \log_2 27 \) | Group two of the logarithms. Use the order of operations.
| Solution | \( = \log_2 36 - \log_2 27 \) | Quotient Property
| Solution | \( = \log_2 \left(\frac{36}{27}\right) \) | Product Property
| Solution | \( = \log_2 2 \) | Simplify

As a single logarithm, \( 2 \log_6 6 - \log_2 27 = \log_2 2 \).

7-4

Enrichment

Properties of Logarithms

Scotsman John Napier and Joost Burgi from Switzerland are credited for being the first to introduce the concept of a logarithm. While the logarithms they described were quite different than the ones we use today, both men used logarithms to simplify mathematical calculations. Arithmetic operations of addition and subtraction are relatively easy to compute, but without the modern calculation, multiplication and division of powers and roots can be time-consuming. Before calculations, logarithms were used to simplify expressions in an addition or subtraction problem. The logarithm values could be found in extensive tables and the calculations were much easier completed.

1. Consider the relation \( y = \log_5 x \) as an example. Without using a calculator, determine the value for \( y \) when \( x = 10 \).

2. While the calculations in Exercise 1 are impossible, they are certainly time-consuming, and, with men involved, are inaccurate. If you take the \( \log \) of both sides, the equation becomes \( \log y = \frac{\log x}{\log 5} \)

3. The properties of logarithms to rewrite \( \log y = \frac{\log x}{\log 5} \).

4. Use the properties of logarithms to rewrite \( y = \log_2 (x+5) \).

5. Use the properties of logarithms to rewrite \( \log_2 (3x+2) \).

ANSWERS

7. Write each logarithmic expression as a single logarithm.
   - \( \log_2 13 + \log_2 10 \)
   - \( \log_3 (x^2 \cdot y^3) \)
   - \( \log_2 6 - \log_2 3 \)
   - \( \log_4 \left(\frac{a}{b}\right) \)
   - \( \log_5 (x^2 + y^3) \)
   - \( \log_2 (x^2 \cdot y^3) \)
   - \( \log_3 (x^2 \cdot y^3) \)

8. Write each logarithmic expression as a quotient of two logarithms.
   - \( \log_2 100 \)
   - \( \log_3 (x^2 \cdot y^3) \)
   - \( \log_2 (x^2 \cdot y^3) \)
   - \( \log_3 (x^2 \cdot y^3) \)
   - \( \log_2 (x^2 \cdot y^3) \)

9. Write each logarithmic expression as a quotient of two logarithms.
   - \( \log_2 100 \)
   - \( \log_3 (x^2 \cdot y^3) \)
   - \( \log_2 (x^2 \cdot y^3) \)
   - \( \log_3 (x^2 \cdot y^3) \)
   - \( \log_2 (x^2 \cdot y^3) \)
Exponential and Logarithmic Equations

Think Cards

Write the equation in exponential form.

First, apply the Product Property of logarithms.

Simplify to a quadratic equation in standard form.

Write the equation in exponential form.

Apply the Product Property of logarithms.

Factor the trinomial.

Second, apply the Product Property of logarithms.

Then, simplify to a quadratic equation in standard form.

Finally, solve for x. Check for extraneous solutions.

Solve for x. Check for extraneous solutions.

Exponential and Logarithmic Equations

Solve each equation.

Simplify to a quadratic equation in standard form.

Simplify to a quadratic equation in standard form.

Solve for x. Check for extraneous solutions.

Solve each equation.

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7-5 Practice (continued) Form K
Exponential and Logarithmic Equations

Convert from Logarithmic Form to Exponential Form to solve each equation.

<table>
<thead>
<tr>
<th>Logarithmic Form</th>
<th>Exponential Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log_{10} 1000$</td>
<td>$10^3 = 1000$</td>
</tr>
<tr>
<td>$\log_{10} 0.001$</td>
<td>$10^{-3} = 0.001$</td>
</tr>
<tr>
<td>$\log_{10} 1$</td>
<td>$10^0 = 1$</td>
</tr>
</tbody>
</table>

Use the properties of logarithms to solve each equation.

11. $ \log_{10} (2x + 4) = 3$
   $2x + 4 = 10^3$
   $2x = 1000$
   $x = 500$

12. $ \log_{10} (2 - 3) = 2$
   $2 - 3 = 10^2$
   $x = 100$

13. $ \log_{10} (x - 6) = 2$
   $x - 6 = 10^2$
   $x = 100 + 6$
   $x = 106$

Enrichment

Exponential and Logarithmic Equations

When solving logarithmic equations, you primarily use the Product Property, Quotient Property, and Power Property to simplify the equation. Here is an interesting, lesser-known property of logarithms to explore.

1. Determine the value of each pair of expressions.
   - $\log_b x + \log_b y$
   - $\log_b (xy)$
   - $\log_b x - \log_b y$
   - $\log_b \left(\frac{x}{y}\right)$
   - $\log_b x^p$
   - $p \log_b x$

2. How are the values of each pair of expressions related?
   - Answers may vary. Sample: When the base and argument are switched, the expressions are reciprocals.

3. This reciprocal property states that $\log_b \frac{1}{x} = -\log_b x$. To prove this property, assume $r = \log_b x$ and $s = \log_b a$. Rewrite each of these equations in exponential form. $x^r = b^s$.

4. Next, use one equation to substitute an equivalent expression in for $a$. What is your new equation? $y^q = b^s$

5. Use the laws of exponents to simplify. $y^q = b^s$

6. Because the bases are the same, what equation can you write for the exponents? $q = s$

7. What must be true about $x$ and $y$ if the product equals $1$? $x$ and $y$ must be reciprocals.

8. Use this new property to solve the equation $\log_b x + \log_b y = \log_b 1$. $x + y = 1$
7-5 Reteaching
Exponential and Logarithmic Equations

Use logarithms to solve exponential equations.

Problem:
What is the solution of \( 2^{x-1} - 4 = 0 \)?

- First isolate the term that has the variable in the exponent. Begin by subtracting 4 from each side.
- \( 2^{x-1} = 4 \)

Multiply each side by \( \log_2 \),

\( \log_2(2^{x-1}) = \log_2(4) \)

Because the variable is in the exponent, use logarithms. Take \( \log_2 \) of each side because it is the base of the exponent.

\( x - 1 = 2 \)

Add 1 to each side.

\( x = 3 \)

Divide each side by 2.

Exercise:
Solve each equation. Round the answer to the nearest hundredth.

1. \( 2^{x} - 2 = 1 \)
2. \( 3^{x} = 9 \)
3. \( 5^{x} = 25 \)

7-6 Think About a Plan
Natural Logarithms

Archaeology
A fossil bone contains 25% of its original carbon-14. What is the approximate age of the bone?

Understanding the Problem:
1. What is the amount of carbon-14 remaining in the fossil bone? 25% of the original amount.
2. If it is the amount of carbon-14 originally in an object and \( t \) is the object's age in years, what equation gives the amount of carbon-14 in the object? \( y = a \cdot e^{-0.00012t} \)
3. What is the problem asking you to determine? the approximate age of the fossil bone.

Planning the Solution:
4. What number should you substitute for \( y \) in the equation above? 0.25a
5. Write an equation you can use to determine the approximate age of the bone. \( 0.25a = a \cdot e^{-0.00012t} \)

Getting an Answer:
6. How can logarithms help you solve your equation? After dividing each side by \( a \), take the natural log of both sides of the equation to eliminate \( e \).

7. Solve your equation to find the approximate age of the bone.
\( \ln(0.25) = \ln(a \cdot e^{-0.00012t}) \)
\( \ln(0.25) = \ln(a) - 0.00012t \)
\( -0.00012t = \ln(0.25) - \ln(a) \)
\( t = \frac{\ln(0.25) - \ln(a)}{-0.00012} = 15,522 \) years
Write each expression as a single natural logarithm.

1. \( \ln 6 - \ln 8 - \ln 2 \)
2. \( 3 \ln 3 - \ln 9 + \ln 240 \)
3. \( \ln 4 - \ln 0.0098 \)
4. \( \ln 2 - 3 \ln \frac{3}{2} \)
5. \( \ln 9 + \ln 3 - \ln 9 \)
6. \( 4 \ln x - 2 \frac{3}{4} \ln x^2 \)
7. \( \ln 8 + \ln x - \ln 2x \)
8. \( 3 \ln x - b - 2 \ln \left( \frac{b}{x^2} \right) \)

Solve each equation. Check your answers. Round your answer to the nearest hundredth.

9. \( \ln x = -2.89 \)
10. \( \ln (3x - 4) = 12.37 \)
11. \( 3 \ln x - 4 = 7 \)
12. \( \ln (x - 4) = -6 - 1.58 \)
13. \( -7 = \ln (4x + 1) \)
14. \( 3 - 4 \ln (x - 1) = -0.12 \)
15. \( 8x = x - 3 \)
16. \( 2 \ln x + 2 = 3.22 \)
17. \( \ln x + 4 = 2.18 \)
18. \( \ln (x - 1) = -1.04 \)
19. \( 5 \ln x^3 = 12 \)
20. \( 3 \ln x^2 = 17.12 \)
21. \( 10 \ln (x - 2) = 8 - 0.83 \)
22. \( 5 \ln (3x - 4) = 30.15 \)
23. \( 2 \ln x^2 - 2 = 15.73 \)
24. \( 7 \ln (2x - 5) = 8 - 0.31 \)
25. \( \ln (3x + 4) = 5 - 40.14 \)
26. \( 2 \ln x^2 + 5 = 151.68 \)
27. \( 2 \ln (x - 1)^2 = 4.61 \)

Simplify each expression.

28. \( \ln^2 2 + 2.71 \)
29. \( \ln 6 - 10.82 \)
30. \( e^x - 3 = 5 \ln x \)
31. \( \ln x^6 - 7 \)
32. \( \ln 3 + 0.69 \)
33. \( 0.69 e^{2x} + 4.5 \)
34. \( y = 1 \)
35. \( x^2 = 22.73 \)
36. \( x^2 = 2.65 \)
37. \( 0.2x^{0.3} = 0.12 \)
38. \( x = 25 \)
39. \( 4x = 25 \)
40. \( \ln x^6 = 20.9 \)
41. \( 3x = 21 \)
42. \( x^2 = 21 \)
43. \( x^2 + 1 = 1 \)
44. \( e^x = 11 \)
45. \( x = 0 \)
46. \( y = 2 \)
47. \( x^2 = 3 \)
48. \( y = 21 \)
49. \( 2x = 3 \)
50. \( 2x + 1 = 2 \)
51. \( 3x = 53.6 \)
52. \( 6x = 17.5 \)

Use the following formula to complete Exercises 7 and 8.

Maximum Velocity of a Rocket:

\[ v = -0.0039t + x \ln R \]

- \( v \) = maximum velocity
- \( t \) = rocket’s firing time
- \( c \) = velocity of exhaust
- \( R \) = mass ratio of the rocket

7. A rocket has a mass ratio of 24. The rocket’s exhaust has a velocity of 2.4 km/s. The rocket’s firing time is 32 seconds. Approximately what is the rocket’s maximum velocity? Round to the nearest tenth. \( v \approx 3.5 \) km/s

8. The rocket in Exercise 7 was changed to prepare it for a new mission. The new mass ratio is 26, and the new exhaust velocity is 2.3 km/s. Will these changes increase or decrease the rocket’s maximum velocity? What is the difference between the maximum velocities? The changes will decrease the maximum velocity. The difference is approximately 0.1 km/s.

Solve each equation. Use natural logarithms to solve each equation. Round your answer to the nearest hundredth.

7. The rocket in Exercise 7 was changed to prepare it for a new mission. The new mass ratio is 26, and the new exhaust velocity is 2.3 km/s. Will these changes increase or decrease the rocket’s maximum velocity? What is the difference between the maximum velocities? The changes will decrease the maximum velocity. The difference is approximately 0.1 km/s.
7-6 Standardized Test Prep
Natural logarithms

Multiple Choice
For Exercises 1–4, choose the correct letter. Do not use a calculator.
1. What is \( \ln 5 \) as a written as a single natural logarithm? D
   \( x \) \( \ln 7.5 \) \( \ln 27 \) \( \ln \left( \frac{3}{2} \right) \) \( \ln 62.5 \)
2. What is the solution of \( 2^x = 10 \)? G
   \( x = \ln 10 \) \( 2 \times \ln 10 \) \( 2 \pm \sqrt{10} \) \( 2 \pm \sqrt{10} \)
3. What is the solution of \( \ln (x - 2) = 6 \)? C
   \( x = 2 \ln 6 + 2 \) \( 2 - e^6 \) \( 2 \pm e^6 \) \( 2 \pm e^6 \)
4. What is the solution of \( 2^x + 3 = 87 \)? G
   \( x = 3 \ln 5 - 1 \) \( x = 2 \ln 5 - 2 \) \( x = 2 \ln 4 \) \( x = \frac{1}{2} \ln 5 - 1 \)

Short Response
5. The maximum velocity of a rocket is \( v = -0.0098u + v \ln R \). The rocket fires for 2 seconds and the velocity of the exhaust is \( 2 \text{km/s} \). The ratio of the mass of the rocket filled with fuel to the mass of the rocket without fuel is R.
   a. What is the velocity of a spacecraft whose booster rocket has a mass ratio of 10, an exhaust velocity of 5.2 km/s, and a firing time of 40 s?
   b. Can this rocket attain a stable orbit 300 km above Earth? Explain in words or show work for how you determined your answer.

Exercises
4. What is the solution of \( 4^{2x} = 2 \)?
   a. 8.48 km/s
   b. 7.7 km/s
   Therefore, the spacecraft can attain a stable orbit 300 km above Earth.

5. Use a calculator to fill in the blanks in the following chart to four decimal places. Then compare your results with the value of \( \ln x \) obtained directly.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \ln x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.0953</td>
</tr>
<tr>
<td>1.5</td>
<td>0.4055</td>
</tr>
<tr>
<td>2.2</td>
<td>0.8051</td>
</tr>
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<td>3.7</td>
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</tr>
<tr>
<td>5.9</td>
<td>2.2231</td>
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<tr>
<td>10.0</td>
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<td>13.2</td>
<td>2.5753</td>
</tr>
<tr>
<td>20.0</td>
<td>2.9957</td>
</tr>
</tbody>
</table>

7-6 Enrichment
Natural logarithms

Calculating Natural Logarithms
You can compute natural logarithms with
\[ \ln x = \ln (x^{1/2}) = \frac{1}{2} \ln x \]
where the pattern continues forever. Notice that there are actually three patterns involved as the terms progress.

1. What is the pattern of the sequence?
   The signs alternate as the terms progress beginning with a positive term.

2. What is the pattern of the exponents?
   The exponents increase by 1 as the terms progress beginning with 1.

3. What is the pattern of the denominators?
   The denominators increase by 1 as the terms progress beginning with 1.

4. For \( x = 1 \), what is the sum of the series? 0

5. Use a calculator to fill in the blanks in the following chart to four decimal places. Then compare your results with the value of \( \ln x \) obtained directly.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \ln x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.10000</td>
<td>0.09530</td>
</tr>
<tr>
<td>1.50000</td>
<td>0.50000</td>
</tr>
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<td>2.20000</td>
<td>0.80510</td>
</tr>
<tr>
<td>3.70000</td>
<td>1.51140</td>
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<td>5.90000</td>
<td>2.22320</td>
</tr>
<tr>
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<td>2.30260</td>
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<td>13.20000</td>
<td>2.57530</td>
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<tr>
<td>20.00000</td>
<td>2.99570</td>
</tr>
<tr>
<td>10.18200</td>
<td>10.18200</td>
</tr>
<tr>
<td>14.18200</td>
<td>14.18200</td>
</tr>
</tbody>
</table>

7-6 Reaching [continued]
Natural logarithms

The natural logarithmic function is a logarithm with base e, an irrational number.
You can write the natural logarithmic function as \( y = \log_e x \), but you usually write it as \( y = \ln x \).

To solve a natural logarithmic equation:

1. If the term containing the variable is an exponential expression, rewrite the equation in logarithmic form.

2. If terms containing the variable is a logarithmic expression, rewrite the equation in exponential form.

Problem
What is the solution of \( 4^{2x} = 2 \)?

Step 1 Isolate the term containing the variable on one side of the equation.
\[ 4^{2x} = 2 \]
\[ 2x = \ln 2 \]

Step 2 Take the natural logarithm of each side of the equation.
\[ \ln (2x) = \ln 2 \]
\[ 2x = \ln 2 \]

Definition of natural logarithm.

Step 3 Solve for the variable.
\[ x = \frac{1}{2} \ln 2 \]

Use a calculator.

Step 4 Check the solution.
(continued)
13. **Do you UNDERSTAND?**
   a. What is the student’s mistake?
   b. Is $x = 5$ or $x = -5$ a valid solution?

14. **Writing:** Explain how you would graph an exponential function.

15. **Reasoning:** Find the value of $\log_2 64$ without using a calculator. Justify your answer.
   a. Answers may vary. Sample: Write $\log_2 64$ as an equation $\log_b a = x$; rewrite in exponential form $b^x = a$; rewrite each side using a common base $2^x = 2^4$; solve for $x = 4$.
   b. Answers may vary. Sample: Write a common base that has powers equaling 16 and 64, $2^4 = 16$ and $2^6 = 64$; using the common base, apply the Change of Base Formula and simplify; $\log_2 64 = \log_2 2^6 = \frac{\log 2^6}{\log 2} = \frac{6 \log 2}{\log 2} = \frac{6}{1} = 6$.

16. **Writing:** Explain how you could find the value of $\log_6 24$ without using a calculator.
   a. Answers may vary. Sample: Find a common base that has powers equaling 6 and 24. $6^2 = 36$ and $24^2 = 576$. Using the common base, apply the Change of Base Formula and simplify; $\log_6 24 = \log_6 576 = \frac{\log 576}{\log 6} = \frac{2.778}{0.778} = 3.58$.

17. **Vocabulary:** What is the base of the natural logarithmic function $y = \ln x$?

18. **Reasoning:** Explain how you could find the value of $\log_5 125$ without using a calculator.
   a. Answers may vary. Sample: Find a common base that has powers equaling 5 and 125. $5^3 = 125$. Using the common base, apply the Change of Base Formula and simplify; $\log_5 125 = \log_5 5^3 = \frac{\log 5^3}{\log 5} = \frac{3 \log 5}{\log 5} = 3$.

---

**Chapter 7 Quiz 2**

---

**Do you know HOW?**

1. Solve each equation. Round your answer to the nearest hundredth.
   a. $\ln x = 6$
   b. $2 \ln x = 4$
   c. $\ln (x - 4) = 2$

2. Write each expression as a single logarithm.
   a. $\log_4 3 + \log_4 6$
   b. $2 \log_3 x + \log_4 6$

---

**Do you UNDERSTAND?**

13. **Vocabulary:** What is an exponential equation?

14. **Open-Ended:** Write log 27 as a sum or difference of two logarithms. Simplify if possible.

15. **Vocabulary:** What is the base of the natural logarithmic function $y = \ln x$?

16. **Reasoning:** Explain how you could find the value of $\log_4 32$ without using a calculator.

---

**Chapter 7 Chapter Test**

---

**Do you know HOW?**

1. Solve each equation.
   a. $x - 3 = -2$
   b. $\log_2 (x - 3) = 2$
   c. $\log_2 (x + 1) = 4$

2. Write each expression as a single logarithm.
   a. $\log_4 3 + \log_4 6$
   b. $2 \log_3 x + \log_4 6$

---

**Do you UNDERSTAND?**

13. **Vocabulary:** What is an exponential equation?

14. **Open-Ended:** Write log 27 as a sum or difference of two logarithms. Simplify if possible.

15. **Vocabulary:** What is the base of the natural logarithmic function $y = \ln x$?

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---

**Chapter 7 Chapter Test**

---

**Do you know HOW?**

1. Solve each equation. Round your answer to the nearest hundredth.
   a. $\ln x = 6$
   b. $2 \ln x = 4$
   c. $\ln (x - 4) = 2$

2. Write each expression as a single logarithm.
   a. $\log_4 3 + \log_4 6$
   b. $2 \log_3 x + \log_4 6$
Evaluate each logarithm.

1. \( \log_3 27 \) 2. \( \log_4 64 \) 3. \( \log_{10} 1000 \)

Do you understand?

5. In March, a town was hit by an earthquake with a magnitude of 7.3 on the Richter scale. The same town was hit by an earthquake with a magnitude of 4 in June. How many times more intense was the first earthquake?

6. Fran has $2400 invested in a savings account. The account pays 4% annual interest. How much money will be in the account after 8 years? Use the formula $A = P \cdot (1 + r)^t$

7. A scientist is calculating the pH levels of vinegar and dish detergent. He uses the formula pH = \(-\log[H^+]\) [molarity]. For vinegar, the pH is 2.7 and for dish detergent the pH is 10. What is the difference of the pH levels for vinegar and dish detergent?

Do you know HOW?

10. Error Analysis A math test contained the equation \( 25^{2x} = 625 \). The student answered \( 2x = 3 \). What error did the student make? Sample answer: The student used the equation \( 25^2 = 625 \) to find his answer of \( 2 \).

Chapter 7 Quiz 2

Lessons 7–4 through 7–5

Do you know HOW?

Write each expression as a single logarithm.

1. \( \log_2 4 + \log_2 8 \) 2. \( \log_2 12 - \log_2 3 \) 3. \( 3 \log_2 x + 5 \log_2 y \)

Expand each logarithm.

4. \( \log_3 \frac{27}{9} \) 5. \( \log_2 (x + 4) \) 6. \( \log_5 \frac{25}{5} \)

Use the Change of Base Formula to evaluate each expression.

7. \( \log_{10} 25 \) 8. \( \log_5 25 \) 9. \( \log_3 27 \)

Solve each equation.

10. \( 2^{2x} = 9 \) 11. \( \log (x - 2) = 3 \) 12. \( \log 4 + \log x = 2 \)

Do you understand?

13. The population of fish in a lake is decreasing. There are currently 24,000 fish in the lake. The population is decreasing by 5% each year. In how many years will there be 60% of the current number of fish in the lake?

14. A scientist is calculating the pH levels of vinegar and dish detergent. He uses the formula pH = \(-\log[H^+]\) [molarity]. For vinegar, the pH is 2.7 and for dish detergent the pH is 10. What is the difference of the pH levels for vinegar and dish detergent?

Chapter 7 Test

Lessons 7–1 through 7–3

Do you know HOW?

Without graphing, determine whether the function represents exponential growth or decay.

1. \( y = (0.8)^x \) - decay 2. \( y = 10(1.03)^x \) - growth 3. \( y = 5(0.25)^x \) - decay

Identify the parent of each function. Then graph each function as a transformation of its parent function.

4. \( y = -0.25 \cdot 4^{-x} \) - growth 5. \( y = 2^{-x^2} - 2 \) - growth

Evaluate each logarithm.

6. \( \log_2 125 \) 7. \( \log_{10} 8 \) 8. \( \log_3 27 \)

Do you understand?

5. In March, a town was hit by an earthquake with a magnitude of 7.3 on the Richter scale. The same town was hit by an earthquake with a magnitude of 4 in June. How many times more intense was the first earthquake?

6. Fran has $2400 invested in a savings account. The account pays 4% annual interest. How much money will be in the account after 8 years? Use the formula $A = P \cdot (1 + r)^t$

7. A scientist is calculating the pH levels of vinegar and dish detergent. He uses the formula pH = \(-\log[H^+]\) [molarity]. For vinegar, the pH is 2.7 and for dish detergent the pH is 10. What is the difference of the pH levels for vinegar and dish detergent?

Do you know HOW?

10. Error Analysis A math test contained the equation \( 25^{2x} = 625 \). A student used the equation \( 25^2 = 625 \) to find his answer of \( 2 \). What error did the student make? Sample answer: The student used a common base to solve the equation, but 25 and 25 do not have a common base. The correct answer is \( x = 3 \).

11. A fireworks display in Downsville last night. The first burst during the display made a sound with an intensity of 3.14 W/m². How many decibels louder was the last burst?

12. The population of a bee colony is growing at a rate of 2.3% each year. There are currently 3400 bees in the colony. At this rate, in how many years will there be 10,200 bees in the colony? Use the formula $P = P_0 \cdot (1 + r)^t$
Chapter 7 Performance Tasks

Give complete answers.

Task 1
a. Write an exponential function that could model the information in the graph.
b. Describe a business, scientific (not mathematical), or economic situation in which you would apply one or more of the properties. Show how you would use them. Check students’ work.

c. How will the graph and situation change when you change the base of this exponential function?
d. Describe the conditions under which the function represents a growth or decay situation. Check students’ work.

Task 2
a. Write a detailed explanation of what the graph might represent. Student includes solutions that are provided.
b. Describe a business, scientific (not mathematical), or economic situation in which you would apply one or more of the properties. Show how you would use them. Check students’ work.
c. How will the graph and situation change when you change the base of this exponential function?
d. Describe the conditions under which the function represents a growth or decay situation. Check students’ work.

Chapter 7 Performance Tasks [continued]

Task 3
a. State the three Properties of Logarithms: log MN = log M + log N, \( \log (xy) = \log x + \log y \), and \( \log (\frac{x}{y}) = \log x - \log y \). Check students’ work.
b. Give an example for using each property. Check students’ work.
c. Describe a real-life situation in which you would apply one or more of the properties. Show how you would use them. Check students’ work.
[...]

Extended Response
14. You and your friend are saving for college. You have $50 and are adding $10 each week to your savings. Your friend has $20 and is adding $20 each week to his savings.
a. What systems of equations would be a good model for this situation? Let x be the number of weeks and y be the number of dollars saved.
b. Graph the system of equations.
c. Sketch the graph to determine when you and your friend will have the same amount of money saved. Explain your answer.

Chapter 7 Cumulative Review

Multiple Choice
For Exercises 1–6, choose the correct letter.

1. Which of the following systems is dependent?
   a. \( y = 2x - 1 \)
   b. \( y = 2x + 1 \)
   c. \( y = 2x - 1 \)
   d. \( y = 2x + 1 \)

2. The electric current I in an amperes (A) of a circuit is given by the formula \( I = \frac{v}{R} \). Find the current when \( v = 20 \) volts and \( R = 4 \) ohms.
   a. 5 A
   b. 10 A
   c. 15 A
   d. 20 A

3. Which of the following functions represents exponential growth?
   a. \( y = 50(0.50)^x \)
   b. \( y = 50(1.50)^x \)
   c. \( y = 50(0.60)^x \)
   d. \( y = 50(0.80)^x \)

4. Which is the next number in the pattern 2, 4, 8, 16, ...?
   a. 32
   b. 24
   c. 30
   d. 24

5. Use the Change of Base Formula to rewrite \( \log_6 18 \) using common logarithms.
   a. \( \log 18 \)
   b. \( \log 18 \)
   c. \( \log 18 \)
   d. \( \log 18 \)

6. Which of the following is the vertex of the function \( y = -2x^2 + 6x - 4 \)?
   a. (0, -16)
   b. (0, 6)
   c. (1, 0)
   d. (2, 0)

7. What is the simplified form of the expression \( 2x^2 + 3y^2 + z^2 \)?
   a. \( 2x^2 + 3y^2 + z^2 \)
   b. \( 2x^2 + 3y^2 + z^2 \)
   c. \( 2x^2 + 3y^2 + z^2 \)
   d. \( 2x^2 + 3y^2 + z^2 \)

8. Which of the following represents the polynomial \( 4x^2 + 3y^2 - 2z^2 \) in standard form?
   a. \( 4x^2 + 3y^2 - 2z^2 \)
   b. \( 4x^2 + 3y^2 - 2z^2 \)
   c. \( 4x^2 + 3y^2 - 2z^2 \)
   d. \( 4x^2 + 3y^2 - 2z^2 \)

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Chapter 7 Project Teacher Notes: Crime Time

About the Project
The Chapter Project gives students the opportunity to explore how mathematics is used in forensic science. Students investigate the nature of the victim of a suspect of a crime by using Newton’s Law of Cooling. They verify formulas by using natural logarithms to solve equations. Students use these equations to calculate information that will help them prove or disprove the suspect’s alibi. They present their conclusions in an investigative report.

Introducing the Project
- Ask students if they are familiar with the field of forensic science.
- Discuss information that can help identify a crime suspect.
- Have students review direct variation.

Activity 1: Investigating
Students consider information that can help prove or disprove a crime suspect’s alibi.

Activity 2: Writing
Students use direct variation to write an equation for Newton’s Law of Cooling.

Activity 3: Solving
Students use natural logarithms to solve equations.

Activity 4: Calculating
Students calculate how long ago a car’s engine was running at its normal operating temperature.

Finishing the Project
You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results. Ask students to review their project work and update their folders.

- Have students review any information needed in order to be able to prove or disprove the alibi. Have them summarize how they solved the formulas used in the project.
- Ask groups to share their insights that resulted from completing the project, such as any shortcuts they found for solving formulas or for researching information.

Chapter 7 Project: Crime Time

Beginning the Chapter Project
Forensic science is the application of science to law. A forensic scientist investigates evidence that can help place a suspect at the scene of a crime. Each piece of forensic evidence may help build a successful case against a suspect in a court of law.

In this project, you will examine how mathematics can be used by forensic scientists to help indict people suspected of criminal actions.

List of Materials
- Calculator

Activities
Activity 1: Investigating
A major crime occurred at approximately 10:15 p.m. Shortly thereafter, a certain makes and models car, along with its license plate number, were recorded by a witness who reportedly saw the car speeding in the vicinity of the crime scene. Police and forensic scientists immediately went to the home of the person to whom the car was registered, where they found the car parked in the driveway. The investigation team noted that it had taken them 30 minutes to travel to the suspect’s home from the crime scene. They also noted that the engine of the car was still warm when they arrived at 11:00 p.m. When confronted, the suspect claimed to have been at a friend’s house earlier that night and had returned home at about 11:05 p.m. The suspect’s friend confirmed the alibi, and reported seeing the suspect at home at about 11:00 p.m., and having a conversation with the suspect until about 11:20 p.m. What information can investigators use to help prove or disprove the suspect’s alibi? Explain. Check students’ work.

Activity 2: Writing
As police questioned the suspect, the team of forensic scientists began to take temperature measurements of the vehicle’s engine coolant. Knowing that this information could help determine how long it had been since the engine had been running. In order to determine this, forensic scientists use Newton’s Law of Cooling. The law states that the change in temperature of an object over time t, denoted T, varies directly with the difference between the temperature of the object T and the temperature of the surrounding environment, or ambient temperature, A. Letting –k represent the constant of variation for a positive value of k, write an equation to represent Newton’s Law of Cooling.

T = A – Ae(−kt)

Activity 3: Solving
Using calculate and the equation you wrote in Activity 2, the team determined that the temperature of an object after time t is given by the formula T(t) = A + (To − A)e(−kt), where To represents the temperature at time t = 0. Verify that the constant of variation can be found by using the formula k = ln(T0 − T)/t0, where T0 represents the temperature at some later time t0. Then, verify that T(t0) is the time the engine stopped running at a normal operating temperature, given by Tn = A + (To − A)e(−ktn), where Tn represents the normal operating temperature of the engine.

Activity 4: Calculating
When the team began investigating at 11:00 p.m., the initial temperature of the engine’s coolant was 97°F. By 11:10 p.m., the temperature had dropped to 95°F. The ambient temperature was 78°F. The normal running temperature of the car’s engine, based on its make and model, is about 200°F. Let t = 0 represent 11:00 p.m.

- Using this information and the equations from Activity 1, first determine the value of k. Then determine the value of T0. What do these values tell you? Explain.
- If the suspect’s car had been turned off at 10:00p.m., what would the temperature reading have been at 11:00 p.m.? A = 95°F, t = 60 min; the engine stopped running at its normal operating temperature about 12 min before 11:00 p.m., or at about 10:48 p.m. The suspect could have been in the area around the time the crime occurred. About 97°F

Finishing the Project
The activities should help you complete your project. Prepare a presentation in the form of an investigative report, providing evidence that will help clear or indict the suspect. Present your report to your classmates. Then discuss the data that helped you reach your conclusion(s).

Reflect and Revise
Ask a small group of classmates to review your report. Is your information presented clearly? Have you sufficiently explained why the evidence helps to prove or disprove the suspect’s alibi? Make any necessary changes and improvements before presenting your project to the class.

Extending the Project
Discuss other evidence that an investigator could use to place a suspect at a crime scene. Research how mathematics can be applied to other crime scene evidence. If possible, interview a forensic scientist.

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### 8-1 ELL Support

**Inverse Variation**

Choose the expression from the list that best matches each sentence.

<table>
<thead>
<tr>
<th>Combined Variation</th>
<th>Constant of Variation</th>
<th>Inverse Variation</th>
<th>Joint Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( xy = k )</td>
<td>( y = \frac{k}{x} )</td>
<td>( y = \frac{k}{x} )</td>
<td>( y = \frac{k}{x} )</td>
</tr>
</tbody>
</table>

1. equations of the form \( xy = k \)
2. when one quantity varies with respect to two or more quantities
3. when one quantity varies directly with two or more quantities
4. the product of two variables in an inverse variation

### Multiple Choice

9. Which function would be used to model the relationship "x and y vary inversely"? C
   - \( y = \frac{k}{x} \)
   - \( y = \frac{x}{k} \)
   - \( y = x \cdot k \)
   - \( y = \frac{k}{x} \)

10. Which function would be used to model the relationship "x varies jointly with y and z"? G
    - \( y = kx \)
    - \( y = kxz \)
    - \( y = kx \cdot z \)
    - \( y = \frac{k}{xz} \)

### Planning the Solution

3. What does it mean that the data can be modeled by an inverse variation?
   - The product of each \( P \) – \( V \) pair is the same constant.

4. How can you estimate the constant of the inverse variation?
   - Find \( PV \) for each row of the data. It should be approximately the same for each row.

5. What is the constant of the inverse variation? About 14,000

6. Write an equation that you can use to find \( P \) when \( V = 62 \).
   - \( 62P = 14,000 \)
   - \( P = \frac{14,000}{62} \)

### Getting an Answer

7. Solve your equation.
   - \( 62P = 14,000 \)
   - \( P = \frac{14,000}{62} \approx 226 \)

8. What is an estimate for \( P \) when \( V = 62 \)? About 226

### 8-1 Practice Form G

Is the relationship between the values in each table a direct variation, an inverse variation, or neither? Write equations to model the direct and inverse variations.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

- \( x \) and \( y \) vary inversely. Write a function that models each inverse variation. Graph the function and find \( y \) when \( x = 10 \).

- \( x = 10 \) when \( y = 0.3 \)
- \( y = \frac{3}{10} \) or \( 0.3 \)

10. A student club decides to raise money by selling hats with the school mascot on them. The table below shows how many hats they can expect to sell based on how much they charge per hat in dollars.

<table>
<thead>
<tr>
<th>Price per Hat (p)</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hats Sold (h)</td>
<td>72</td>
<td>60</td>
<td>50</td>
<td>45</td>
<td>40</td>
</tr>
</tbody>
</table>

- What is a function that models this data? \( h = 3600 + 60p \)
- How many hats can they expect to sell if they charge $7.50 per hat? 30

11. The minimum number of carpet rolls \( n \) needed to carpet a house varies directly as the house’s square footage \( A \) and inversely with the square footage \( x \) in one roll. It takes a minimum of two 1200-ft\(^2\) carpet rolls to cover 2300-ft\(^2\) of floors. What is the minimum number of 1200-ft\(^2\) carpet rolls you would need to cover 2500-ft\(^2\) of floors? Round your answer up to the nearest whole roll. 2.5

### 8-1 Practice (continued) Form G

12. On Earth, the mass \( m \) of an object varies directly with the object’s potential energy \( E \) and inversely with its height above the Earth’s surface \( h \). What is an equation for the mass of an object on Earth? \( m = \frac{k}{h} \) where \( k \) is a constant of variation.

Each ordered pair is from an inverse variation. Find the constant of variation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-13</th>
<th>-22</th>
<th>-1</th>
<th>-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>268</td>
<td>52</td>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

- Each pair of values is from an inverse variation. Find the missing value.
- \( z \) varies inversely with \( y \) and \( x \). When \( x = 7 \) and \( y = 2 \), \( z = 6 \)
- \( z = 6 \) when \( y = 2 \)

- Each pair of values is from an inverse variation. Find the missing value.
- \( x \) varies directly with \( y \) and inversely with the cube of \( z \). When \( x = 8 \) and \( y = 2 \), \( z = 2 \)
- \( x = \frac{64}{z^3} \)

- A load of gravel contains 240 ft\(^3\) of gravel. The area \( A \) that the gravel will cover is inversely proportional to the depth \( d \) of the gravel’s spread.
  - Write a model for the relationship between the area and depth for one load of gravel. \( A = \frac{C}{d} \)
  - \( C \) is the constant of variation.

- A designer plans a playground with gravel 6 in. deep over the entire play area. If the play area is a rectangle 40 ft wide and 24 ft long, how many loads of gravel will be needed? 2 loads.
8-1 Practice Form K

Inverse Variation

Is the relationship between the values in each table a direct variation, an inverse variation, or neither? Write an equation to model the direct and inverse variations.

1. \( \begin{array}{cc}
x & y \\
0.1 & 1 \\
0.2 & 2 \\
0.4 & 5 \\
0.6 & 18 \\
\end{array} \)

inverse variation; \( y = \frac{100}{x} \)

direct variation; \( y = 10x \)

neither

2. \( \begin{array}{cc}
\text{Time (h)} & \text{Speed (mph)} \\
\frac{1}{5} & 60 \\
\frac{1}{4} & 80 \\
\frac{1}{3} & 100 \\
\frac{1}{2} & 150 \\
\end{array} \)

\( \frac{1}{t} = \frac{50}{s} \)

3. \( \begin{array}{cc}
x & y \\
-4 & 5 \\
0 & 1 \\
3 & 0.5 \\
6 & 0.166 \\
\end{array} \)

inverse variation; \( y = \frac{5}{x} \)

direct variation; \( y = 5x \)

neither

Suppose that \( x \) and \( y \) vary inversely. Write a function that models each inverse variation. Graph the function and find \( y \) when \( x = 10 \).

4. \( x = -2 \) when \( y = -4 \)

5. \( x = -9 \) when \( y = -1 \)

6. \( x = 1.5 \) when \( y = 10 \)

7. Suppose the table at the right shows the time \( t \) it takes to drive home when you travel at various average speeds \( s \).

a. Write a function that models the relationship between the speed and the time it takes to drive home: \( s = \frac{50}{t} \)

b. At what speed would you need to drive to get home in 50 min or \( \frac{1}{6} \) h?

8-1 Enrichment Form K

Inverse Variation

Each situation below can be modeled by a direct variation, inverse variation, joint variation, or combined variation equation. Decide which model to use and explain why.

1. The circumference \( C \) of a circle is about 3.14 times the diameter \( d \).

   - Direct variation; \( C = 3.14d \)

2. The number of cans that develop in a patient’s teeth depends on the total number of minutes spent brushing.

   - Inverse variation; \( C = \frac{360}{T} \)

3. The time it takes to build a bridge depends on the number of workers.

   - Inverse variation; \( T = \frac{40}{w} \)

4. The number of minutes it will take to solve a problem set depends on the number of problems and the number of people working on the set problem.

   - Combined variation; \( T = \frac{40}{w} + 2 \)

5. The current \( I \) in an electrical circuit decreases as the resistance \( R \) increases.

   - Inverse variation; \( I = \frac{k}{R} \)

6. Charles’s Law states the volume \( V \) of an enclosed gas at a constant pressure will increase as the absolute temperature \( T \) increases.

   - Direct variation; \( V = kT \)

7. Boyle’s Law states the volume \( V \) of an enclosed gas at a constant temperature is related to the pressure. The pressure of 3.41 L of neon gas is 0.026 atmospheres (atm). At the same temperature, the pressure of 0.22 L of neon gas is 1.42 atm.

   - Inverse variation; \( V = \frac{k}{P} \)

   - As the pressure increases, the volume decreases. This suggests an inverse variation.
The chart below shows how to decide whether a relationship between two variables is a direct variation, inverse variation, or neither.

**Does the table represent a direct variation, inverse variation, or neither?**

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

As the value of x increases, the value of y decreases, so the table values in the inverse variation model: \( y = \frac{k}{x} \).

**Exercises**

Does the table represent a direct variation, inverse variation, or neither?

1. direct variation
2. inverse variation

---

**8-2 ELL Support**

For Exercises 1–4, draw a line from each word in Column A to its definition in Column B.

**The Reciprocal Function Family**

1. reciprocal
2. branches
3. reciprocal function
4. reflection of the reciprocal function
5. transformation.

How is the graph of each function a transformation of the parent graph of \( f(x) = \frac{1}{x} \)?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tbody>
<tr>
<td>f(x) = \frac{1}{x}</td>
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<td>A</td>
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<td>C</td>
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**8-1 Reteaching (continued)**

To solve problems involving inverse variation, you need to solve for the constant of variation \( k \) before you can find an answer.

**Problem**

The time it takes to complete a task varies inversely as the number of people \( p \) working. If it takes 4 h for 12 people to paint the exterior of a house, how long does it take for 3 people to do the same job?

\[ \frac{t}{p} = k \]

1. \( t = \frac{4}{12} \)
2. Multiply both sides by 12 to solve for \( k \), the constant of variation.
3. \( 4 \times 12 = k \)
4. \( k = 48 \)
5. \( t = \frac{48}{3} \)
6. \( t = 16 \)

It takes 3 people 16 h to paint the exterior of the house.

**Exercises**

3. The time \( t \) needed to complete a task varies inversely as the number of people \( p \). It takes 5 h for seven men to install a new roof. How long does it take ten men to complete the job? 3.5 h

4. The time \( t \) needed to drive a certain distance varies inversely as the speed \( r \). It takes 7.5 h at 40 mi/h to drive a certain distance. How long does it take to drive the same distance at 60 mi/h? 5 h

5. The cost of each item bought is inversely proportional to the number of items \( m \) when spending a fixed amount. When 40 items are bought, each costs $3.46. Find the number of items when each costs $2.16. about 28 items

6. The length \( L \) of a rectangle varies inversely as the width \( W \). The length of a rectangle is 9 cm when the width is 6 cm. Determine the length if the width is 8 cm. 4.25 cm

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**ANSWERS**

Prentice Hall Algebra 2 • Teaching Resources

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Given value of

Sketch the asymptotes and the graph of each function. Identify the domain and range.

Graph each function. Identify the x- and y-intercepts and asymptotes of the graph. Also, state the domain and range of the function.

Write an equation for the translation of \( y = \frac{1}{x} \) that has the given asymptotes.

Sketch the asymptotes and the graph of each function. Identify the domain and range.

Write each equation in the form \( y = \frac{1}{x} \).

Sketch the graph of each function.

13. The length of a pipe is a function of its pitch and its length. The pitch of a pipe is inversely proportional to its length. The inverse variation is modeled by the equation \( y = \frac{1}{x} \). Find the length of a pipe with a pitch of 220 ft.

Find the length required to produce a pitch of 220 ft, 2.25 ft.

Write each equation in the form \( y = \frac{1}{x} \).

Sketch the graph of each function.

17. Sketch the graph of each function.

19. The junior class is buying keepsakes for Class Night. The price of each keepsake is inversely proportional to the number of keepsakes bought. The keepsake company also offers a 10% discount in addition to the class’s color. The equation \( p = \frac{324}{n} \) gives the price for each one.

a. If the class buys 36 keepsakes, what is the price for each one? $7.20
b. If the class pays $11.36 for 10 keepsakes, how many can they get, including the free keepsake? 36

c. If the class buys 400 keepsakes, what is the price for each one? $4.30

Write each pair of functions. Find the approximate point(s) of intersection.

Graph each pair of functions. Find the approximate point(s) of intersection.

If the class buys 85 keepsakes, what is the price for each one? $5.50

Write each equation in the form \( y = \frac{1}{x} \).

Sketch the graph of each function.

16. Writing Explain the difference between what happens to the parent function \( y = \frac{1}{x} \) when \( |a| > 1 \) and what happens to the parent function \( y = \frac{1}{x} \) when \( 0 < |a| < 1 \).

When \( |a| > 1 \), the parent function is stretched by the factor of \( a \). When \( 0 < |a| < 1 \), the parent function is compressed by the factor of \( a \).

17. Suppose you are the teacher and receive a gift of a weekend package at your favorite spa. The package costs $250. Let \( c \) equal the cost each student needs to pay and \( n \) equal the number of students.

a. If there are 22 students, how much will each student need to pay? $11.36
b. Using the information, how many total students (including those from other classes) need to contribute to the teacher’s gift? If a student wants to pay more than $37, 36 students

c. Reasoning Did you need to round your answer up or down? Explain: up, because this is a real-world situation, you can only have whole numbers of students.
8-2 Enrichment
The Reciprocal Function Family
Understanding Horizontal Asymptotes
The line $y = \frac{1}{x}$ is a horizontal asymptote for the graph of the function $y = \frac{4}{x}$. By using long division, you can rewrite this function in the form $\frac{q(x)}{x}$, where $q(x)$ is a polynomial of degree less than the degree of the denominator, or in the form $\frac{r}{x^n}$, where $n$ is a positive integer.

Examine what happens to the remainder divided by the divisor and the value of $y$ as the value of $x$ gets larger. Fill in the following table to four decimal places.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{4}{x}$</th>
<th>$y = \frac{1}{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>2.0000</td>
<td>0.5000</td>
</tr>
<tr>
<td>3</td>
<td>1.3333</td>
<td>0.3333</td>
</tr>
<tr>
<td>4</td>
<td>1.0000</td>
<td>0.2500</td>
</tr>
<tr>
<td>5</td>
<td>0.8000</td>
<td>0.2000</td>
</tr>
</tbody>
</table>

Note that as $x$ gets larger, both the remainder and the value of $y$ get smaller. Although the value of $y$ is always greater than $\frac{1}{x}$, it gets closer to $\frac{1}{x}$ as a gets larger. As $x$ gets infinitely large, $y$ approaches $\frac{1}{x}$ from above. Write this as $\lim_{x \to \infty} \frac{4}{x} = \frac{1}{x}$.

Examine what happens as $x$ gets smaller. Fill in the following table to four decimal places.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{4}{x}$</th>
<th>$y = \frac{1}{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-4.0000</td>
<td>-1.0000</td>
</tr>
<tr>
<td>-2</td>
<td>-2.0000</td>
<td>-0.5000</td>
</tr>
<tr>
<td>-3</td>
<td>-1.3333</td>
<td>-0.3333</td>
</tr>
<tr>
<td>-4</td>
<td>-1.0000</td>
<td>-0.2500</td>
</tr>
<tr>
<td>-5</td>
<td>-0.8000</td>
<td>-0.2000</td>
</tr>
</tbody>
</table>

Here the value of $y$ is always less than $\frac{1}{x}$ but it gets closer to $\frac{1}{x}$ as a gets smaller (more negative). Write this as $\lim_{x \to -\infty} \frac{4}{x} = \frac{1}{x}$.

In both cases, $y$ approaches $\frac{1}{x}$, the horizontal asymptote.

8-2 Retraching
The Reciprocal Function Family
A Reciprocal Function in General Form

The general form is $y = \frac{1}{x^2} + k$, where $a > 0$ and $x > 0$.

The graph of this equation has a horizontal asymptote at $y = k$ and a vertical asymptote at $x = 0$.

Problem
What is the graph of the inverse variation function $y = \frac{4}{x^2}$?

Step 1
Rewrite in general form and identify $a$, $k$, and $b$.

$y = \frac{4}{x^2}$

$a = \frac{1}{4}$, $k = 0$, $b = 0$.

Step 2
Identify and graph the horizontal and vertical asymptotes.

Horizontal asymptote: $y = 0$ Vertical asymptote: $x = 0$.

Step 3
Make a table of values for $y = \frac{4}{x^2}$. Plot the points and then connect the points in each quadrant to make a curve.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0.05</td>
</tr>
<tr>
<td>-1</td>
<td>0.10</td>
</tr>
</tbody>
</table>
| 0   | -
| 1   | 0.10|
| 2   | 0.05|

Exercises
Graph each function. Include the asymptotes.

4. $y = \frac{1}{x^2}$

5. $y = -\frac{1}{x^2}$

6. $y = \frac{1}{x^2} + 2$
### Concept List

<table>
<thead>
<tr>
<th>continuous</th>
<th>discontinuous</th>
<th>factors</th>
<th>point of discontinuity</th>
<th>horizontal asymptote</th>
<th>non-removable discontinuity</th>
<th>removable discontinuity</th>
<th>vertical asymptote</th>
</tr>
</thead>
<tbody>
<tr>
<td>rational function</td>
<td>non-removable</td>
<td>polynomial functions</td>
<td>as a hole in the graph</td>
<td>increases in absolute value</td>
<td>Sketch the graph of each rational function.</td>
<td>Determine whether the discontinuities are removable or non-removable.</td>
<td>Sketch the graph of each rational function.</td>
</tr>
</tbody>
</table>

1. The line that a graph approaches as it increases in absolute value
   vertical asymptote
2. In the denominator, these reveal the points of discontinuity: factors
3. This type of discontinuity appears as a hole in the graph, removable discontinuity
4. This type of graph has no jumps, breaks, or holes: continuous
5. A function that you can write in the form $f(x) = \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomial functions: rational function
6. A graph that has a corner point or a vertical asymptote: discontinuous
7. This type of discontinuity appears as a vertical asymptote on the graph: non-removable discontinuity
8. The graph of $f(x)$ is not continuous at this point: point of discontinuity
9. The line that a graph approaches as $x$ increases in absolute value: horizontal asymptote

### Practice

#### 8-3 Rational Functions and Their Graphs

**Form G**

Find the domain, points of discontinuity, and $x$- and $y$-intercepts of each rational function. Determine whether the discontinuities are removable or non-removable.

1. $y = \frac{x - 4}{x^2 - 9}$
   - all real num except $-3, 3$
   - removable
2. $y = \frac{x}{x^2 - 1}$
   - all real num except $-1, 1$
   - nonremovable
3. $y = \frac{x - 1}{x^2 - 3x}$
   - all real num except $0$
   - removable
4. $y = \frac{x}{x^2 - 1}$
   - all real num; none; $0$
   - nonremovable

Find the vertical asymptotes and holes for the graph of each rational function.

5. $y = \frac{x^2 - 1}{x - 1}$
   - vertical asymptotes at $x = 1$ and $x = 2$
   - hole at $x = 0$
6. $y = \frac{x^2 - 1}{x}$
   - vertical asymptote at $x = 0$
   - hole at $x = 2$
7. $y = \frac{x^2 - 1}{x - 1}$
   - vertical asymptote at $x = 1$
   - hole at $x = 0$
8. $y = \frac{x^2 - 1}{x - 2}$
   - vertical asymptote at $x = 0$
   - hole at $x = -1$
9. $y = \frac{x^2 - 1}{x - 2}$
   - vertical asymptote at $x = 1$
   - hole at $x = 0$
10. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - no vertical asymptotes or holes
11. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - vertical asymptote at $x = 0$
    - hole at $x = -1$

Find the horizontal asymptote of the graph of each rational function.

12. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
13. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
14. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
15. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
16. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
17. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
18. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
19. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$
20. $y = \frac{x^2 - 1}{x^2 - 2x}$
    - horizontal asymptote $y = 1$

#### 8-3 Think About a Plan

**Rational Functions and Their Graphs**

**Grades**

A student earns an 82% on her first test. How many consecutive 100% test scores does she need to bring her average up to 95%? Assume that each test has equal impact on the average grade.

**Understanding the Problem**

1. One test score is $\frac{82}{1}$.
2. The average of all the test scores is $\frac{95}{1}$.
3. What is the problem asking you to determine?
   - the number of tests with scores of 100% the student needs to have an average of 95%

**Planning the Solution**

4. Let $x$ be the number of 100% test scores. Write an expression for the total number of test scores.
   - $x + 1$
5. Write an expression for the sum of the test scores.
   - $100x + 82$
6. How can you model the student’s average as a rational function?
   - $A = \frac{100x + 82}{x + 1}$

**Getting an Answer**

7. How can a graph help you answer this question?
   - Answers may vary. Sample: I can graph the rational function and the function $y = 95$ at the same time. The intersection is the solution.
8. What does a fractional answer tell you? Explain.
   - I would have to round a fractional answer up to the nearest whole number, because the number of test scores must be a whole number.
9. How many consecutive 100% test scores does the student need to bring her average up to 95%?
   - 3

### Practice [continued]

#### 8-3 Rational Functions and Their Graphs

21. How many milliliters of a 15% sugar solution must be added to 100 mL of a 3% sugar solution to form a 12% sugar solution? 50 mL
22. A soccer player has made 3 of his last 24 shots on goal, or 12.5%. How many more consecutive goals does he need to raise his shots-on-goal average to at least 20%? 3

**Error Analysis**

A student listed the asymptotes of the function $y = \frac{1}{x}$ as shown at the right. Explain the student’s error(s). What are the correct asymptotes?

The horizontal asymptote should be $y = 0$, because the degree of the numerator is less than the degree of the denominator. The zeros of the denominator are $x = 0$ and $x = -1$, so there should also be vertical asymptotes at $x = 0$ and $x = -1$.

**Sketch the graph of each rational function.**

24. $y = \frac{1}{x}$
25. $y = \frac{1}{x + 1}$
26. $y = \frac{1}{x + 2}$
27. $y = \frac{1}{x + 3}$

28. You start a business by word-processing papers for other students. You spend $5.00 for a computer system and office furniture. You figure additional costs at $4\$ per page.
   - Write a rational function modeling the total average cost per page.
   - Graph the function $y = \frac{5 + 4x}{x}$, where $x = \text{number of pages}$.
   - What is the total average cost per page if you type 100 pages?
   - $\frac{5}{100} + \frac{400}{100} = \frac{405}{100} = 4.05$
   - a. Write a rational function modeling the total average cost per page.
   - b. What is the total average cost per page if you type 100 pages?
   - c. How many pages must you type to bring your total average cost to less than $1.50 per page?
   - d. How many consecutive 100% test scores does the student need to bring her average up to 95%?
   - e. How can you model the student’s average as a rational function?
   - f. How can a graph help you answer this question?
   - g. What does a fractional answer tell you? Explain.
   - h. How many consecutive 100% test scores does the student need to bring her average up to 95%?
Find the domain, points of discontinuity, and x- and y-intercepts of each rational function. Determine whether the discontinuities are removable or non-removable.

To start, factor the numerator and denominator, if possible.

1. \( y = \frac{x^2 - 2x}{x - 2} \)  
   - Domain: all real numbers except 2; non-removable point of discontinuity at \( x = 2 \); a removable point of discontinuity at \( x = -1 \); a slant asymptote: none; y-intercept: \( y = 0 \)

2. \( y = \frac{x^2 - 4x + 3}{x - 1} \)  
   - Domain: all real numbers except 1; non-removable point of discontinuity at \( x = -1 \); a removable point of discontinuity at \( x = 2 \); a slant asymptote: none; hole at \( x = 1 \)

Find the vertical asymptotes and holes for the graph of each rational function.

4. \( y = \frac{x^2 - x - 2}{x - 2} \)  
   - Vertical asymptote: \( x = -1 \)  
   - Vertical asymptote: none; hole at \( x = 2 \)

5. \( y = \frac{x^2 - 4}{x - 2} \)  
   - Vertical asymptote: \( x = -1 \)  
   - Vertical asymptote: \( x = 2 \)

Find the horizontal asymptote of the graph of each rational function.

7. \( y = \frac{x^2 + 3}{x + 1} \)  
   - Degree of numerator: 2  
   - Degree of denominator: 1  
   - Vertical asymptote: \( x = -1 \)  
   - Horizontal asymptote: \( y = \frac{1}{1} \)  
   - No horizontal asymptote

8. \( y = \frac{x^2}{x^2 + 1} \)  
   - Degree of numerator: 2  
   - Degree of denominator: 2  
   - Horizontal asymptote: \( y = \frac{1}{1} \)  
   - No horizontal asymptote

Sketch the graph of each rational function.

10. \( y = \frac{x^2 - 4}{x^2 - 2x + 1} \)  
11. \( y = \frac{x^3 - 2x + 5}{x^2 - 2x + 1} \)  
12. \( y = \frac{x^2 - 1}{x^2 + 1} \)

Multiple Choice

1. What function has a graph with a removable discontinuity at \( x = 2 \)?
   - A: \( y = \frac{x^2 - 4}{x - 2} \)
   - B: \( y = \frac{x^2 - 4}{x + 2} \)
   - C: \( y = \frac{x^2 - 4}{x^2 - 2x + 4} \)
   - D: \( y = \frac{x^2 - 4}{x^2 + 2} \)

2. What is the vertical asymptote of the graph of \( y = \frac{x - 3}{x^2 - 2x + 4} \)?
   - A: \( x = -3 \)
   - B: \( x = -2 \)
   - C: \( x = 0 \)
   - D: \( x = 3 \)

3. What best describes the horizontal asymptote(ies) if any, of the graph of \( y = \frac{x^2 + 2x}{x + 4} \)?
   - A: \( y = 0 \)
   - B: \( y = 1 \)
   - C: The graph has no horizontal asymptote.

4. Which rational function has a graph with vertical asymptotes at \( x = 2 \) and \( x = -4 \) and a horizontal asymptote at \( y = 0 \)?
   - A: \( y = \frac{2x + 4}{x^2 - 2x} \)
   - B: \( y = \frac{2x + 4}{x^2 + 2x} \)
   - C: \( y = \frac{2x + 4}{x^2 - 2x} \)

Short Response

5. How many milliliters of 0.3% sugar solution must you add to 75 mL of 4% sugar solution to get a 0.5% sugar solution? Show your work.
   2. \% \( \frac{0.3\%}{0.5\%} \) = 0.6 \( x = 0.6 \times 75 \) \( x = 45 \) mL

6. What is the equation of the graph of a rational function with a vertical asymptote at \( x = 2 \) and a horizontal asymptote at \( y = 0 \)?
   - \( y = \frac{2}{x} \)

7. Which rational function has a graph with vertical asymptotes at \( x = 2 \), \( x = -4 \), and a horizontal asymptote at \( y = 0 \)?
   - A: \( y = \frac{x^2 - 2x + 4}{x^2 - 4} \)
   - B: \( y = \frac{x^2 - 2x + 4}{x^2 + 4} \)
   - C: \( y = \frac{x^2 - 2x + 4}{x^2 - 4} \)
   - D: \( y = \frac{x^2 - 2x + 4}{x^2 + 4} \)

8. Which points are on the graph of a rational function with a vertical asymptote at \( x = 2 \) and a horizontal asymptote at \( y = 0 \)?
   - A: (2, 0)
   - B: (0, 2)
   - C: (2, 2)

9. What is the equation of the graph of a rational function with a vertical asymptote at \( x = 2 \) and a horizontal asymptote at \( y = 0 \)?
   - A: \( y = \frac{1}{x^2 - 4} \)
   - B: \( y = \frac{1}{x^2 + 4} \)
   - C: \( y = \frac{1}{x^2 - 4} \)
   - D: \( y = \frac{1}{x^2 + 4} \)

10. How many discs must be produced to bring the average cost under $8? Show your work.
   - Let \( C(x) \) be the total cost function.
   - Let \( x \) be the number of discs produced.
   - Average cost: \( \frac{C(x)}{x} \)
   - Development cost: $124,000
   - First 100 discs: samples
   - If 2000 discs are produced, the average cost is $10.80 per disc. If 16,104 discs are produced, the average cost is $7.68 per disc.
   - The average cost is under $8 when \( x > 16,104 \) discs are produced.

11. What is the equation of the graph of a rational function with a vertical asymptote at \( x = 2 \) and a horizontal asymptote at \( y = 0 \)?
   - A: \( y = \frac{1}{x^2 - 4} \)
   - B: \( y = \frac{1}{x^2 + 4} \)
   - C: \( y = \frac{1}{x^2 - 4} \)
   - D: \( y = \frac{1}{x^2 + 4} \)

12. What is the equation of the graph of a rational function with a vertical asymptote at \( x = 2 \) and a horizontal asymptote at \( y = 0 \)?
   - A: \( y = \frac{1}{x^2 - 4} \)
   - B: \( y = \frac{1}{x^2 + 4} \)
   - C: \( y = \frac{1}{x^2 - 4} \)
   - D: \( y = \frac{1}{x^2 + 4} \)
8-3 Reteaching (continued)  
Rational Functions and Their Graphs

Before you try to sketch the graph of a rational function, get an idea of its general shape by identifying the graph’s holes, asymptotes, and intercepts.

**Problem:**
What is the graph of the rational function \(y = \frac{4x}{x^2 - 2x - 3} = \frac{4x}{(x - 3)(x + 1)}\)?

**Step 1** Identify any holes or asymptotes.
No holes; vertical asymptote at \(x = -1\), horizontal asymptote at \(y = \frac{4}{3}\).

**Step 2** Identify any x- and y-intercepts.
Intercepts occur when \(x = 0\): \(y = 0\) when \(x = 0\).
No y-intercepts occur when \(x = -1\), \(y = \frac{4}{3}\) when \(x = -1\).

**Step 3** Sketch the asymptotes and intercepts.

**Step 4** Make a table of values, plot the points, and sketch the graph.

**Exercises**
Graph each function. Include the asymptotes.

4. \(y = \frac{3}{x - 2}\)
5. \(y = \frac{x^2 - 4}{x^2 - 4x + 4}\)

8-4 Think About a Plan  
Rational Expressions

**Manufacturing** A toy company is considering a cube or sphere-shaped container for packaging a new product. The height of the cube would equal the diameter of the sphere. Compare the ratios of the volumes to the surface areas of the containers. Which packaging will be more efficient? For a sphere, \(V = \frac{4}{3}\pi r^3\).

**Understanding the Problem**
1. Let \(x\) be the height of the cube. What are expressions for the cube’s volume and surface area?
Volume: \(V = x^3\)
Surface area: \(A = 6x^2\)

2. Let \(x\) be the diameter of the sphere. What are expressions for the sphere’s volume and surface area?
Volume: \(V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{x}{2}\right)^3\)
Surface area: \(A = 4\pi r^2 = 4\pi \left(\frac{x}{2}\right)^2\)

3. What is the problem asking you to do?
Find the ratios of volume to surface area for the cube and the sphere. Compare the ratios to decide which is a more efficient package.

**Planning the Solution**
4. Write an expression for the ratio of the cube’s volume to its surface area. Simplify your expression.
\[\frac{V}{A} = \frac{x^3}{6x^2} = \frac{x}{6}\]

5. Write an expression for the ratio of the sphere’s volume to its surface area. Simplify your expression.
\[\frac{V}{A} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}\]

**Getting an Answer**
6. Compare the ratios of the volumes to the surface areas of the containers. Which packaging will be more efficient?
The ratios are the same. The package shapes are equally efficient.
8-4 Practice  
Rational Expressions

Simplify each rational expression. State any restrictions on the variables.

1. \( \frac{2x - 4}{xy} \)  
   \[ x \neq 0, y \neq 0 \]
2. \( \frac{x^2 - 4x + 4}{x^2 - 9} \)  
   \[ x \neq 3, x \neq 2 \]
3. \( \frac{x^2 - 6x + 9}{x^2 - 9} \)  
   \[ x \neq 3, x \neq 2 \]

Multiply. State any restrictions on the variables.

4. \( \frac{2x}{x - 3} \cdot \frac{x^2 - 9}{x - 2} \)  
   \[ x \neq 3, x \neq 2, 3 \]
5. \( \frac{x - 2}{x^2 - 4} \cdot \frac{x + 2}{x - 2} \)  
   \[ x \neq 2, x \neq -2 \]
6. \( \frac{2x}{x^2 - 1} \cdot \frac{x^2 - 1}{x} \)  
   \[ x \neq 0, x \neq \pm 1 \]

Divide. State any restrictions on the variables.

7. \( \frac{2x^2 - 3x - 2}{x^2 - 4} \div \frac{x - 2}{x + 2} \)  
   \[ x \neq 2, x \neq 3, x \neq -3 \]
8. \( \frac{x^2 - 4}{x - 2} \div \frac{x + 2}{x - 2} \)  
   \[ x \neq 2, y \neq -2 \]
9. \( \frac{2x^2 + x - 6}{x^2 - 4} \div \frac{2x - 3}{x - 2} \)  
   \[ x \neq 2, x \neq 3, x \neq -3 \]

8-4 Practice  
Rational Expressions

10. Your school wants to build a courtyard surrounded by a low brick wall. It wants the maximum area for a given amount of brick wall. The courtyard can be either a circle or an equilateral triangle. Which shape provides the most efficient use of brick wall: a circle or an equilateral triangle?  
   Circle

11. \( \frac{x^2 - 4}{x + 2} \)  
   \[ x \neq -2 \]

12. \( \frac{x^2 - 4}{x + 2} \)  
   \[ x \neq -2 \]

13. \( \frac{x^2 - 4}{x + 2} \)  
   \[ x \neq -2 \]

14. Writing How can you tell whether a rational expression is in simplest form?  
   Include an example with your explanation.
   Answer may vary. Sample: A rational expression is in simplest form when the numerator and denominator have no common factor.

15. The width of a rectangle is given by the expression \( \frac{1}{x^2} \). The area can be represented by \( \frac{1}{x^2} \). What is the length of the rectangle?
   \( x = 1 \)

14. Multiple Choice Which expression can be simplified to \( \frac{1}{x^2} \)?
   D. \( \frac{x^2}{x^4} \)

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Multiple Choice
For Exercises 1–4, choose the correct letter.

1. Which expression equals \( \frac{2y^2 + 4y}{2y^2 - 3y} \)?
   \[ \text{A}\ \frac{2y + 2}{2y - 3} \quad \text{B}\ \frac{y}{2y - 3} \quad \text{C}\ \frac{2y}{y - 3} \]  
   \[ \text{D}\ \frac{y}{y + 3} \quad \text{E}\ \frac{y}{2y + 3} \]  
   Answer: \text{C} \quad \text{Explanation: Factor out the greatest common factor from the numerator and denominator, then divide.}

2. Which expression equals \( \frac{x^2 - 16}{x^2 - 9} \)?
   \[ \text{A}\ \frac{(x - 4)(x + 4)}{(x - 3)(x + 3)} \quad \text{B}\ \frac{x - 4}{x + 3} \quad \text{C}\ \frac{x^2 - 4}{x^2 - 9} \]  
   \[ \text{D}\ \frac{x^2}{x^2 - 3} \quad \text{E}\ \frac{x}{x - 3} \]  
   Answer: \text{A} \quad \text{Explanation: Factor each expression and cancel common factors.}

3. Which expression equals \( \frac{5x^2 - 20x}{10x^2 - 30x} \)?
   \[ \text{A}\ \frac{5x}{10x} \quad \text{B}\ \frac{x}{x - 3} \quad \text{C}\ \frac{1}{2} \]  
   \[ \text{D}\ \frac{x}{x} \quad \text{E}\ \frac{5}{10} \]  
   Answer: \text{B} \quad \text{Explanation: Factor out the greatest common factor from the numerator and denominator, then divide.}

4. What is the area of the triangle shown on the right?
   \[ \text{A}\ \frac{1}{2} \quad \text{B}\ 2 \quad \text{C}\ 3 \]  
   \[ \text{D}\ 4 \quad \text{E}\ 5 \]  
   Answer: \text{B} \quad \text{Explanation: Use the formula for the area of a triangle, \( \frac{1}{2} \times 	ext{base} \times 	ext{height} \).}

Short Response

5. What is the question \( x = \frac{a}{b} \) if \( x = \frac{c}{d} \) expressed in simplest form? State any restrictions on the variable. Show your work.
   \[ a, b, c, d \quad \text{are not equal to zero} \]  
   Answer: \( \frac{ac}{bd} \)

6. Evaluate \( f(x) = \frac{2x^2 - 3x - 2}{x - 2} \) for \( x = 3 \).
   \[ f(3) = \frac{2(3)^2 - 3(3) - 2}{3 - 2} = \frac{18 - 9 - 2}{1} = \frac{7}{1} = 7 \]  
   Answer: 7

7. For the function \( f(x) = \frac{1}{x} + 2 \), the function is undefined at what value(s)?
   \[ x \neq 0 \]  
   Answer: 0

8. For rational functions, vertical asymptotes are lines located at the value(s) of \( x \) that make the denominator 0. Use your graph to check if your vertical asymptotes are correct.
   \[ \text{From the graph of the function in Exercise 4, you saw that even though } x = 2 \text{ made the denominator 0, it was not an asymptote.} \]  
   Answer: See graph

9. Reciprocal of 7
   \[ \frac{1}{7} \]  
   Answer: \( \frac{1}{7} \)

10. Use a graphing calculator to graph the rational function \( f(x) = \frac{x^2 - 1}{x - 1} \).
    \[ \text{Use your graph to check if your vertical asymptotes are correct.} \]  
    Answer: See graph

Rational Expressions
For rational functions, vertical asymptotes are lines located at the value(s) of \( x \) that make the denominator 0. From the graph of the function in Exercise 4, you saw that even though \( x = 2 \) made the denominator 0, it was not an asymptote. Values such as this are called a removable discontinuity. The value is not part of the domain, yet it is not an asymptote. Factor the rational expression \( \frac{x^2 - 1}{x - 1} \) and use your results to explain why \( x = 2 \) is not an asymptote.

Exercises

1. Simplify each rational expression. State any restrictions on the variable.
   \[ \frac{x^2 - 4}{x^2 - 1} = \frac{(x - 2)(x + 2)}{(x - 1)(x + 1)} , \quad x \neq \pm 1 \]  
   \[ \frac{x^2 - 4}{x^2 - 1} = \frac{(x - 2)(x + 2)}{(x - 1)(x + 1)} , \quad x \neq \pm 1 \]  
   \[ \frac{x^2 - 4}{x^2 - 1} = \frac{(x - 2)(x + 2)}{(x - 1)(x + 1)} , \quad x \neq \pm 1 \]  
   \[ \frac{x^2 - 4}{x^2 - 1} = \frac{(x - 2)(x + 2)}{(x - 1)(x + 1)} , \quad x \neq \pm 1 \]  
   \[ \frac{x^2 - 4}{x^2 - 1} = \frac{(x - 2)(x + 2)}{(x - 1)(x + 1)} , \quad x \neq \pm 1 \]  
   Answer: See graph

2. Substitute the given value into the simplified expression and evaluate if possible.
   \[ f(2) = \frac{2x^2 - 3x - 2}{x - 2} \]  
   \[ f(2) = \frac{2(2)^2 - 3(2) - 2}{2 - 2} = \frac{8 - 6 - 2}{0} \]  
   \[ f(2) = \frac{0}{0} \]  
   Answer: See graph

3. Evaluate \( f(x) = \frac{2x^2 - 3x - 2}{x - 2} \) for \( x = 3 \).
   \[ f(3) = \frac{2(3)^2 - 3(3) - 2}{3 - 2} = \frac{18 - 9 - 2}{1} = \frac{7}{1} = 7 \]  
   Answer: 7

4. For the function \( f(x) = \frac{1}{x} + 2 \), the function is undefined at what value(s)?
   \[ x \neq 0 \]  
   Answer: 0

5. Use a graphing calculator to graph the rational function \( f(x) = \frac{x^2 - 1}{x - 1} \).
    \[ \text{Use your graph to check if your vertical asymptotes are correct.} \]  
    Answer: See graph
8-5 Adding and Subtracting Rational Expressions

The column on the left shows the steps used to subtract two rational expressions. Use the columns on the left to answer each question in the column on the right.

**Problem**: What is the difference of the two rational expressions in simplest form? State any restrictions on the variable.

\[ \frac{x^2 - 36}{x^2 + 6x} - \frac{1}{x + 6} \]

1. Read the problem. What process are you going to use to solve the problem? Find the difference of two rational expressions.

\[ \frac{x^2 - 36}{x^2 + 6x} - \frac{1}{x + 6} \]

2. Why do you factor the denominators? To determine the common denominator.

\[ \frac{(x - 6)(x + 6)}{x(x + 6)} - \frac{1}{x + 6} \]

3. What does LCD stand for? Least Common Denominator

\[ \frac{(x - 6)(x + 6)}{x(x + 6)} - \frac{1}{x + 6} \]

4. Why is the numerator equal to 1? When simplifying, you divide x + 6 by x - 6 to get the answer to 1.

\[ \frac{(x - 6)(x + 6)}{x(x + 6)} - \frac{1}{x + 6} \]

5. Why is the numerator equal to 1? If the denominators are the same, the numerator is 1.

\[ \frac{(x - 6)(x + 6)}{x(x + 6)} - \frac{1}{x + 6} \]

6. Why are 0, 6, and -6 restrictions on the variable? These values make at least one of the original denominators equal zero.

**ANSWERS**

**8-5 Think About a Plan**

Adding and Subtracting Rational Expressions

**Option 1**: To read small print, you use the magnifying lens with the focal length 3 in. How far from the magnifying lens should you place the page if you want to hold the lens at 1 foot from your eye? Use the thin-lens equation.

**Know**

1. The focal length of the magnifying lens is 3 in.

2. The distance from the lens to your eye is 12 in.

3. The thin-lens equation is \( \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \)

**Need**

4. To solve the problem, I need to find:

the distance from the page to the lens

\[ \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \]

5. What are the known variables in the thin-lens equation?

\[ \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \]

6. Solve the thin-lens equation for the unknown variable. \( d_o = \frac{f d_i}{d_i - f} \)

7. Substitute the known values into your equation and solve. \( d_o = \frac{3 \times 12}{12 - 3} = 4 \text{ in.} \)

8. How far from the page should you hold the magnifying lens? 4 in.
Find the least common multiple of each pair of polynomials.

To start, factor each expression.

1. $$4x^2 - 36$$ and $$6x + 30x = 54$$
   
2. $$x - 2x + 3$$ and $$10x - 22x + 3y$$

Simplify each sum or difference. State any restrictions on the variables.

To start, factor the denominators and identify the LCD.

Add or subtract. Simplify where possible. State any restrictions on the variables.

Find the least common multiple of each pair of polynomials.

To start, multiply the numerator and the denominator by the LCD of all the rational expressions.

10. $$\frac{1}{2}$$
   
11. $$\frac{3}{2}$$

12. Reasoning What real numbers are not in the domain of the function $$f(x) = \frac{1}{x}$$? Explain.

If you jog 12 mi at an average rate of 4 mi/h and walk the same route back at an average rate of 3 mi/h, you have traveled 26 mi in 7 h and your overall rate

To start, completely factor each expression.

13. $$2x - x - 6$$
   
14. $$x + 1$$

15. Multiple Choice Simplify $$\frac{2}{y} + \frac{3}{x}$$.

8-5 Practice (continued) Form K

Adding and Subtracting Rational Expressions

Simplify each complex fraction.

8. $$\frac{x^2 - 4}{x^2 + 2x - 3}$$

9. Error Analysis A classroom said that the sum of $$\frac{1}{x} + \frac{1}{x} = \frac{2}{x + 1}$$.

What mistake did your classroom make? What is the correct sum?

10. The harmonic mean of two numbers a and b equals $$\frac{2}{\frac{1}{a} + \frac{1}{b}}$$.

8-5 Enrichment Form K

Adding and Subtracting Rational Expressions

The Superposition Principle

The illumination received from a light source is given by the formula

$$I = \frac{S}{d^2}$$

where I is the illumination at a certain point, S is the strength of the light source, measured in units of kilowatts, and d is the distance of the point from the light source. The superposition principle states that the total illumination received at a given point is equal to the sum of the illuminations from each of the sources.

Suppose a plant is positioned at point A. Copy and complete the following to find the total illumination received by the plant when both lights are on.

L1 = 100 W
L2 = 100 W

1. Add or subtract. Simplify where possible. State any restrictions on the variables. Show your work.

2. $$\frac{1}{x} + \frac{1}{x} = \frac{2}{x}$$

3. $$\frac{1}{x} + \frac{1}{x} = \frac{2}{x}$$

4. The harmonic mean of two numbers a and b equals $$\frac{2}{\frac{1}{a} + \frac{1}{b}}$$.

5. Subtract $$\frac{3}{x}$$.

Write your answer in simplest form. State any restrictions on the variable. Show your work.

2. $$\frac{1}{x} + \frac{1}{x} = \frac{2}{x}$$

3. $$\frac{1}{x} + \frac{1}{x} = \frac{2}{x}$$

4. No answer given
8-5 Retracing
Adding and Subtracting Rational Expressions

Adding and subtracting rational expressions is a little adding and subtracting fractions. Before you can add or subtract the expressions, they must have a common denominator. The easiest common denominator to work with is the least common denominator (LCD).

Problem
What is the LCD of \(\frac{1}{x+3}\) and \(\frac{2}{x+2}\)?

Completely factor each denominator: 
\[x+3 = (x+3)\]
\[x+2 = (x+2)\]

Cross off any repeated factors.

Multiply the remaining factors in the list to form the LCD.

The LCD of \(\frac{1}{x+3}\) and \(\frac{2}{x+2}\) is \((x+3)(x+2)\).

Exercises
Assume that the denominators given are the denominators of rational expressions. Find the LCD of each set.

1. \(x + 3\) and \(x + 4\): \(x(x + 3)(x + 4)\)

2. \(2x - 3\) and \(x + 2\): \(2(x - 1)(x + 2)\)

3. \(x^2 + 3x + 2\) and \(x^2 - 1\): \((x + 1)(x + 2)(x - 1)\)

4. \(x^2 + 5x + 6\) and \(x^2 - 4x + 4\): \((x + 3)(x + 2)(x - 2)\)

5. \(x^2 - 9\) and \(x^2 + 2x + 1\): \((x + 3)(x - 3)(x + 1)\)

6. \(x^2 + 4x + 3\) and \(x^2 - 4x + 3\): \((x + 3)(x - 1)(x + 1)\)

8-6 ELL Support
Solving Rational Equations

Problem
What are the solutions of the rational equation? Justify your steps.

Write original equation.

\(x^2 = 4\)

Factor the denominator to find the LCD.

\((x - 2)(x + 2)\)

Multiply each side by the LCD to clear the denominator.

\(x(x - 2) = 2\)

Distribute and simplify.

\(x^2 - 3x - 2 = 0\)

Factor the quadratic.

\(x = 4\) or \(x = -1\)

Solve for \(x\).

\(x = 4\) causes division by 0, so \(x = 4\) is an extraneous solution. Check for extraneous solutions.

Because \(x = -1\) is not a solution, \(x = -1\) is the solution.

Exercise
What are the solutions of the rational equation? Justify your steps.

\(\frac{1}{x} + \frac{1}{x+1} = 0\)

Write the original equation.

\(\frac{x + 1}{x(x + 1)} = \frac{x}{x + 1}\)

Factor the denominator to find the LCD.

\((x + 1)(x)\)

Multiply each side by the LCD to clear the denominator.

\((x + 1)(x + 1) = 0\)

Distribute and simplify.

\(x + 1 = 0\)

Solve for \(x\).

\(x = \frac{1}{x} + \frac{1}{x+1} = 0\)
Solving Rational Equations

13. \( \frac{1}{x} + \frac{2}{x} = 10 \)
   \( \frac{1}{x} + \frac{2}{x} = \frac{12}{x} \)  

14. \( \frac{1}{x} - \frac{2}{x} = \frac{1}{2} \)
   \( \frac{1}{x} - \frac{2}{x} = \frac{-1}{2} \)

15. \( \frac{1}{x} + \frac{1}{y} = 1 \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{x+y} \)

16. \( \frac{1}{x} - \frac{1}{y} = 1 \)
   \( \frac{1}{x} - \frac{1}{y} = \frac{1}{x-y} \)

17. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{y} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{y} \)

18. \( \frac{1}{x} - \frac{1}{y} = \frac{1}{y} \)
   \( \frac{1}{x} - \frac{1}{y} = \frac{1}{y} \)

19. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \)

20. \( \frac{1}{x} - \frac{1}{y} = \frac{1}{3} \)
   \( \frac{1}{x} - \frac{1}{y} = \frac{1}{3} \)

21. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{2} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{2} \)

22. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \)

23. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{4} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{4} \)

24. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{5} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{5} \)

25. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{6} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{6} \)

26. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{7} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{7} \)

27. \( \frac{1}{x} + \frac{1}{y} = \frac{1}{8} \)
   \( \frac{1}{x} + \frac{1}{y} = \frac{1}{8} \)

Solve each equation. Check each solution.

To start, multiply each side by the LCD.

1. \( \frac{1}{a} + \frac{2}{a} = 10 \)
   \( a = 5 \)

2. \( \frac{1}{b} - \frac{2}{b} = 0 \)
   \( x = -2 \)

3. \( \frac{3}{c} + \frac{1}{c} = \frac{7}{c} \)
   \( x = 6 \)

4. \( \frac{2}{d} + \frac{3}{d} = \frac{5}{d} \)
   \( x = -4 \)

5. \( \frac{1}{e} - \frac{1}{e} = \frac{2}{e} \)
   \( x = -2 \)

6. \( \frac{1}{f} + \frac{1}{f} = \frac{3}{f} \)
   \( x = -6 \)

7. \( \frac{2}{g} - \frac{3}{g} = \frac{4}{g} \)
   \( x = 14 \)

8. \( \frac{1}{h} + \frac{1}{h} = \frac{2}{h} \)
   \( x = -8 \)

9. \( \frac{1}{i} - \frac{1}{i} = \frac{3}{i} \)
   \( x = -10 \)

10. \( \frac{1}{j} + \frac{1}{j} = \frac{4}{j} \)
    \( x = 5 \)

11. \( \frac{1}{k} - \frac{1}{k} = \frac{5}{k} \)
    \( x = -1 \)

12. \( \frac{1}{l} + \frac{1}{l} = \frac{6}{l} \)
    \( x = 5 \)

13. \( \frac{1}{m} - \frac{1}{m} = \frac{7}{m} \)
    \( x = -1 \)

14. \( \frac{1}{n} + \frac{1}{n} = \frac{8}{n} \)
    \( x = 5 \)

15. \( \frac{1}{o} - \frac{1}{o} = \frac{9}{o} \)
    \( x = -1 \)

16. \( \frac{1}{p} + \frac{1}{p} = \frac{10}{p} \)
    \( x = 5 \)

17. \( \frac{1}{q} - \frac{1}{q} = \frac{11}{q} \)
    \( x = -1 \)

Solving each equation. Check each solution.

Using a graphing calculator, solve each equation. Check each solution.

5. \( \frac{3}{x} = \frac{4}{x} \)
   \( x = -2 \)

6. \( \frac{5}{x} = \frac{3}{x} \)
   \( x = -6 \)

7. \( \frac{2}{x} = \frac{1}{x} \)
   \( x = -4 \)

8. \( \frac{4}{x} = \frac{3}{x} \)
   \( x = -6 \)

9. \( \frac{6}{x} = \frac{5}{x} \)
   \( x = -6 \)

10. \( \frac{7}{x} = \frac{6}{x} \)
     \( x = -6 \)

Solve each equation for the given variable.

8. \( F = \frac{\text{mph}}{\text{mph}} \) for \( x = \frac{5}{x} \)
   \( x = \frac{5}{x} \)

9. \( G = \text{Cost} \) for \( d = \frac{\text{Cost}}{\text{Price}} \)
   \( d = \frac{\text{Cost}}{\text{Price}} \)

10. \( R = \frac{\text{Rate}}{\text{Rate}} \) for \( r = \frac{\text{Rate}}{\text{Rate}} \)
    \( r = \frac{\text{Rate}}{\text{Rate}} \)
8-6 Standardized Test Prep

Solving Rational Equations

Grided Response

For Exercises 1–8, what are the solutions of each rational equation? Enter your answer in the grid provided. If necessary, enter your answer as a fraction.

1. \( \frac{3x}{x - 2} = \frac{2x}{x - 3} \)
2. \( \frac{2x}{x - 2} = \frac{4}{x - 2} \)
3. \( \frac{5}{x - 1} = \frac{6}{x - 5} \)
4. \( \frac{4x}{x + 3} = \frac{6}{x + 3} \)
5. \( \frac{7}{x^2} = \frac{4}{x^2 - 1} \)
6. \( \frac{4}{x} = \frac{1}{x - 2} \)
7. \( \frac{12}{x + 2} = \frac{4}{x - 4} \)
8. \( \frac{8}{x + 1} = \frac{2}{x - 3} \)

Answers

1. \( x = \frac{6}{5} \)
2. \( x = 2 \)
3. \( x = 2 \)
4. \( x = 3 \)
5. \( x = -3 \)
6. \( x = 3 \)
7. \( x = 4 \)
8. \( x = 4 \)

8-6 Reteaching

Solving Rational Equations

When one or both sides of a rational equation has a sum or difference, multiply each side of the equation by the LCD to eliminate the fractions.

Problem

What is the solution of the rational equation \( \frac{6}{x} - \frac{2}{x - 3} = \frac{4}{x + 2} \)? Check the solutions.

1. Multiply each term on both sides by the LCD, 2x(x - 3)(x + 2).
2. \( 6(x - 3)(x + 2) - 2x(x - 3)(x + 2) = 4x(x - 3) \)
3. \( 6x^2 - 18x + 36 - 2x^3 + 6x^2 + 6x = 4x^2 - 12x \)
4. \( -2x^3 + 12x^2 + 6x = 0 \)
5. \( x(x - 6)(x - 2) = 0 \)
6. \( x = 0, x = 6, x = 2 \)

Exercises

Solve each equation. Check the solutions.

1. \( \frac{6}{x} - \frac{2}{x - 3} = \frac{4}{x + 2} \)
2. \( \frac{3x}{x - 2} = \frac{2x}{x - 3} \)
3. \( \frac{6}{x} - \frac{2x}{x^2 - 4} = \frac{2}{x - 2} \)
4. \( \frac{7}{x^2} = \frac{1}{x - 2} \)
5. \( \frac{3x}{x^2 - 4} = \frac{5}{x + 2} \)
6. \( \frac{4x - 11}{x^2} = \frac{4}{x + 2} \)
7. \( x + 2 = \frac{3}{x} \)
8. \( \frac{6}{x} + \frac{1}{x - 2} = 1 \)
9. \( \frac{1}{x} + \frac{1}{x + 2} = \frac{3}{x^2 + 2x} \)
10. \( \frac{7}{x + 1} = \frac{2}{x - 3} \)
11. \( \frac{5}{x} + 1 = 3 \)
12. \( \frac{3}{x} - \frac{2}{x + 1} = \frac{3}{x^2 + x - 2} \)

8-6 Enrichment

Solving Rational Equations

Gravitational Attraction

Many physical phenomena obey inverse-square laws. That is, the strength of the quantity is inversely proportional to the square of the distance from the source. Isaac Newton was the first to discover that gravity obeys an inverse-square law. The gravitational force \( F \) between objects of masses \( M \) and \( m \) separated by a distance \( D \) is given by \( F = \frac{GMm}{D^2} \), where \( G \) is a constant.

Suppose that two stars, Alpha Major and Beta Minor, are separated by a distance of 6 light-years. Alpha Major has four times the mass of Beta Minor. Let \( M \) represent the mass of Beta Minor. Suppose that an object of mass \( m \) is placed between the two stars at a distance of 2 light-years from Beta Minor.

1. Write an expression for the gravitational force between this object and Beta Minor.
2. Write an expression for the gravitational force between this object and Alpha Major.
3. What is the distance of the neutral position of the object with mass \( m \) from Beta Minor? Alpha neutral position, both Beta Minor and Alpha Major exert equal force on \( m \).
4. What is the distance of the planet to the mass of the moon? 9 : 1
5. What would be the ratio of their masses if the distance of the spaceship from the planet was 8 times the distance of the spaceship to the moon? 8 : 1

A spaceship is stationary between a planet and its moon, experiencing an equal gravitational pull from each. When measurements are taken, it is determined that the craft is 300,000 km from the planet and 100,000 km from the moon.

Once every 377 y, the two moons of the planet Omega Minor line up in a straight line with the planet. The moons are equal in mass, and the inner moon is equivalent from the outer moon and from the planet. Measurements show that an object two thirds from the distance from the planet to the inner moon, and in the same line as all three, experiences an equal gravitational pull in both directions.

6. What is the ratio of the mass of the planet to the mass of one of its moons? 17 : 4

Jack would take 15 h and Quinn would take 3.75 h to refinish the floor alone. Let \( x \) be the number of hours Quinn takes to refinish the floor alone, and let \( y \) be the number of hours Jack takes to refinish the floor alone.

\[ \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \]

Let \( x \) be the number of hours Jack takes to refinish the floor alone, and let \( y \) be the number of hours Quinn takes to refinish the floor alone.

\[ \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \]

Jack works at the rate of 6.67 ft/h. Quinn works at the rate of 26.67 ft/h. Let \( x \) be the number of hours Jack takes to refinish the floor alone, and let \( y \) be the number of hours Quinn takes to refinish the floor alone.

\[ \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \]

\[ \frac{1}{6.67} + \frac{1}{26.67} = \frac{1}{3} \]

\[ f = 15 \]

\[ q = 3.75 \]

Jack would take 15 h and Quinn would take 3.75 h to refinish the floor alone.

Exercises

12. An airplane flies from its home airport to a city and back in 5 h flying time. The plane travels the 720 mi to the city at 205 mph with no wind. How strong is the wind on the return flight? Is the wind a headwind or a tailwind?
13. Let each student work together to complete the project about 13 days.
Do you UNDERSTAND?

12. Open-Ended Write a rational function that has a removable discontinuity in its graph at x = 3. Check students’ work.

13. Reasoning How many inverse variation functions have (0, 0) as a solution? Explain. None; division by zero is undefined.

14. Reasoning (x, y) is a solution of the equation y = 2. Name one solution of the equation y = \frac{x}{2}. Is the line that contains the point and y-intercept the graph of y = \frac{x}{2}? Answers may vary. Sample: (x, y); the functions are reflections of each other over the x-axis.

15. How many milliliters of 0.65% saline solution must be added to 100 mL of 3% saline solution to make 1 L of saline solution?

4600 mL

16. Becky and Kendra start a business painting fences. They can paint 200 ft\(^2\) of fence in 40 min if they work together. If Kendra paints four times faster than Becky, how long would it take each of them to paint a 500-ft\(^2\) fence working alone?

Kendra: 125 min, Becky: 500 min

17. Writing Explain how to simplify \(\frac{4x}{y} + \frac{5x}{y}\). Add the terms in the numerator, rewrite division as multiplication of the numerator by the reciprocal of the denominator, factor \(x\) out of the denominator, and divide out common factors.

\(\frac{4x}{y} + \frac{5x}{y} = \frac{9x}{y}\)

Chapter 8 Quiz 2

Do you know HOW?

Simplify each expression.

1. \(\frac{2x^2 + 4x}{x^2 - 1} \div \frac{x + 2}{x^2 - 2x + 1}\)
2. \(\frac{x + 1}{x - 1} \div \frac{x - 1}{x^2 - 1}\)
3. \(\frac{5x}{3x - 6} \div \frac{10}{x - 2}\)
4. \(\frac{x^2 + 3x + 2}{x + 1} \div \frac{x^2 - 4}{x - 2}\)
5. \(\frac{2x^2 + x - 3}{x - 3} \div \frac{2x^2 + 5x + 3}{x + 1}\)
6. \(\frac{x^2 + 4}{x + 2} \div \frac{x^2 + 10x + 24}{x + 6}\)
7. \(\frac{x^2 - 16}{x - 4} \div \frac{x^2 - 4}{x + 2}\)
8. \(\frac{3x^2 - 11x + 6}{x - 2} \div \frac{3x^2 - 12x}{x - 3}\)

Solve each equation. Check each solution.

11. \(\frac{1}{x + 2} = \frac{3}{x - 2}\)
12. \(\frac{1}{x} = \frac{2}{x + 3}\)
13. \(\frac{x + 2}{x - 3} = \frac{x - 1}{x - 2}\)
14. \(\frac{2}{x} = \frac{4}{x + 2}\)

Do you UNDERSTAND?

10. Error Analysis A student claims that \(x^2 - 9 = 0\) is in simplest form. Is the student correct? Explain. No; \(x^2 - 9\) can be factored into \((x + 3)(x - 3)\).

11. Writing Explain how to simplify \(\frac{2x}{x + 1} + \frac{3}{x - 2}\). Add the terms in the numerator, rewrite division as multiplication of the numerator by the reciprocal of the denominator, and divide out common factors.

\(\frac{2x}{x + 1} + \frac{3}{x - 2} = \frac{2x(x - 2) + 3(x + 1)}{(x + 1)(x - 2)}\)

Chapter 8 Chapter Test (continued)

Do you know HOW?

Simplify each complex fraction.

10. \(\frac{3}{4} \div \frac{5}{6}\)
20. \(\frac{1}{2} \div \frac{3}{4}\)

Solve each equation. Check each solution.

21. \(\frac{2}{3} = \frac{1}{x + 2}\)
22. \(\frac{x}{3} = \frac{1}{x + 1}\)
23. \(\frac{3}{2} = \frac{x}{x - 2}\)
24. \(\frac{4}{3} = \frac{1}{x - 2}\)
25. \(\frac{1}{x} = \frac{2}{x - 2}\)
26. \(\frac{1}{x - 2} = \frac{2}{x - 3}\)

Chad can paint a room in 2h. Cassie can paint the room in 3h. How long would it take them to paint the room working together? 1.2h

28. How many milliliters of 0.65% saline solution must be added to 100 mL of 3% saline solution to get a 0.7% solution? 400 mL

Do you UNDERSTAND?

25. Writing Explain what it means to simplify a rational expression. The numerator and denominator have no common factors.

26. Reasoning \(x\) and \(y\) are solutions of the equation \(x^2 + xy + y^2 = 0\). Find the least common multiple of each pair of polynomials.

27. Simplify each rational expression. State any restrictions on the variable.

28. Simplify each sum or difference.

29. Writing Describe how the variables in the given equation are related. \(y = \frac{3}{x} - 5\) varies directly with the square of \(x\) and the difference \(x - 2\), and varies inversely with \(x\).
Chapter 8 Quiz 1

Do you know HOW?

If \( z = 15 \) when \( x = 4 \) and \( y = 5 \), write a function that models the relationship.
1. \( z \) varies directly with \( y \) and inversely with \( x \). \( z = \frac{20}{x} \)
2. \( z \) varies jointly with \( x \) and \( y \). \( z = 5xy \)
3. \( z \) varies inversely with the product of \( x \) and \( y \). \( z = \frac{25}{xy} \)

Sketch the graph of each rational function. Then identify the domain and range.
4. \( y = \frac{4}{x} \)
   - Domain: all reals except \( x = 2 \); range: all reals
5. \( y = \frac{2x}{x^2 + 1} \)
   - Domain: all reals; range: \( y \) is all reals
6. \( y = \frac{x^3}{x^2 - 1} \)
   - Domain: all reals except \( x = 2 \); range: all reals

Do you UNDERSTAND?

7. Writing: Explain how to find the asymptotes of \( y = \frac{1}{x} + 10 \).

8. Your school is renting an indoor water park for a school outing. The water park costs \( $1200 \) for the day.
   a. How many students need to go to the water park so that each person pays \( \leq \$25 \)? Each 250 students
   b. Reasoning: The water park can hold up to 450 people. Does the park hold enough students so that each person pays only \$25? Explain.
   No, to pay \$25 each, 580 students would need to go to the park.

Chapter 8 Quiz 2

Do you know HOW?

Simplify each rational expression. State any restrictions on the variables.
1. \( \frac{3x - 6}{x - 2} \) \( x \neq 2 \)
2. \( \frac{2x + 10}{x - 4} \) \( x \neq 4, 0, -4, -5 \)
3. \( \frac{3x + 5}{x - 6} \) \( x \neq 6 \)
4. \( \frac{3}{y^2 + 3y} \) \( y \neq 0 \)
5. \( \frac{3x - 12}{x^2 + 2x - 3} \) \( x \neq 1, 3 \)

Solve each equation. Check your solutions.
7. \( \frac{y - 2}{y + 3} \) \( x \neq 2, \text{or} -3 \)
8. \( \frac{x + 2}{x - 2} \) \( x = 2 \)
9. \( \frac{x}{x - 1} = \frac{1}{2} \) \( x = \frac{2}{3} \)

Do you UNDERSTAND?

10. A large snowplop can clear a parking lot in 4 h. A small snowplop needs more time to clear the lot. Working together, they can clear the lot in 3 h. How long would it take the small snowplop to clear the lot by itself?
   12 h

11. Error Analysis Your classmate says that the simplified form of the complex fraction \( \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} \) is \( \frac{x + 1}{x - 1} \). What mistake did he make? What is the correct answer?
   He multiplied the numerator by \( x \) and the denominator by \( y \); instead of multiplying both by \( xy \).

Chapter 8 Test

Do you know HOW?

Write a function that models each variation.
1. \( x = 5 \) when \( y = 15 \), \( y \) varies inversely with \( x \). \( y = \frac{75}{x} \)
2. \( x = 6 \) when \( x = 4 \) and \( y = 3 \), \( z \) varies directly with \( y \) and inversely with \( x \). \( z = \frac{54}{y} \)
3. \( x = 8 \) when \( x = 2 \) and \( y = 4 \), \( z \) varies jointly with \( x \) and \( y \). \( z = 8xy \)

Write and graph an equation of the translation of \( y = \frac{1}{x} \) that has the given asymptotes.
4. \( y = \frac{1}{x + 3} \)
5. \( y = \frac{1}{x} - 3 \)

For each rational function, identify any holes or horizontal or vertical asymptotes of the graph.
6. \( y = \frac{x^2 - 4}{x^2 - 2x} \)
   - Hole: \( x = 2 \);
   - Horizontal asymptote: none
   - Vertical asymptote: \( x = 0 \)
7. \( y = \frac{x^2 + 1}{x^2 - 2x} \)
   - Horizontal asymptote: none
   - Vertical asymptote: \( x = 0 \)
8. \( y = \frac{x^2 - 2x - 3}{x^2 - 3x - 4} \)
   - Horizontal asymptote: \( x = -1 \)
   - Vertical asymptote: \( x = 3 \)

Simplify each rational expression. State any restrictions on the variable.
9. \( \frac{2x + 1}{x - 3} \) \( x \neq 3 \)
10. \( \frac{x^2 - 9}{x - 3} \) \( x \neq 3 \)
11. \( \frac{x^2 - 9}{x^2 - 4} \) \( x \neq 2, -2 \)
12. \( \frac{1}{x} + \frac{1}{x} \) \( x \neq 0 \)
13. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
14. \( \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
15. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
16. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
17. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
18. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
19. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
20. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)
21. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x \neq 0 \)

Chapter 8 Test [continued]

Solve each equation. Check your solutions.
13. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
14. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
15. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
16. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
17. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
18. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
19. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
20. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)
21. \( \frac{1}{x} + \frac{1}{x} - \frac{1}{x} \) \( x = 0 \)

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Chapter 8 Performance Tasks

Task 1
a. Write a function that models an inverse variation situation.
   b. Find the constant of variation.
   c. Determine the domain and range.
   d. Graph the function, making sure to indicate any asymptotes.
   e. Find the values of any asymptotes.
   f. Check students’ work. Student writes an appropriate function and correctly identifies constant of variation, independent and dependent variables, domain and range, and values of asymptotes. Graph is accurate, but work presented is correct.
   g. Check students’ work. Student attempts to identify constant of variation, independent and dependent variables, domain and range, and values of asymptotes. Graph is incomplete, but work presented is correct.
   h. Check students’ work. Student writes a rational function that is not an inverse variation. Uses appropriate methods to identify constant of variation, independent and dependent variables, domain and range, and values of asymptotes. The only errors are minor computational or copying errors.
   i. Check students’ work. Student writes a rational function that is not an inverse variation. Correctly identifies constant of variation, independent and dependent variables, domain and range, and values of asymptotes. The only errors are minor computational or copying errors.
   j. Check students’ work. Student uses appropriate methods to write correct expressions, and note any minor errors.

Task 2
a. Write a function that models a combined variation situation.
   b. Let the dependent variable be f. Find the value of the independent variable.
   c. Let the independent variable be f. Find the value of the dependent variable.
   d. What value does the dependent variable approach when the independent variable approaches infinity?
   e. What value does the independent variable approach when the dependent variable approaches infinity?
   f. Check students’ work. Student writes an appropriate function and correctly identifies constant of variation, independent and dependent variables, domain and range, and values of asymptotes. Graph is accurate, but work presented is correct.
   g. Check students’ work. Student attempts a solution, but shows little understanding of the problem.
   h. Check students’ work. Student attempts a solution, but shows understanding of the problem. The only errors are minor computational or copying errors.
   i. Check students’ work. Student attempts a solution, but shows little understanding of the problem.
   j. Check students’ work. Student attempts a solution, but shows little understanding of the problem.

Task 3
To answer each question, use the function f(x) = x^3 where x is the time in hours, d is the distance in miles, and v is the rate in miles per hour.
   a. Sydney drives 10 mi at a constant rate and then drives 10 mi at a rate 5 mi/h faster than the initial rate. Write expressions for the time along each part of the trip. Add these times to write an equation for the total time in terms of the initial rate, v.
   b. Determine the reasonable domain and range and describe any discontinuities of f(x). Graph f(x) on your graphing calculator.
   c. At what rate, to the nearest mi/h, must Sydney drive if the entire 30 mi must be covered in about 45 min? Find the answer using the graph and algebraic methods.
   d. How long will Sydney take to drive the entire 30 mi if the car’s initial rate varies faster than the initial rate. Write expressions for the time along each part of the trip. Add these times to write an equation for the total time in terms of the initial rate, v.

Chapter 8 Cumulative Review
Multiple Choice
For Exercises 1–18, choose the correct letter.

1. Which matrix represents the system of equations?
   \[ \begin{align*}
   2a + 3b &= 1 \\
   3a - 4b &= 0 \\
   0 &= 0
   \end{align*} \]
   a. \[ \begin{bmatrix} 3 & 1 \\ -4 & 0 \\ 0 & 0 \end{bmatrix} \]
   b. \[ \begin{bmatrix} 2 & 3 \\ 3 & 1 \\ 0 & 0 \end{bmatrix} \]
   c. \[ \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]
   d. \[ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]
   2. Which point lies on the graph of 2x + y = z = 37?
   a. \((2, 4, 3)\)  b. \((0, 0, 0)\)  c. \((0, 0, 5)\)
   3. Which of these shows a trend line for the plotted data? A
   a. \[ y = 2x + 3 \]
   b. \[ y = 3x - 2 \]
   c. \[ y = x + 2 \]
   d. \[ y = x - 2 \]
   4. Which of these is a binomial quadratic? F
   a. \(x^2 + 4\)  b. \(x^2 + 3x - 2\)  c. \(2x^2 + 2x + 2\)  d. \(x^2 - 2x\)
   5. What is the inverse of \(y = 6x - 37\)?
   a. \(y = 6x + 37\)  b. \(y = \frac{1}{6x} + \frac{37}{6}\)  c. \(y = \frac{1}{6x} - \frac{37}{6}\)  d. \(y = 6x - 37\)
   6. Which is a zero of the function \(f(x) = x^2 - 36x + 10\)?
   a. \(x = 2\)  b. \(x = 5\)  c. \(x = 18\)  d. \(x = 36\)
   7. Which number equals \(\log_2 36 + \log_2 20 - \log_2 5/7\)?
   a. \(\log_2 3\)  b. \(\log_2 4\)  c. \(\log_2 6\)  d. \(\log_2 4\)
   8. Which matrix contains \(a_{34} = 467\)?
   a. \[ \begin{bmatrix} 4 & 4 \end{bmatrix} \]
   b. \[ \begin{bmatrix} 4 & -4 \end{bmatrix} \]
   c. \[ \begin{bmatrix} -4 & 4 \end{bmatrix} \]
   d. \[ \begin{bmatrix} 4 & -4 \end{bmatrix} \]
   9. Which point cannot lie on the line \(y = 2x + 37\)?
   a. \((1, 8)\)  b. \((1, 7)\)  c. \((0, 4)\)  d. \((0, 0)\)
   10. Which system has no solution?
   a. \(y = 2x + 3\)  b. \(y = 4x - 3\)  c. \(y = 4x - 2\)  d. \(y = 3x + 2\)
   Short Response
   11. The graph of a certain direct variation includes the point \((2, 6)\). Write the equation of the direct variation. Show your work. \(y = 3x\)
   12. Identify any vertical or horizontal asymptotes on the graph of \(y = \frac{2x}{x-3}\). Justify your answer. Vertical asymptote at \(x = 3\); horizontal asymptote at \(y = 2\)
   Extended Response
   13. The height of a triangle is 9 mm less than the square of its base length. The height of a similar triangle is 6 mm more than twice the length of the base of the first triangle. Write an expression for the area of the second triangle in simplified form. State any restrictions on the variables.
   14. Solve the inequality \(\frac{x + 2}{x - 4} > 0\). Use a number line to graph the solution set.
   15. Identify the domain and range.
   16. Students use appropriate strategies, but misunderstood part of the problem or ignored a condition in the problem.
   17. Students make no attempt or no response is given.
   18. Students make no attempt or no response is given.
About the Project
The Chapter Project gives students an opportunity to explore safety issues related to scuba diving. They use inverse proportions to find volumes of air in lungs and to find the sizes of tanks needed to hold enough air to dive to various depths.

Introducing the Project
• Ask students how they think a graph of the distance below the surface versus the volume of air in the lungs might look.
• Review the project with students and instruct students to make a list of questions they will need to answer to complete the project.

Activity 1: Graphing
Students make tables and graphs to find how the volume of air in their lungs varies with depth and pressure.

Activity 2: Writing
Students explain why they must exhale while ascending from diving.

Activity 3: Solving
Students use Boyle’s Law to find how large a scuba diving tank needs to be.

Activity 4: Solving
Students use inverse variation to find how long the air in a tank lasts at various depths.

Finishing the Project
You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results.

• Ask students to review their project and update their folders.
• Ask students to review their methods for finding, recording, and solving formulas, and for making the tables and graphs used in the project.
• Ask groups to share their insights that resulted from completing the project, such as any shortcuts they found for solving formulas or making graphs.

Chapter 8 Project: Under Pressure

Activity 3: Solving
A popular size of scuba-diving tank holds the amount of compressed air that would occupy 7.5 ft³ at a normal surface pressure of 1 atm. The air in the tank is at a pressure of about 2250 lb/in.², so the tank itself can have a volume much less than 7.5 ft³.

Activity 4: Solving
The rate at which a scuba diver uses air in the tank depends on many factors, like the diver’s age and lung capacity. Another important factor is the depth of the dive.

Finishing the Project
Design a poster or brochure explaining what you learned about scuba diving safety in this chapter. Use graphs, tables, and examples to support your conclusions.

Reflect and Revise
Work with a classmate to review your poster or brochure. Check that your graphs and examples are correct and that your explanations are clear. If necessary, refer to a book on scuba diving. Discuss your poster or brochure with an adult who works in the area of sports safety, such as a lifeguard, coach, physical education teacher, or recreation director. Ask for their suggestions for improvements.

Extending the Project
What other safety issues must scuba divers consider? Ask a scuba diving or refer to a book to find other things a scuba diver must consider to dive safely.

Chapter 8 Project Teacher Notes: Under Pressure

Chapter 8 Project Manager: Under Pressure

Getting Started
Read the project. As you work on the project, you will need a calculator, materials on which you can record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist
☐ Activity 1: determining air volume vs. depth and pressure
☐ Activity 2: understanding breathing while diving
☐ Activity 3: determining tank size
☐ Activity 4: determining duration of air supply
☐ project display

Suggestions
Make a table and a graph for each comparison.
The Boyle’s Law equation
First, find the pressure at each depth.

Scoring Rubric
4 Calculations are correct. Graphs are neat, accurate, and clearly show the relationship between the variables. Explanations are clear and complete.
3 Calculations are mostly correct. Graphs are neat and mostly accurate with minor errors. Explanations are not complete.
2 Calculations contain both minor and major errors. Graphs are not accurate. Explanations and the poster or brochure are not clear or are incomplete.
1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project
Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of Project
9.1 Think About a Plan

Mathematical Patterns

Geometry: Suppose you are stacking boxes in layers that form squares. The numbers of boxes in successive squares form a sequence. The figure at the right shows the top four levels as viewed from above.

1. How many boxes of equal size would you need for the next lower level?
2. How many boxes of equal size would you need to add three levels?
3. Suppose you are stacking a total of 285 boxes. How many levels will you have?
4. How many boxes would you need to add three levels?
5. What is a recursive or explicit formula that describes the total number of boxes in a stack of n levels?

Mathematical Patterns

For a sequence that is described by a recursive formula, the first term in the ordered list of numbers is the initial condition.

5. In the sequence 2, 4, 6, 8, the number 2 is the second term in the sequence.

6. The position of a term in a sequence can be represented by using the subscript number.

7. The formula
   \[ a_n = 3n + 2 \]
   is an explicit formula. Each term in the sequence can be found by using the subscript number of the term.

8. The equation
   \[ a_n = n(n+1) \]
   is a recursive formula.

9. An ordered list of numbers is called a sequence.

10. The formula
    \[ a_{n+1} + a_n = 5 + a_n \]
    is a recursive formula.

Write a recursive definition for each sequence.

11. \[ a_n = 4 + 2n \ where \ a_1 = 6 \]
12. \[ a_n = 3 + 2n \ where \ a_1 = 5 \]
13. \[ a_n = 2^n \ where \ a_1 = 2 \]
14. \[ a_n = 3^n \ where \ a_1 = 1 \]
15. \[ a_n = a_{n-1} + 2 \ where \ a_1 = 1 \]
16. \[ a_n = 2n \ where \ a_1 = 2 \]

Write an explicit formula for each sequence. Find the twelfth term.

17. \[ a_n = 3n + 2 \ where \ a_1 = 5 \]
18. \[ a_n = 2n + 1 \ where \ a_1 = 3 \]
19. \[ a_n = 4n - 1 \ where \ a_1 = 3 \]
20. \[ a_n = 3n \ where \ a_1 = 1 \]

Find the first ten terms of each sequence.

21. \[ a_n = 3 \]
22. \[ a_n = 2 \]
23. \[ a_n = 3 \]
24. \[ a_n = 4 \]
25. \[ a_n = 2 \]
26. \[ a_n = 3 \]

27. A man enters a 1.5 mi on Monday, 1.6 mi on Tuesday, 1.8 mi on Wednesday, 2.1 mi on Thursday, and 2.5 mi on Friday. If the pattern continues, how many miles will he run on Saturday? 3.0 mi

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9-1 Practice Form K
Mathematical Patterns

Find the first five terms of each sequence.

1. \(a_n = 4n - 1\)
   - \(5, 9, 13, 17, 21\)

2. \(a_n = x^2 + 4\)
   - \(5, 8, 13, 20, 29\)

3. \(a_n = \frac{2}{n + 1}\)
   - \(2, 5, 1, 1.25, 1.5\)

4. \(a_n = \frac{n}{n + 1}\)
   - \(2, \frac{3}{2}, rac{1}{2}, \frac{1}{3}, \frac{1}{4}\)

5. \(a_n = 2^{n-1}\)
   - \(1, 2, 4, 8, 16\)

6. Write an explicit formula for a sequence with 3, 5, 7, 9, and 11 as its first five terms.

7. Write a recursive definition for each sequence.

   a. \(a_1 = 2\)
   - \(a_n = a_{n-1} + 2n\)

   b. \(a_1 = 5, a_2 = 3\)
   - \(a_n = a_{n-1} + a_{n-2}\)

12. Writing Explain the difference between a recursive definition and an explicit formula. An explicit formula describes the nth term of a sequence using the number n. A recursive formula defines a sequence by the relationship between successive terms.

9-1 Standardized Test Prep Form K
Mathematical Patterns

Multiple Choice

For Exercises 1–6, choose the correct letter.

1. What are the first five terms of the sequence? \(c\)
   - \(a_1 = 3\), \(a_2 = 4\), \(a_3 = 5\), \(a_4 = 6\), \(a_5 = 7\)
   - \(a_1 = 3\), \(a_2 = 4\), \(a_3 = 5\), \(a_4 = 6\), \(a_5 = 7\)

2. The formula \(a_n = 3n + 1\) best represents which sequence? \(G\)
   - \(2, 5, 8, 11, 14\)
   - \(3, 6, 9, 12, 15\)
   - \(4, 7, 10, 13, 16\)
   - \(5, 8, 11, 14, 17\)

3. Which pattern can be represented by \(a_n = 2^n - 1\)? \(D\)
   - \(0, 1, 2, 3, 4\)
   - \(1, 2, 3, 4, 5\)
   - \(2, 3, 4, 5, 6\)
   - \(3, 4, 5, 6, 7\)

4. The sequence 4, 16, 36, 60, 100, ... can best be represented by which formula? \(G\)
   - \(a_1 = 4\), \(a_n = 4n^2\)
   - \(a_1 = 4\), \(a_n = 4n^3\)
   - \(a_1 = 4\), \(a_n = 4n^4\)

5. For the sequence 0, 5, 10, 15, 20, ... what is the 40th term? \(A\)
   - \(100\)
   - \(200\)
   - \(300\)
   - \(400\)

6. A student saved up a savings plan to transfer money from his checking account to his savings account. The first week $10 is transferred, the second week $12 is transferred, and the fourth week $20 is transferred. How much will he transfer in the 10th week? \(G\)
   - $30
   - $40
   - $50
   - $60

7. A student saves $10 the first week, $20 the second week, $30 the third week, and so on. How many weeks will it take before his balance is zero? \(0\)
   - 4 weeks
   - 5 weeks
   - 6 weeks
   - 7 weeks

Short Response

7. After training for and running a marathon, an athlete wants to reduce her daily run by half each day. The marathon is about 26 mi. How many days will it take her to run less than a 10 mi? Show your work.

9-1 Enrichment Form K
Mathematical Patterns

You can define the terms in a sequence using an explicit formula or a recursive definition. You can use another method, called iteration, to form a sequence. The word iteration means to repeat an action. In mathematics, a sequence of numbers is generated through iteration when the same procedure is performed on each output.

1. Consider the function \(f(x) = x^2 + 1\). Let the first term of a sequence be 0. What is the fourth term of the sequence? Write the sequence. 0, 1, 2, 5, 26

2. To create more terms of this sequence through iteration, continue to apply \(f(x) = x^2 + 1\) to each output. The third term in this sequence can be described as \(f(f(f(0)))\). What is the third term? \(f(f(0)) = f(1) = 2\)

3. Determine the first 10 terms of this sequence. You already have the first 3 terms. 0, 1, 2, 5, 26, 677, 11,797, 20,172, 126,765, 236,190

4. Determine the first 5 terms of the sequence formed through iterations of \(f(x) = x^2 + 1\). Begin with \(x = 2\). Describe the sequence. 2, 5, 26, 677, 11,797

5. Will you get the same type of sequence if you start with a different number? No, for example, if you start with \(x = 0\), the sequence is 0, 1, 1, 2, 5, 26, ...

6. Iterations use other than to form numerical sequences. Consider this iterative process, which forms a sequence of a set of three integers. Make a set of any three integers. Compare the absolute value of the difference between each pair of integers in the set. This produces a new set of three integers. Continue this process on each new set of three integers. Describe what eventually happens. No matter what three integers you choose to start with, the set will eventually repeat itself. In combinations of the set \((4, 5, 1)\), where \(n > 0\) is a positive integer.

7. A sequence is a list of numbers. A sequence can be represented in two ways: Explicit formula for the first few terms of a sequence. Then find the tenth term.

13. 7, 10, 13, 16, ...
   - \(a_n = 3n + 1\)
   - \(16\)

14. 4, 8, 12, 16, ...
   - \(a_n = n + 7\)
   - \(17\)

15. \(n = 4, 8, 12, 16, ...
   - \(a_n = 2 \cdot \frac{1}{2}\)
   - \(4\)

16. 1, 4, 9, 16, ...
   - \(a_n = n^2\)
   - \(100\)

17. 1, 3, 5, 7, 9, ...
   - \(a_n = \frac{n}{2} + 1\)

18. 1, 7, 13, 19, 25, ...
   - \(a_n = \frac{n}{2}\)
   - \(50, 047\)
9.1 Reaching Mathematical Patterns

Some patterns are much easier to determine than others. Here are some tips that can help with unfamiliar patterns:
- If the terms become progressively smaller, subtraction or division may be involved.
- If the terms become progressively larger, addition or multiplication may be involved.

**Problem**

What is the next term in the sequence 6, 8, 11, 15, 20, ...?

6 8 11 15 20

Spread the numbers in the sequence apart, leaving space between numbers.

+2 +3 +4 +5

Beneath each space, write what can be done to get the next number in the sequence.

In each term, the number that is added to the previous term increases by one. Find a pattern.

If the pattern is continued, the next term is 20 + 6, or 26.

**Exercises**

Describe the pattern that is formed. Find the next three terms.

1. 5, 6, 8, 11, 15, ...
   
2. 3, 6, 12, 24, 48, ...
   
3. 2, 6, 14, 28, 56, ...
   
4. 1, 3, 5, 7, 9, ...
   
5. 10, 14, 18, 22, 26, ...
   
6. 7, 14, 21, 28, ...
   
7. 3, 6, 12, 24, ...

Find the 104th term in the sequence that begins 5, 9, 13, ...

Find the missing number in the arithmetic sequence. This number is the arithmetic mean of the two given numbers.

8. …, 12, 26, ...
   
9. …, 15, 33, ...
   
10. …, 3, 11, ...

9.2 Ell Support

### Arithmetic Sequence

An arithmetic sequence is a sequence where the difference between consecutive terms is constant.

\[ a, a + d, a + 2d, a + 3d, \ldots \]

**Sample**

2, 5, 8, 11, ...

Determine whether or not each sequence is arithmetic.

1. 1, 3, 4, 7, 9, 11, ...
   - not arithmetic
   
2. 3, 9, 27, ...
   - arithmetic
   
3. 5, 10, 30, 60, ...
   - arithmetic
   
4. 1, 2, 3, 5, ...
   - not arithmetic

Use the formula \( a_n = a + (n - 1)d \) to find the indicated term in each arithmetic sequence.

5. Find the 12th term in the sequence that begins 3, 6, 9, ...
   - 36
   
6. Find the 16th term in the sequence that begins 4, 10, 16, ...
   - 224
   
7. Find the 104th term in the sequence that begins 5, 9, 13, ...
   - 417

### Arithmetic Mean

The arithmetic mean is the average of a set of numbers. The arithmetic means of two numbers \( a \) and \( b \) is found using the formula displayed below.

\[ \text{Mean} = \frac{a + b}{2} \]

**Sample**

The arithmetic mean of 4 and 6 is \( \frac{4 + 6}{2} = 5 \).

Find the missing number in the arithmetic sequence. This number is the arithmetic mean of the two given numbers.

8. …, 12, 26, ...
   
9. …, 15, 33, ...
   
10. …, 3, 11, ...

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## 9.2 Practice Form G

### Arithmetic Sequences

Determine whether each sequence is arithmetic. If so, identify the common difference.

<table>
<thead>
<tr>
<th>n</th>
<th>a_n</th>
<th>a_{n+1}</th>
<th>a_{n+2}</th>
<th>a_{n+3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2, 3, 4, 5, ...</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2, 4, 6, 8, ...</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>3, 6, 9, 12, ...</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>4, 8, 12, 16, ...</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>5, 10, 15, 20, ...</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>6, 12, 18, 24, ...</td>
<td>12</td>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>

The sequence is arithmetic.

The common difference is 3.

7. **Reasoning:** Is the sequence represented by the formula \(a_n = 4n + 8\) arithmetic? Explain.

Yes; the difference between consecutive terms is 4.

Find the 24th term of each arithmetic sequence.

<table>
<thead>
<tr>
<th>n</th>
<th>a_n</th>
<th>a_{n+1}</th>
<th>a_{n+2}</th>
<th>a_{n+3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>8, 10, 12, 14, ...</td>
<td>10</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>9, 12, 15, 18, ...</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>10, 15, 20, 25, ...</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

The sequence is arithmetic.

The common difference is 3.

The formula represents an arithmetic sequence.

The common difference is -3.

14. **Error Analysis:** Noah used the formula \(a_n = a + (n - 1)d\) to find the 12th term in the sequence 2, 4, 7, 11, ... . Did Noah find the correct term? How do you know? No. Noah applied the explicit formula for arithmetic sequences to a sequence that is not arithmetic.
5. Find the 25th term of each sequence.

Exercises

Check the answer. Write 20, 16.5, and 13.

9-2

1. Which sequence is an arithmetic sequence? A
   
   7, 10, 13, 16, 19, ...
   8, 14, 20, 26, 32, 38, ...
   2, 4, 8, 16, 32, ...
   1, 2, 4, 8, 16, 32, ...
   6. Which sequence represents an arithmetic sequence?
   A
   8, 14, 20, 26, 32, 38, ...
   1, 2, 4, 8, 16, 32, ...
   7. What are the missing terms of the arithmetic sequence 5, __, __, 62, __?

9-2

6. For Exercises 1–6, choose the correct letter.

Multiple Choice

9-2

1. What is the 100th term in the arithmetic sequence beginning with 3, 19, 24, 28, ...
   a. 5, 2, 1
   b. 5, 2, 1
   c. 5, 2, 1
   d. 5, 2, 1

9-2

58

31 rows

52...
9-3 ELL Support
Geometric Sequences

Use the chart below to review vocabulary. These vocabulary words will help you complete this page.

<table>
<thead>
<tr>
<th>Vocabulary Words</th>
<th>Explanations</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric sequence</td>
<td>A sequence in which the ratio of any term (after the first) to its preceding term is a constant value.</td>
<td>The sequence 1, 6, 12, 24... is geometric because all of the consecutive terms have a ratio of 2.</td>
</tr>
<tr>
<td>Common ratio</td>
<td>The ratio of each term to its preceding term in a geometric sequence.</td>
<td>The common ratio in the sequence 1, 4, 16, 64... is 4.</td>
</tr>
<tr>
<td>Geometric mean</td>
<td>The geometric mean of two numbers a and b is . The geometric mean of the numbers 4 and 9 is .</td>
<td></td>
</tr>
</tbody>
</table>

1. The terms in the sequence 2, 6, 18, 54, 162,... all share a common ratio.
2. The numbers 2 and 3 have a geometric mean of 4.
3. The consecutive terms in a geometric sequence all share a common ratio.

Identify each sequence as arithmetic or geometric:

4. 2, 5, 8, 11, 14, ...
5. 1, 2, 4, 8, 16, ...
6. 1, 2, 6, 18, ...

Identify the common ratio for each geometric sequence.

7. 5, 12, 40, 162 ...
8. 12, 36, 108, ...

Find the missing term in the geometric sequence:

9. , ________, 8
10. , ________, 25

Planning the Solution
6. Write a formula for the sequence: 

Getting an Answer
7. a. Find the number of miles you run during your twelfth week of training.
   b. Find the 12th term of a geometric sequence that represents the number of miles you run each week.

ANSWERS

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Find the ninth term of each geometric sequence.

3.

1.

Find the missing terms of each geometric sequence.

7.

Short Response

5.

4.

1.

For Exercises 1−6, choose the correct letter.

Multiple Choice

9-3

Standardized Test Prep

9-3

Practice

Form K

Geometric Sequences

Determine whether each sequence is geometric. If so, find the common ratio.

1. 1, 3, 9, 27, ...

Find the ratio between consecutive terms. not geometric.

2. 2, 4, 8, 16, ...

The sequence is geometric.

The common ratio is 2.

3. −2, −4, −8, −16, ...

4. 4, 50, 5, 0.5, ...

The sequence is geometric.

The common ratio is 2.

5. 0, 25, 50, 75, 100, ...

6. 6, 25, 96, 301, ...

The sequence is geometric.

The common ratio is 4.

7.

Open-Ended

Write a geometric sequence with a common ratio of \( \frac{1}{2} \). Explain how you developed the sequence.

Assumes may vary. Sample: 6, 16, 6, 16, 8, 8, 4, 2, 1, 8, 1, 16, 32, 64, ...

I divided the first term by 2 to get the second term. Then I divided the second and third terms by 4.

Find the sixth term of each geometric sequence.

5.

4.

1.

Find the missing terms of each geometric sequence.

10. \( \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{40}, \frac{1}{80}, \frac{1}{160}, \frac{1}{320}, \frac{1}{640}, \frac{1}{1280}, \frac{1}{2560}, \frac{1}{5120} \), ...

Identify the common ratio.

\( a_1 = \frac{1}{5} \)

\( a_2 = \frac{1}{10} \)

\( a_3 = \frac{1}{20} \)

\( a_4 = \frac{1}{40} \)

\( a_5 = \frac{1}{80} \)

\( a_6 = \frac{1}{160} \)

\( a_7 = \frac{1}{320} \)

\( a_8 = \frac{1}{640} \)

\( a_9 = \frac{1}{1280} \)

\( a_{10} = \frac{1}{2560} \)

\( a_{11} = \frac{1}{5120} \)

\( a_{12} = \frac{1}{10240} \)

\( a_{13} = \frac{1}{20480} \)

\( a_{14} = \frac{1}{40960} \)

\( a_{15} = \frac{1}{81920} \)

The sixth term is \( \frac{1}{320} \).

The third term is \( \frac{1}{80} \).

Find the geometric mean of 5 and 45.

\( \sqrt{5 \cdot 45} = \sqrt{225} = 15 \)

Find the missing term of the geometric sequence.

9-3

Practice (continued)

Form K

Geometric Sequences

Find the missing term of each geometric sequence. It could be the geometric mean or its opposite.

13. \( \ldots, 45, \ldots \)

Find the geometric mean of 5 and 45.

\( \sqrt{5 \cdot 45} = \sqrt{225} = 15 \)

14. \( 2, \ldots, 72, \ldots \)

Error Analysis

On a recent math test, your classmate was asked to find the missing term in the geometric sequence \( 4, \ldots, 256 \). Her answer was 128. What error did your classmate make? What is the correct answer?

She found the arithmetic mean of 256 and 4 rather than the geometric mean; 32

19. The bacteria population in a petri dish was 14 at the beginning of an experiment. After 30 min, the population was 28, and after an hour the population was 56.

a. Write an explicit definition to represent this sequence. \( a_n = 14 \cdot 2^{n/30} \)

b. If this pattern continues, what will be the bacteria population after 4 hr? \( 70368 \)

20. A corporation earned a profit of \$420,000 in its first year of operation. Over the next 10 years, the company’s CEO hopes to increase the profit by 8% each year. If the CEO reaches her goal, what will be the company’s profit in its seventh year, to the nearest dollar? \$586,487

Doubling Periods in Geometric Sequences

Consider the geometric sequence \( 3, 6, 12, 24, \ldots \) to determine the doubling period of the geometric sequence.

1. Describe how the terms of the sequence are related.

The terms of the sequence are related by multiplication by 2.

2. What is the doubling period of a geometric sequence?

The doubling period of a geometric sequence is the number of terms needed to reach a term at least twice as large as a given term. What is the doubling period for the given sequence? 2 terms

3. Write the first ten terms of the geometric sequence \( a_0 = 2 \) in two decimal places. \( 2, 3.3, 3.63, 3.99, 4.39, 4.83, 5.31, 5.85, 6.43, 7.07 \)

4. What is the doubling period for \( a_0 = 35 \) for \( a_n = 3 \cdot 2^n \)? 8 terms; 2 terms

Although the doubling period does not depend on which term is given, it does depend on the common ratio. For what value(s) of \( r \) is the doubling period of a geometric sequence greater than \( 7 \)? \( 0 < r < 2 \)

The idea of a doubling period applies to certain everyday situations. For example, under optimum conditions, bacteria reproduce by splitting in two. Their numbers increase geometrically over time. Suppose at noon on a certain day, there are 1000 bacteria in a dish. At 6 p.m. on the same day, there are 9000 bacteria.

5. If a count is taken every hour, how many terms are in the geometric sequence? What is the common ratio? What is the doubling period? 3; \( 2 \); 2 terms

6. If a count is taken every 40 min, how many terms are in the sequence? What is the common ratio? What is the doubling period? 10; \( 2 \); 2 terms

7. In both cases, how many hours does it take the bacteria to double? 2
1. **Exercises**

Find the 12th term of the geometric sequence $5, 15, 45, \ldots$.

**Solution:**

The common ratio, $r$, is $15/5 = 3$.

The explicit formula for the $n$th term of a geometric sequence is $a_n = a_1 \cdot r^{(n-1)}$.

Using this formula to find the 12th term:

$$a_{12} = 5 \cdot 3^{(12-1)} = 5 \cdot 3^{11} = 5 \cdot 177147 = 885735$$

2. **Exercises**

Identify the following series as finite or infinite.

3. **Exercises**

Compute. Round to the nearest hundredth.

4. **Exercises**

Know the number of seats in a row increases by three with each successive row. The first row has 18 seats.

**Problem:**

Find the total seating capacity of the theater.

**Solution:**

The number of seats in the $n$th row is $18 + 3(n-1)$. The total seating capacity is the sum of the first $n$ terms of this arithmetic sequence.

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

where $a_1 = 18$, $d = 3$, and $n$ is the number of rows.

5. **Exercises**

Your friend’s rent was $1304.77 in 2009.

**Problem:**

By clipping coupons and eating more meals at home, your family plans to decrease her rent in 2010 by $5. What is the total amount of money generated by a full house?

**Solution:**

The rental cost in 2010 is $1304.77 - 5 = 1300.77$.

6. **Exercises**

Your friend’s rent in 2003 was $1092.73. This means that her rent is increasing by 5% each year. What will be her rent in 2009?

**Solution:**

Use the formula $a_n = a_1 \cdot r^{(n-1)}$ where $a_1 = 1092.73$, $r = 1.05$, and $n = 7$ (from 2003 to 2009).

$$a_7 = 1092.73 \cdot 1.05^{(7)} = 1092.73 \cdot 1.4775 = 1612.08$$

7. **Exercises**

What are the ticket prices for each set of 5 rows?

**Solution:**

The ticket prices can be found by using the explicit formula for an arithmetic sequence.

8. **Exercises**

How can you use a graphing calculator to evaluate the series?

**Solution:**

Use the sum command and the sequence command.

9. **Exercises**

What is the total amount of money generated by a full house?

**Solution:**

The total amount of money generated by a full house is $46,950.

10. **Exercises**

What is the total amount of money generated by a full house?

**Solution:**

The total amount of money generated by a full house is $50,350.

11. **Exercises**

What is the total amount of money generated by a full house?

**Solution:**

The total amount of money generated by a full house is $54,650.
Find the sum of each finite arithmetic series.

- (continued)
Short Response

7. What is the sum of the finite arithmetic series 2 + 4 + 6 + ··· + 50? Show your work.

[Solution: 1050; Sn = 50(105)]

8. Which of the following is an infinite series?

- 9. How many series are there? 16

10. Express the sum of all the pairs using multiplication. n(2an + 0) = 150n

9-4

Reteaching (continued)

Arithmetic Series

Problem

The debate club is offering a prize at the end of 10 weeks to a current member who brings three new members for the first meeting, and then increases the number of new members they bring each week by two thereafter. One member qualified for the prize with the minimum number of new members. How many new members did the member bring at Week 10? For all 10 weeks?

Step 1 Identify key information in the problem.
To win the prize, a member must bring three members to the first meeting, so n = 3. A member must also bring two more new members to each meeting thereafter. With the help of a tutor, a student's weekly quiz scores have increased during...
**Geometric Series**

**Problem**

What is the sum of the geometric series \(2 + 6 + 18 + 54 + \ldots + 14587\)?

\[
\begin{align*}
\frac{18}{2} &+ \frac{18}{2^2} + \frac{18}{2^3} + \ldots + \frac{18}{2^7} \\
\text{Identify the common ratio and the 1st term.} \\
\frac{9}{2} &+ \frac{9}{2^2} + \frac{9}{2^3} + \ldots + \frac{9}{2^7} \\
\text{Use the explicit formula.} \\
729 &+ 364.5 + 182.25 + \ldots + 2.8125 \\
\text{Divide each side by 2.} \\
364.5 &+ 182.25 + 91.125 + \ldots + 1.40625 \\
\text{Use the sum formula.} \\
S_n &+ \ldots + S_5 \\
\text{Substitute 1 for } a_n \text{ for } n, 5 \\
S_5 &+ \ldots + S_5 \\
\text{Substitute 2 for } a_n, 3 \text{ for } r, \text{ and } 7 \text{ for } n.
\end{align*}
\]

**Solution**

The sum of the geometric series \(2 + 6 + 18 + 54 + \ldots + 14587\) is 14,587.

**Exercise**

What is the sum of the geometric series \(1 + 3 + 9 + 27 + 81 + 243 + \ldots + 10247\)?

\[
\begin{align*}
2 &+ \frac{2}{3} + \frac{2}{9} + \ldots + \frac{2}{3^6} \\
\text{Identify the common ratio and the 1st term.} \\
1 &+ \frac{1}{3} + \frac{1}{9} + \ldots + \frac{1}{3^6} \\
\text{Use the explicit formula.} \\
3 &+ \frac{3}{9} + \frac{3}{27} + \ldots + \frac{3}{243} \\
\text{Divide each side by 1.} \\
9 &+ \frac{9}{27} + \frac{9}{81} + \ldots + \frac{9}{729} \\
\text{Use the sum formula.} \\
S_n &+ \ldots + S_5 \\
\text{Substitute 1 for } a_n, 3 \text{ for } r, \text{ and } 7 \text{ for } n.
\end{align*}
\]

**Solution**

The sum of the geometric series \(1 + 3 + 9 + 27 + 81 + 243 + \ldots + 10247\) is 10,247.

---

**Think About a Plan**

**Geometric Series**

**Communications** Many companies use a telephone chain to notify employees of a closing due to bad weather. Suppose a company’s CEO calls three people. Then each of these people calls three others, and so on.

1. Make a diagram to show the first three stages in the telephone chain. How many calls are made at each stage?
2. Write the series that represents the total number of calls made through the first six stages.
3. How many employees have been notified after stage six?

**Solution**

1. The number of calls at each stage is given by the geometric sequence \(3, 9, 27, 81, 243, 729\). Each term is three times the previous term.

2. The sum of the series for the first six stages is given by the formula \(S_n = \frac{a_1(1 - r^n)}{1 - r}\), where \(a_1 = 3\) and \(r = 3\). Therefore, the total number of calls made through the first six stages is \(S_6 = \frac{3(1 - 3^6)}{1 - 3} = 1215\).

3. The number of employees notified after stage six is the sum of the series for the first six stages, which is 1215.

---

**Open Ended** Write an infinite geometric series that converges to 2. Show your work. Check students’ work.

**Solution**

An example of an infinite geometric series that converges to 2 is \(\sum_{n=0}^{\infty} 0.5^n\). The sum of this series is 2.

---

**Error Analysis** Your friend says that an infinite geometric series cannot have a sum because it’s infinite. You say that it is possible for an infinite geometric series to have a sum. Who is correct? Explain.

**Solution**

Both friends are correct. An infinite geometric series can have a sum if the common ratio is between -1 and 1. The sum of the series \(\sum_{n=0}^{\infty} r^n\) converges to \(\frac{1}{1-r}\) if \(|r| < 1\).

---

**Writing** Describe in general terms how you would find the sum of a finite geometric series. Identify the first term, common ratio, and nth term. Use the explicit formula to find the sum. Then, use the sum formula with the first term, common ratio, and n to find the sum of the series.

**Solution**

To find the sum of a finite geometric series, identify the first term (\(a_1\)), common ratio (\(r\)), and nth term (\(a_n\)). Use the explicit formula \(a_n = a_1r^{n-1}\) to find \(a_n\). Then, use the sum formula \(S_n = \frac{a_1(1 - r^n)}{1 - r}\) to find the sum of the series.
9-5 Practice
Geometric Series

Find the sum of each finite geometric series.

1. 2 + 6 + 18 + ... + 4374

   Find the number of terms.
   \[ a_n = a_1 r^{n-1} \]
   Use the sum formula.
   \[ S_n = \frac{a_1(1 - r^n)}{1 - r} \]
   \[ 2 + 6 + 18 + ... + 4374 \]
   \[ a_1 = 2 \]
   \[ r = 3 \]
   \[ n = 10 \]
   \[ S_{10} = \frac{2(1 - 3^{10})}{1 - 3} = 4374 \]

2. 1 + 2 + 4 + ... + 2048

   \[ S_{10} = \frac{2(1 - 3^{10})}{1 - 3} = 4374 \]

3. 4 + 8 + 16 + ... + 2048

   \[ S_{10} = \frac{2(1 - 3^{10})}{1 - 3} = 4374 \]

4. 3 + 6 + 12 + ... + 192

   \[ S_{6} = \frac{3(1 - 2^6)}{1 - 2} = 192 \]

5. 2 + 4 + 8 + ... + 1024

   \[ S_{11} = \frac{2(1 - 2^{11})}{1 - 2} = 1024 \]

6. Find the sum of the geometric series 2 – 4 + 8 – 16 + ... + 8192. Explain how you found the sum.

   \[ S_{13} = \frac{2(1 - (-2)^{13})}{1 - (-2)} = 8192 \]

7. A family farm produced 2400 ears of corn in its first year. For each of the next 9 yr, the farm increased its yearly corn production by 15%. How many ears of corn did the farm produce during this 10-yr period?

   \[ 2400 \times (1.15)^9 \approx 8679 \]

9-5 Practice (continued)
Geometric Series

Determine whether each infinite geometric series diverges or converges. Find the sum of the series converges.

8. 1 + \frac{1}{2} + \frac{1}{4} + \ldots

   \[ S = \frac{a_1}{1 - r} \]
   \[ a_1 = 1 \]
   \[ r = \frac{1}{2} \]
   \[ S = \frac{1}{1 - \frac{1}{2}} = 2 \]
   converges

9. 2 + \frac{4}{3} + \frac{8}{9} + \ldots

   \[ S = \frac{a_1}{1 - r} \]
   \[ a_1 = 2 \]
   \[ r = \frac{2}{3} \]
   \[ S = \frac{2}{1 - \frac{2}{3}} = 6 \]
   converges

10. \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots

   \[ S = \frac{a_1}{1 - r} \]
   \[ a_1 = \frac{1}{2} \]
   \[ r = \frac{1}{2} \]
   \[ S = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1 \]
   converges

11. \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \ldots

   \[ S = \frac{a_1}{1 - r} \]
   \[ a_1 = \frac{1}{3} \]
   \[ r = \frac{1}{2} \]
   \[ S = \frac{\frac{1}{3}}{1 - \frac{1}{2}} = \frac{2}{3} \]
   converges

12. 2 + 4 + 8 + 16 + \ldots

   \[ S = \frac{a_1}{1 - r} \]
   \[ a_1 = 2 \]
   \[ r = \frac{1}{2} \]
   \[ S = \frac{2}{1 - \frac{1}{2}} = 4 \]
   diverges

13. Your classroom is trying to cut down on the amount of time of television is spent watching television. In January, he spent a total of 3240 min watching television. He watched television for 3240 min in February and 2916 min in March. If this pattern continues, how many minutes of television will he watch per month?

   \[ 3240 \times (0.9) \approx 2916 \]

14. Your math teacher asks you to choose between two offers. The first offer is to receive one penny on the first day, 3 pennies on the second day, 9 pennies on the third day, and so on, for 14 days. The second offer is to receive 4 pennies on the first day, 12 pennies on the second day, 36 pennies on the third day, and so on, for 14 days. Which offer is better? What is the difference between the total amounts received?

   \[ 2 \times 4^{14} - 1 \]

9-5 Standardized Test Prep
Geometric Series

Gridded Response

Solve each exercise and enter your answer in the grid provided.

1. What is the value of \(a_1\) in the series \(\sum_{n=1}^{7} \frac{1}{2^n}\)?

   \[ a_1 = \frac{1}{2} \]

2. What is the sum of the geometric series \(2 + 4 + 8 + \ldots + 4867\)?

   \[ S_{10} = \frac{2(1 - 3^{10})}{1 - 3} = 4374 \]

3. A community organizes a phone tree in order to alert each family of emergencies. In the first stage, one person calls five families. In the second stage, each of the five families calls another five families, and so on. How many stages need to be reached before 600 families or more are called?

   \[ 5^k \geq 600 \]

4. What is the approximate whole number sum for the infinite geometric series \(\sum_{n=0}^{\infty} \frac{1}{10^n}\)?

   \[ S = \frac{1}{1 - \frac{1}{10}} = 10 \]

5. What is the sum of the geometric series \(1 + \frac{1}{2} + \frac{1}{4} + \ldots\)? Enter your answer as a fraction.

   \[ S = \frac{2}{1} = 2 \]

9-5 Enrichment
Geometric Series

An infinite geometric series converges if the absolute value of the common ratio is less than 1 (|r| < 1). A power series is an infinite series where each term depends on a variable. Each value of x will give you a specific infinite series, which may converge or diverge.

1. Evaluate the expression \(\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}\) for \(x = 2\) and for \(x = \frac{1}{2}\).

2. You can evaluate many expressions with a power series called a Taylor series. To evaluate \(\sum_{n=0}^{\infty} \frac{x^n}{n!}\) you use the Taylor series \(1 + x + x^2 + x^3 + \ldots\) for \(x = 2\) and for \(x = \frac{1}{2}\) approximately.

3. Determine the sum of the first five terms of the Taylor series \(1 + x + x^2 + x^3 + \ldots\). for \(x = 2\) and for \(x = \frac{1}{2}\).

4. For which value of x in your computation above a better approximation for the value of the expression \(\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}\)? What might need to be true about the value of x in order for this Taylor series to converge to the value of this expression? \(\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}\) converges.

5. You can use a different Taylor series to evaluate \(e^x\). Write the Taylor series \(e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\) in summation notation.

6. Evaluate \(\sum_{n=0}^{\infty} \frac{x^n}{n!}\) for \(x = \frac{1}{2}\). Evaluate the first four terms of the Taylor series \(1 + x + x^2 + x^3 + \ldots\) for \(x = \frac{1}{2}\). Round your answers to the nearest thousandth.

7. Does the Taylor series for \(e^x\) converge if \(|x| > 1|\)? How is it that you give the same value as the function \(y = e^x\)? Explain your reasoning.

Answers may vary. Sample: Yes; yes; for any value of \(x\), the terms essentially decrease rapidly to 0. If you add up enough terms, you will get a good approximation.
10. Determine whether each sequence is arithmetic. If so, identify the common difference. Do you know HOW?

3. Find the first five terms of each sequence.

a. $a_n = n^2 + 2n$, for $n = 1, 2, 3, 4, 5$

b. $a_n = n - 6$, for $n = -5, -4, -3, -2, -1$

d. $a_n = 2^n$, for $n = 3, 4, 5, 6, 7$

e. $a_n = n^3$, for $n = 1, 2, 3, 4, 5$

4. Find the sum of each arithmetic sequence.

a. $10, 8, 6, 4, ...$

b. $-8, -12, -16, ...$

c. $5, 9, 13, 17, ...$

d. $3, 9, 15, 21, ...$

5. Determine whether each sequence is arithmetic. If so, identify the common difference.

a. $-2, 2, -2, 2, -2, ...$

b. $-3, -5, -7, -9, ...$

c. $2, 4, 6, 8, 10, ...$

6. Find the missing term of each arithmetic sequence.

a. $23, 28, 33, 38, ...$

b. $-1, 2, 5, 8, ...$

c. $4, 8, 12, 16, ...$

7. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

a. $3, 6, 12, 24, 48, ...$, for $n = 5$

b. $5, 10, 20, 40, 80, ...$, for $n = 6$

c. $2, 4, 8, 16, 32, ...$, for $n = 5$

8. Vocabulary. Explain what it means for a formula to be an explicit formula. An explicit formula describes the $n^{th}$ term in a sequence using $n$.

9. Open-Ended. Give an example of an arithmetic sequence. Check students’ work.
Write a recursive definition and an explicit formula for each sequence. Then find $a_n$.

1. $a_1 = 23, \ a_2 = 10, \ a_3 = -3, \ a_4 = -13, \ \ldots$  
2. $a_1 = 19, \ a_2 = 39, \ a_3 = 67, \ a_4 = 103, \ \ldots$  
3. $a_1 = 1, \ a_2 = 3, \ a_3 = 5, \ a_4 = 7, \ \ldots$

Find the first five terms in each arithmetic sequence.

4. $a_1 = 0, \ a_2 = 6, \ a_3 = 12, \ a_4 = 18, \ a_5 = 24$ 
5. $a_1 = 65, \ a_2 = 130, \ a_3 = 195, \ a_4 = 260, \ a_5 = 325$ 
6. $a_1 = 72, \ a_2 = 144, \ a_3 = 216, \ a_4 = 288, \ a_5 = 360$

Determine whether each sequence is arithmetic, geometric, or neither. Then find the ninth term.

8. $a_1 = 1, \ a_2 = 3, \ a_3 = 6, \ a_4 = 10, \ a_5 = 15, \ a_6 = 21, \ a_7 = 28, \ a_8 = 36, \ a_9 = 45$ 
9. $a_1 = 2, \ a_2 = 3, \ a_3 = 5, \ a_4 = 8, \ a_5 = 13, \ a_6 = 21, \ a_7 = 34, \ a_8 = 55, \ a_9 = 89$ 
10. $a_1 = 1, \ a_2 = 3, \ a_3 = 5, \ a_4 = 7, \ a_5 = 9, \ a_6 = 11, \ a_7 = 13, \ a_8 = 15, \ a_9 = 17$

Determine whether each series is arithmetic; geometric; or neither.

11. $S = 5 + 10 + 15 + 20 + \ldots$ 
12. $S = 2 + 4 + 8 + 16 + \ldots$ 
13. $S = 6 + 12 + 24 + 48 + \ldots$

When she plants the last bulbs, how many bulbs will the gardener plant on October 15?

14. $c_1 = 1, \ c_2 = 2, \ c_3 = 3, \ c_4 = 4, \ c_5 = 5, \ c_6 = 6, \ c_7 = 7, \ c_8 = 8, \ c_9 = 9, \ c_{10} = 10$ 
15. $c_1 = 1, \ c_2 = 2, \ c_3 = 4, \ c_4 = 8, \ c_5 = 16, \ c_6 = 32, \ c_7 = 64, \ c_8 = 128, \ c_9 = 256, \ c_{10} = 512$

Evaluate each infinite geometric series.

16. $S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ 
17. $S = 2 + 4 + 8 + 16 + 32 + \ldots$ 
18. $S = 5 + 10 + 20 + 40 + 80 + \ldots$

Write a recursive definition for each sequence.

19. $c_1 = 1, \ c_2 = 2, \ c_3 = 3, \ c_4 = 4, \ c_5 = 5, \ c_6 = 6, \ c_7 = 7, \ c_8 = 8, \ c_9 = 9, \ c_{10} = 10$ 
20. $c_1 = 1, \ c_2 = 3, \ c_3 = 5, \ c_4 = 7, \ c_5 = 9, \ c_6 = 11, \ c_7 = 13, \ c_8 = 15, \ c_9 = 17, \ c_{10} = 19$

Write an explicit formula to model the number of bulbs she plants each day.

21. $c_1 = 1, \ c_2 = 2, \ c_3 = 3, \ c_4 = 4, \ c_5 = 5, \ c_6 = 6, \ c_7 = 7, \ c_8 = 8, \ c_9 = 9, \ c_{10} = 10$ 
22. $c_1 = 1, \ c_2 = 3, \ c_3 = 5, \ c_4 = 7, \ c_5 = 9, \ c_6 = 11, \ c_7 = 13, \ c_8 = 15, \ c_9 = 17, \ c_{10} = 19$

Find the sum of each finite arithmetic series.

23. $S = 2 + 4 + 6 + 8 + \ldots + 20$ 
24. $S = 3 + 6 + 9 + 12 + \ldots + 27$ 
25. $S = 4 + 8 + 12 + 16 + \ldots + 56$

Find the sum of each finite geometric series.

26. $S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{512}$ 
27. $S = 2 + 4 + 8 + 16 + \ldots + 512$ 
28. $S = 3 + 6 + 9 + 12 + \ldots + 90$ 
29. $S = 4 + 8 + 12 + 16 + \ldots + 192$

Find the 18th term of each arithmetic sequence.

30. $a_1 = 6, \ a_2 = 12, \ a_3 = 18, \ a_4 = 24, \ a_5 = 30$ 
31. $a_1 = 15, \ a_2 = 30, \ a_3 = 45, \ a_4 = 60, \ a_5 = 75$ 
32. $a_1 = 8, \ a_2 = 16, \ a_3 = 24, \ a_4 = 32, \ a_5 = 40$ 
33. $a_1 = 3, \ a_2 = 6, \ a_3 = 9, \ a_4 = 12, \ a_5 = 15$ 
34. $a_1 = 1, \ a_2 = 2, \ a_3 = 3, \ a_4 = 4, \ a_5 = 5$ 
35. $a_1 = 1, \ a_2 = 3, \ a_3 = 5, \ a_4 = 7, \ a_5 = 9$ 
36. $a_1 = 2, \ a_2 = 4, \ a_3 = 6, \ a_4 = 8, \ a_5 = 10$

Find the sum of each finite series.

37. $\sum_{n=1}^{10} 2n$ 
38. $\sum_{n=1}^{5} n^2$ 
39. $\sum_{n=1}^{4} (n + 1)$ 
40. $\sum_{n=1}^{3} (n - 3)$

Write a sequence and describe it using both an explicit definition and a recursive definition.

1. $a_1 = 5, \ a_2 = 10, \ a_3 = 15, \ a_4 = 20, \ a_5 = 25$ 
2. $a_1 = 1, \ a_2 = 2, \ a_3 = 3, \ a_4 = 4, \ a_5 = 5$ 
3. $a_1 = 1, \ a_2 = 3, \ a_3 = 5, \ a_4 = 7, \ a_5 = 9$

Write an arithmetic sequence in summation notation.

4. $\sum_{n=1}^{10} (2n - 1)$ 
5. $\sum_{n=1}^{5} (n^2)$ 
6. $\sum_{n=1}^{8} (n^3)$ 
7. $\sum_{n=1}^{6} (n^4)$ 
8. $\sum_{n=1}^{10} (2n + 1)$ 
9. $\sum_{n=1}^{5} (3n - 2)$ 
10. $\sum_{n=1}^{4} (n^2 + 1)$
### Chapter 9 Test

**Do you know HOW?**

Find the first five terms of each sequence.

1. \(a_n = 3n + 4\)
   - 7, 10, 13, 16, 19

2. \(a_n = n^2 + 2\)
   - 2, 6, 12, 20, 30

3. \(a_n = 2n^2 - 2\)
   - -1, -4, -9, -16, -25

Write an explicit formula for each sequence. Then find the 12th term.

4. \(\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\)
   - \(a_n = \frac{n+1}{n+2}\); \(a_{12} = \frac{13}{14}\)

5. \(0, 1, 2, 4, 8\)
   - \(a_n = 2^n\); \(a_{12} = 4096\)

6. \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\)
   - \(a_n = \frac{1}{n}\); \(a_{12} = \frac{1}{12}\)

Find the 20th term of each arithmetic sequence.

7. \(2, 5, 8, 11, \ldots\)
   - \(a_n = 2n + 1\); \(a_{20} = 41\)

8. \(-3, 0, 3, 6, 9, \ldots\)
   - \(a_n = 3n - 3\); \(a_{20} = 57\)

**Do you UNDERSTAND?**

10. **Writing** Find the missing term in the sequence below. Then explain how you found the term.

    - 12, 24, 44, 64, ...
    - Answers may vary. Sample: First, I found the sum of 12 and 44, which is 56. Then I divided the sum by 2 to find the missing term, 28.

11. **Reasoning** Rita must find the 35th term in the sequence that begins 2, 8, 26, ...

    - She needs to find the answer as fast as possible. Should Rita use a recursive definition or an explicit formula? Why? 
    - Rita uses a recursive definition because it allows her to go from one term to the next without the need for a formula.

12. A bus has 6 people on it as it pulls out of the station to begin its route. After one stop, there are 11 people on the bus. At the second stop, there are 16 people on the bus. If this pattern continues, how many people will be on the bus after 10 stops? 91 people

### Chapter 9 Performance Tasks

**Task 1**

a. Use your graphing calculator to graph the function \(f(x) = 2x\) over the domain \(x \geq 0\).

b. Use the TABLE feature on your calculator to make a table of values of the function when \(x = 1, 2, 3, 4\). 

c. Determine whether the sequence of function values is arithmetic, geometric, or neither. Find its sequence in summation notation.

d. Write a recursive definition and an explicit formula for the sequence of function values.

**Task 2**

a. Determine whether the sequence \(27, 9, 3, \ldots\) is geometric, arithmetic, or neither. 

b. Write a recursive definition and an explicit formula for the sequence.

**Task 3**

a. Determine whether the sequence \(2, 4, 8, 16, \ldots\) is arithmetic, geometric, or neither. 

b. Write a recursive definition and an explicit formula for the sequence. 

**Writing** Find the possible values of the missing term in the following geometric sequence, and explain how you found the answer.

- 6, 24, 96, ...
- Answers may vary. Sample: First, I found the product of 6 and 4, which is 24. Then I found the square root of 24, which is 4.
Multiple Choice

1. What is the y-intercept of \( y = -0.5x^2 + 7 \)?
   - A \((0, 7.5)\)
   - B \((0, 2.5)\)
   - C \((0, 0)\)
   - D \((4, 0)\)
   - E \((4, 2)\)

2. What is \( \frac{2}{3} \) in simplest form?
   - A \(\frac{2}{3}\)
   - B \(\frac{4}{5}\)
   - C \(\frac{6}{12}\)
   - D \(\frac{5}{9}\)
   - E \(\frac{3}{5}\)

3. Which expression is equivalent to \( \sqrt[5]{x^3} \)?
   - A \(\sqrt[5]{x^3}\)
   - B \(\sqrt[3]{x^5}\)
   - C \(\sqrt[15]{x^3}\)
   - D \(\sqrt{3x}\)
   - E \(\sqrt{x^3}\)

4. How is the polynomial \( x^2 - x^2 + 4x + 15 \) classified by degree?
   - A linear
   - B quadratic
   - C cubic
   - D quartic
   - E quintic

5. The discriminant of a quadratic equation has a value of 13. Which of the following is true?
   - A There is one real solution.
   - B There are two real solutions.
   - C There are no real solutions.
   - D There is one complex solution.
   - E There are two complex solutions.

6. Which of these does not have the same value as the others?
   - A \(\log_2(8)\)
   - B \(\log_3(27)\)
   - C \(\log_4(128)\)
   - D \(\log_5(125)\)
   - E \(\log_8(256)\)

7. Which inequality is graphed?
   - A \(y \leq x + 4\)
   - B \(y = x + 4\)
   - C \(y < x + 4\)
   - D \(y > x + 4\)
   - E \(y \geq x + 4\)

8. If \(f(x) = 4x + 1\) and \(g(x) = 2^x\), what is the value of \(f(3) - g(2)\)?
   - A \(1022\)
   - B \(21\)
   - C \(377\)
   - D \(-127\)
   - E \(-21\)

9. Graph the system of inequalities:
   - \(y < 2x + 1\)
   - \(y > -x + 3\)

10. Describe how the graph of \( y = \log_2(a - 2) + 5 \) compares to the graph of the parent function.
    - The graph of the parent function is shifted right two units and up five units.
    - The graph of the parent function is flipped over the x-axis.
    - The graph of the parent function is stretched vertically.
    - The graph of the parent function is compressed vertically.
    - The graph of the parent function is shifted left two units and up five units.

11. How can the relationship between variables in the table be described?
    - The variables in the table, x and y, have an inverse variation relationship. As x-values increase, y-values decrease, but the product of their pairs remains constant.

12. Use the sequence: 100, 95, 90, 85, ... This is an arithmetic sequence in which each term is a.
    - b. Describe the sequence as words.
    - c. Find the next three terms.
    - d. 80, 75, 70

13. Water leaks from a 10,000-gal tank at a rate of 5 gal/h. Write a linear model for the situation and use it to find the amount of water in the tank after 24 h. \(w = -5t + 10,000\), 9880 gal

Extended Response

14. You have a coupon for $10 off a CD. You also get a 20% discount if you show your membership card in the CD club. How much more would you pay if the cashier applies the coupon first? Use composite functions. Show your work.
   - a. Find the cost of the CD after the 20% discount.
   - b. Find the cost of the CD after applying the coupon.
   - c. Determine the total cost of the CD after applying both discounts.
   - d. Compare the total cost with and without the discount. Which method is more cost-effective?

Activity 2: Designing

Students use grid paper to enlarge designs. They use the same ratios sequentially to draw lengths which form geometric sequences. They then write explicit or recursive formulas for their sequences.

Beginning the Chapter Project

When a book is being made, artists, designers, and photographers work with writers and editors to make the pages visually attractive. These professionals often work with patterns involving arithmetic and geometric sequences.

In this project, you will see how perspective affects perceived lengths and distances. You will see grids to change the sizes of drawings. You will also learn how a designer crops a photo, then enlarges or reduces it.

Activities

Activity 1: Researching

Research the concepts of one- and two-point perspective and vanishing points in art.

- What is the relationship between these lengths? How does this relate to your research on perspective?
- Trace the four arrows at the right, moving the paper to the left after tracing the longest arrow so that it is farther away from the others that are in view. What do you notice?
- Make a simple drawing of three or more similar objects whose lengths can be represented by an arithmetic sequence. Write the corresponding arithmetic sequence, and a recursive or explicit formula for that sequence.
- Check student work. Answers may vary. Sample: The lengths form an arithmetic sequence; answers may vary. Lengths: These lines are the same length. The arrows have an arithmetic sequence; answers may vary. Lengths: These lines are the same length. There is no longer a vanishing point.
- Check student work.

Activity 2: Designing

When a book is made, a designer or artist may change the size of an original sketch to fit the space available on a page. One way to change the dimensions of a sketch is to use graph paper with different size squares.

- Draw a figure or design on a sheet of graph paper. Label this and record its dimensions.
- Enlarge the original figure by copying each portion of the design so that it is further away from the others that are in view.
- Measure the lengths of the arrows shown at the right. The arrows move to the left after tracing the longest arrow so that it is farther away from the others that are in view.
- Check students’ work. Answers may vary. Lengths: These lines are the same length. There is no longer a vanishing point.
- Check student work.

Answers
Chapter 9 Project: Get the Picture

Activity 3: Analyzing

Photographs are often cropped so that only part of the photograph remains. Then, this cropped portion can be reduced or enlarged. Choose a photograph in a textbook. Place a piece of paper over the photograph, trace its original size, and draw a rectangle to indicate a portion of the photograph that you would like to crop. Draw a diagonal from the lower left corner to the upper right corner of the rectangular cropped area. If this diagonal is extended through the upper right corner of the cropped area, and a point selected anywhere along the diagonal or its extension, then the rectangle having the chosen point as its upper right corner (and the same lower left corner as the original cropped area) will have dimensions that are proportional to the dimensions of the cropped area.

- Measure the dimensions and the length of the diagonal of the cropped area.
- Write the first four terms of an arithmetic sequence that has the length of the diagonal of the cropped area as its first term. Using the terms of your sequence as diagonal lengths, find the four corresponding photo widths. What do you notice about this list of widths?
- Write the first four terms of a geometric sequence that has the length of the diagonal of the cropped area as its first term. Using the terms of your sequence as diagonal lengths, find the four corresponding photo widths. What do you notice about this list of widths?

Finishing the Project

The answers to the activities should help you complete your project. Prepare a presentation or demonstration that summarizes how an artist, a designer, or a photographer uses sequences. Present this information to your classmates. Then discuss the sequences you made.

Reflect and Revise

Review your summary. Are your drawings clear and correct? Are your sequences accurate? Practice your presentation in front of at least two people before presenting it to the class. Ask for their suggestions for improvement.

Extending the Project

Geometric and arithmetic patterns are used in other aspects of design and in other careers. Research other areas where sequences are applied.

Checklist Suggestions

☐ Activity 1: relating perspective and arithmetic sequences
  - Use art books from the school library or the Internet.

☐ Activity 2: relating dimensions and geometric sequences
  - Use grid paper to draw simple geometric designs.

☐ Activity 3: relating photo-cropping and sequences
  - Measure directly or use proportions to find the widths.
  - Does your display include examples of both arithmetic and geometric sequences? What artists or work of art with which you are familiar best demonstrate the concepts of one-point perspective, two-point perspective, or vanishing points?

Scoring Rubric

4 Calculations, sequences, and formulas are correct. Drawings are neat, accurate, and clearly show the sequences. Explanations are thorough and well thought out.
3 Calculations, sequences, and formulas are mostly correct with some minor errors. Drawings are neat and mostly accurate. Explanations lack detail or are not completely accurate.
2 Calculations contain both minor and major errors. Drawings are not accurate.
1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project

Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of the Project

Your project demonstrates your understanding of geometric and arithmetic patterns and their applications in design. The incorporation of sequences in your presentation shows a thorough knowledge of the subject. Your drawings and explanations are clear and accurate, making your project a valuable learning tool. Keep up the good work!
Problem
Graph \(x^2 - y^2 = 25\). What are the lines of symmetry? What are the domain and range?

1. Read the problem. What process are you going to use to solve the problem?
   Answers may vary. Sample:
   Graph and find lines of symmetry, domain, and range.

2. Why do the \(y\)-values have the \(-2\) symbol in front of them?
   Answers may vary. Sample: You must take both the positive and negative square root.

3. Why are there no \(y\)-values for the \(x\)-values between \(-5\) and \(5\)?
   Answers may vary. Sample: The equation is undefined for those values of \(x\).

4. The graph is a hyperbola with its center at the origin. What points could he use to determine a sketch of the graph?
   Answers may vary. Sample: the center and one point on the circle.

5. Explain why the \(x\)-axis and \(y\)-axis are the lines of symmetry.
   Answers may vary. Sample: Both divide the graph into two mirror images.

6. How can you determine the \(x\)-axis and \(y\)-axis?
   Answers may vary. Sample: The \(x\)-values of the points on the graph make up the domain.

---

Graph each equation. Identify the conic section and describe the graph and its lines of symmetry. Then find the domain and range. See graphs below.

Graph each equation. Describe the graph and its lines of symmetry. Then find the domain and range.

- \(x^2 + y^2 = 25\)
- \(x^2 = y^2 = 16\)
- \(-2x + y = 2\)
- \(-x - 3y = 5\)
- \(x^2 + y^2 = 36\)
- \(x^2 = y^2 = 9\)

Match each equation with a graph in Exercises 4–9.

10. \(x^2 = y^2 = 4\)
11. \(x^2 = y^2 = 12\)
12. \(x^2 = y^2 = 9\)
13. \(x^2 = y^2 = 4\)
14. \(x^2 = y^2 = 36\)
15. \(x^2 - y^2 = 9\)

---

The ground near the airplane is shaped like a cone. This is heard by people on the ground as a sonic boom. What is the shape of the path on the ground?

1. The pressure disturbance is shaped like a cone. What is the shape of the path on the ground?
2. The ground near the airplane is shaped like a cone. What is the shape of the path on the ground?

Need
3. To solve the problem I need to find all possible intersections of the cone-shaped pressure disturbance and the plane of the ground.

Plan
4. How can a drawing or model help you solve this problem?
   Answers may vary. Sample: A model could help me see the possible orientations of a pressure cone formed by an airplane in flight.

5. Is there only one possible path on the ground? Explain. Yes; answers may vary. Sample: The airplane could be flying in more than one orientation, creating different intersections of a cone and a plane.

6. Sketch the possible orientations to the ground of the airplane and its pressure cone.

---

Sound An airplane flying faster than the speed of sound creates a cone-shaped pressure disturbance in the air. This is heard by people on the ground as a sonic boom. What is the shape of the path on the ground?

- \(x^2 + y^2 = 25\)
- \(x^2 = y^2 = 16\)
- \(-2x + y = 2\)
- \(-x - 3y = 5\)
- \(x^2 + y^2 = 36\)
- \(x^2 = y^2 = 9\)

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---

10.1 Think About a Plan
Exploring Conic Sections

Sound An airplane flying faster than the speed of sound creates a cone-shaped pressure disturbance in the air. This is heard by people on the ground as a sonic boom. What is the shape of the path on the ground?

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Think About a Plan
Exploring Conic Sections

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6. Sketch the possible orientations to the ground of the airplane and its pressure cone.

---

ANSWERS
10-1 Practice  
Exploring Conic Sections

Graph each equation, identify the conic section, and describe the graph and its lines of symmetry. Then find the domain and range.

1. \( x^2 + y^2 = 9 \)
   - Make a table of values.
     - If \( y = 0 \), then \( x = 3 \) or \( x = -3 \).
     - If \( y = 3 \), then \( x = 0 \).
   - Plot the points in the table. Connect the points with a smooth curve. The conic section is a circle.
   - The conic section has two lines of symmetry: x-axis and y-axis.
   - The smallest \( x \)-value is -3. The largest \( x \)-value is 3.
   - The domain is the set of all real numbers with \(-3 \leq x \leq 3\).
   - The smallest \( y \)-value is -3. The largest \( y \)-value is 3.
   - The range is the set of all real numbers with \(-3 \leq y \leq 3\).

2. \( x^2 + y^2 = 10 \)
   - Hyperbola; x-axis, y-axis; all real numbers.

3. \( x^2 - y^2 = 9 \)
   - Ellipse; x-axis, y-axis; -3 < x ≤ 3, -3 < y ≤ 3.

4. \( x^2 + y^2 = 9 \)
   - Hyperbola; center, -3 < x < 3, -3 < y < 3.

5. \( x^2 + y^2 = 36 \)
   - Ellipse; x-axis, y-axis; -3 < x ≤ 3, -3 < y ≤ 3.

6. \( 25x^2 + 4y^2 = 100 \)
   - Hyperbola; center, -4 < x ≤ 4, -4 < y ≤ 4.

7. \( 12x^2 + 12y^2 = 112 \)
   - Hyperbola; center, -4 < x ≤ 4, -4 < y ≤ 4.

8. Open-Ended: Describe the figures you can see that can be formed by the intersection of a plane and a rectangular prism. rectangle, parallelogram, line, point

10-1 Practice (continued)  
Exploring Conic Sections

Graph each circle with the given radius or diameter so that the center is at the origin. Then write the equation for each graph.

9. Radius 7
   - To begin, plot several points that are 7-units away from the origin. For example, plot \((7, 0), (-7, 0), (0, 7), \) and \((0, -7)\). Draw a circle through these points.
   - The equation of a circle is \(x^2 + y^2 = r^2\). The equation for this circle is \(x^2 + y^2 = 49\).

10. Radius 6
    - Ellipse
    - Obtuse triangle
    - Hyperbola

11. Radius 2
    - Circle
    - Parabola
    - Hyperbola

Mental Math

Each given point is on the graph of the given equation. Use symmetry to find at least one more point on the graph.

14. \((0, 3), (2, 3), (2, 1)\)
   - The graph is a parabola that consists of two branches. Its center is the origin.
   - The lines of symmetry are the y-axis and the x-axis. The domain is made up of all x-values with \(x \leq 0\) and \(x \geq 0\). The range is all real numbers.

15. \((2, 3), (2, 1), (0, 3), (0, 1)\)
   - The graph is a hyperbola that consists of two branches. Its center is the origin.
   - The lines of symmetry are the y-axis and the x-axis. The domain is made up of all x-values with \(x \leq 0\) and \(x \geq 0\). The range is all real numbers.

16. \((3, 0), (0, 3), (-3, 0), (0, -3)\)
   - Ellipse
   - Right triangle
   - Parabola

17. \((3, 0), (0, 3), (-3, 0), (0, -3)\)
   - Hyperbola
   - Obtuse triangle
   - Parabola

10-1 Standardized Test Prep  
Exploring Conic Sections

Multiple Choice
For Exercises 1–6, choose the correct letter.

1. What shape is the conic section \(x^2 + y^2 = 16\)? A circle, B ellipse, C parabola, D hyperbola
   - B

2. Which line is not a line of symmetry for \(x^2 + y^2 = 16\)? A \(y = x\), B \(y = 2\), C \(y = 3\), D \(y = 4\)
   - B

3. Which equation represents the graph at the right? A \(x^2 + y^2 = 16\), B \(x^2 - y^2 = 16\), C \(x^2 + y^2 = 16\), D \(x^2 + y^2 = 25\)
   - C

4. What are the lines of symmetry of a circle with the center at the origin? A the x-axis, B the y-axis, C all lines that pass through the center
   - C

5. What is the range of \(x^2 + 9y^2 = 144\)? A \(-3 \leq y \leq 3\), B \(-4 \leq y \leq 4\), C \(-16 \leq y \leq 16\), D \(-144 \leq y \leq 144\)
   - A

6. What is the domain of \(x^2 + y^2 = 16\)? A all real numbers, B \(-8 \leq x \leq 8\), C \(-6 \leq y \leq 6\), D \(-64 \leq x \leq 64\)
   - A

Short Response

7. Describe the graph of \(x^2 - y^2 = 16\). What is the center? What are the lines of symmetry? What are the domain and range?
   - The graph is a hyperbola that consists of two branches. Its center is the origin. The lines of symmetry are the x-axis and the y-axis. The domain is made up of all x-values with \(x = -4\) and \(x = 4\). The range is all real numbers.

10-1 Enrichment  
Exploring Conic Sections

In previous chapters, you solved linear systems of equations, linear-quadratic systems of equations, and quadratic systems of equations. Now you can solve other types of systems.

1. How many solutions are possible for a linear system of two equations?
   - Describe what type of lines will result in each solution.
     - One solution is possible if two lines intersect at one point. There is no solution if two lines are parallel. There are an infinite number of solutions if two lines coincide.

2. Consider a system of equations formed by a line and a parabola. How many solutions are possible? Sketch a graph to show each possibility. 2, 3, or 4 solutions are possible.

3. How many solutions are possible for a system of equations formed by a parabola and a circle? Sketch a graph to show each possibility. 0, 1, 2, or 4 solutions are possible.

4. How many solutions are possible for a system of equations formed by a parabola and a hyperbola? Sketch a graph to show each possibility. 0, 1, 2, or 4 solutions are possible.

5. How many solutions are possible for a system of equations formed by an ellipse and a hyperbola? Sketch a graph to show each possibility. 0, 1, 2, or 4 solutions are possible.

Solve each system of nonlinear equations by graphing and estimating the solutions.

6. \(x^2 + y^2 = 1\)
   - Form \(K\)

7. \(x^2 + y^2 = 16\)
   - Form \(K\)
There are four types of special curves called conic sections: parabolas, circles, ellipses, and hyperbolas. You previously learned about parabolas in Chapter 4. Information about the other three conic sections are organized in the table. In this lesson, you will work with conic sections whose centers are at the origin.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Circle</th>
<th>Ellipse</th>
<th>Hyperbola</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines of Symmetry</td>
<td>All lines through the center</td>
<td>through the center</td>
<td>to the center</td>
</tr>
<tr>
<td>Domain and Range</td>
<td>Any real numbers</td>
<td>All real numbers</td>
<td>All real numbers</td>
</tr>
</tbody>
</table>

**Exercises**

Identify the conic section and describe its lines of symmetry. Then find the domain and range.

1. Ellipse: 
   - Equation: $x^2 + y^2 = 9$
   - Symmetry: All lines through the center
   - Domain: $-3 \leq x \leq 3$, $-3 \leq y \leq 3$

2. Hyperbola: 
   - Equation: $x^2 - y^2 = 1$
   - Symmetry: All lines through the center
   - Domain: All real numbers, $y \neq 0$

3. Ellipse: 
   - Equation: $x^2 + 4y^2 = 16$
   - Symmetry: All lines through the center
   - Domain: $-4 \leq x \leq 4$, $-2 \leq y \leq 2$

4. Hyperbola: 
   - Equation: $x^2 - y^2 = 1$
   - Symmetry: All lines through the center
   - Domain: All real numbers, $y \neq 0$

5. Hyperbola: 
   - Equation: $x^2 - y^2 = 9$
   - Symmetry: All lines through the center
   - Domain: All real numbers, $y \neq 0$

6. Ellipse: 
   - Equation: $x^2 + 9y^2 = 25$
   - Symmetry: All lines through the center
   - Domain: $-5 \leq x \leq 5$, $-\frac{5}{3} \leq y \leq \frac{5}{3}$

7. Hyperbola: 
   - Equation: $x^2 - y^2 = 16$
   - Symmetry: All lines through the center
   - Domain: All real numbers, $y \neq 0$

8. Ellipse: 
   - Equation: $x^2 + 4y^2 = 16$
   - Symmetry: All lines through the center
   - Domain: $-4 \leq x \leq 4$, $-\frac{4}{2} \leq y \leq \frac{4}{2}$

9. Hyperbola: 
   - Equation: $x^2 - y^2 = 9$
   - Symmetry: All lines through the center
   - Domain: All real numbers, $y \neq 0$

**Reteaching**

Exploring Conic Sections

10-1

**Problem**

What is the graph of $4x^2 + 9y^2 = 36$? Identify the conic section and its lines of symmetry. Then find the domain and range.

**Step 1** Solve the equation for $y$.

$$4x^2 + 9y^2 = 36 \quad \Rightarrow \quad 9y^2 = 36 - 4x^2 \quad \Rightarrow \quad y^2 = \frac{36 - 4x^2}{9}$$

**Step 2** Substitute values of $x$ into the equation to make a table of values.

**Step 3** Plot the points in the table and connect them with a smooth curve. The graph should be symmetric.

The graph shows that the graph of the equation is an ellipse. It has two lines of symmetry: the $x$-axis and the $y$-axis.

The domain is all values of $x$ with $-3 \leq x \leq 3$. The range is all values of $y$ with $-2 \leq y \leq 2$.

**Exercises**

Graph each equation. Identify the conic section and its lines of symmetry. Then find the domain and range.

5. $25x^2 + 4y^2 = 100$
6. $x^2 + y^2 = 36$
7. $4x^2 - y^2 = 16$

**ELL Support**

Parabolas

What are the vertex, focus, and directrix of the parabola with equation $y = x^2 - 5x + 11$?

You wrote these steps to solve the problem on the note cards, but they got mixed up.

1. Find the vertex: 
   - Use $c = -\frac{b}{2a}$ to determine the distance from the vertex to the focus and from the vertex to the directrix.
2. Find the focus: 
   - The vertex is $(h, k)$, so the focus is $(h, k + \frac{1}{4a})$.
3. Find the directrix: 
   - Use $y = k - \frac{1}{4a}$ to determine the distance from the vertex to the directrix.
4. Complete the square to get the equation in vertex form.

Use the note cards to write the steps in order.

1. First: complete the square to get the equation in vertex form.
2. Second: use $c = -\frac{b}{2a}$ to determine the distance from the vertex to the focus and from the vertex to the directrix.
3. Next: the vertex is $(h, k)$, so the focus is $(h, k + \frac{1}{4a})$.
4. Finally: the directrix is $y = k - \frac{1}{4a}$.

**ANSWERS**

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12. Write an equation of a parabola with vertex at the origin and the given focus.

10-2 Practice Parabolas

Write an equation of a parabola with vertex at the origin and the given focus.

1. Focus (2, 0); $y = -x^2$
2. Focus (3, 0); $y = -x^2$
3. Focus at (0, 1); $y = -x^2$

Identify the vertex, the focus, and the directrix of the parabola with the given equation. Then sketch the graph of the parabola.

7. $y = \frac{1}{2}x^2$
8. $x = \frac{1}{2}y^2$

Write an equation of a parabola with vertex at the origin and the given directrix.

11. Directrix $x = 3$ $y = -\frac{1}{2}x^2$
12. Directrix $y = 4$ $x = -\frac{1}{2}y^2$
13. Directrix $x = -3$ $x = y^2$
14. Directrix $y = -3$ $x = -\frac{1}{2}y^2$

The center of a pipe with a diameter of 3 in. is located 10 in. from a mirror with a parabolic cross section used as a solar collector. The center of the pipe is at the focus of the parabola.

Write an equation to model the cross section of the mirror $y = \frac{1}{8}x^2$. The mirror exceeds 20 mm in every sunlight it would without the mirror. The amount of light collected by the mirror is directly proportional to its diameter. Find the width of the mirror. 10.5 in.

27. Writing What is the relationship between the focus of a parabola and the directrix of a parabola?

Answers may vary. The equation for a horizontal parabola should be in the form $x = ay^2$ while the equation for a vertical parabola should be in the form $y = bx^2$.

28. Error Analysis A student writes the equation of a parabola with vertex (6, 4) and focus (4, 4) $y = 2(x - 6)^2 + 4$. This is the incorrect equation for the parabola because it begins with $y$. The correct equation is $y = -2(x - 6)^2 + 4$.

30. Reasoning How can you find the value for $c$ for the parabola $x = \frac{1}{4}y^2 + 3$?

The value of $c$ is 2.5 because $c = \frac{9}{4}$ or $c = 2.5$.
10-2 Enrichment
Parabolas

Quadratic Maxima and Minima
Consider the equation of a parabola in standard form, \( y = a(x - h)^2 + k \). If \( a > 0 \), then the parabola opens upward and the vertex \((h, k)\) represents the lowest point, or minimum value, on the graph. Similarly, if \( a < 0 \), the parabola opens downward and the vertex \((h, k)\) represents the highest point, or maximum value, on the graph.

Suppose that a rancher has 100 yd of fencing with which to construct a rectangular field in such a way that the total area enclosed is a maximum.

1. If \( f \) represents the length of the field and \( w \) its width, what equation expresses the area \( A \) of the field?

2. How would you express the fact that the perimeter \( P \) of the field must be 100 yd? \( P = 2f + 2w = 100 \)

3. Solve the perimeter equation for \( f \) in terms of \( w \), and substitute into the area equation. What is the equation for \( A \) in terms of \( w \)?

4. Write your equation in standard form for a parabola. \( A = -w^2 - 25w + 625 \)

5. For which value of \( w \) is the area a maximum? \( w = 25 \text{ yd} \)

6. What is the corresponding length? \( f = 25 \text{ yd} \)

7. What is the area of the field? \( 625 \text{ yd}^2 \)

8. What is the shape? square

Repeat the exercises above assuming that the rancher has 200 yd of fencing.

9. What are the dimensions of the field enclosing the maximum area? 50 yd by 50 yd

10. On the basis of your results, what might you infer?

The largest rectangle with a fixed perimeter is a square.

11. What type of geometric figure might enclose the most area given a fixed perimeter? Explain. A circle, its ratio of area to perimeter is even greater than a square's.
Choose the concept from the list below that best represents the item in each box.

<table>
<thead>
<tr>
<th>Concept List</th>
<th>10-3</th>
<th>23.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10-3</td>
<td>21.</td>
</tr>
<tr>
<td></td>
<td>16.</td>
<td>20.</td>
</tr>
<tr>
<td></td>
<td>15.</td>
<td>14.</td>
</tr>
<tr>
<td></td>
<td>13.</td>
<td>12.</td>
</tr>
</tbody>
</table>

Choose the concept from the list below that best represents the item in each box.

1. \( x^2 + y^2 = r^2 \)  
   - parent graph of a circle
2. the set of all points in a plane that are a distance \( r \) from a given point
3. the method used to change an equation of a circle into standard form
4. \( x, y \) in the equation of a circle parameters
5. \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)  
   - distance formula
6. all points on a circle are equivalent from this point
7. \( (x - h)^2 + (y - k)^2 = r^2 \)  
   - standard form of a circle
8. movement of the parent graph horizontally or vertically
9. the center of a circle is a point on the circle
10. the radius of the circle

Write an equation for each translation.

- What do you need to find an equation for each gear?
- What is the problem asking you to determine?
- What is the radii of the gears?
- Describe how to change an equation of a circle.

Planning the Solution

- What do you need to find an equation for each gear?
- What is the problem asking you to determine?
- What is the radii of the gears?
- Describe how to change an equation of a circle.

Getting an Answer

- Fill in the table to the right to find the equation of the circle that represents each gear.

<table>
<thead>
<tr>
<th>Gear</th>
<th>( h, k )</th>
<th>( r )</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(-4, 0)</td>
<td>4</td>
<td>( (x + 4)^2 + y^2 = 16 )</td>
</tr>
<tr>
<td>B</td>
<td>(0, 0)</td>
<td>3</td>
<td>( x^2 + y^2 = 9 )</td>
</tr>
<tr>
<td>C</td>
<td>(4, 0)</td>
<td>1</td>
<td>( (x - 4)^2 + y^2 = 1 )</td>
</tr>
</tbody>
</table>

Write the equation of a circle with the center at \((8.5, 0)\) and diameter 25 as an equation with the center at the origin and radius 5 in a circle with the center is at right and down 2 units. Answers may vary. Sample: Write the equation of a circle with the center at the origin and radius 5. Use an equation in standard form with \( h = 3 \) and \( k = 2 \) to translate the circle (\( x^2 + y^2 = 25 \)).

Open-Ended Write an equation for a circle with the center at the origin and an equation for another circle that is a translation of the first. Answers may vary. The circle with the center at the origin should be in the form \( x^2 + y^2 = r^2 \) and the circle that is translated should have the same value for \( r \) as the original circle.

Error Analysis A classmate sets the equation of a circle with the center at \((4, 5)\) and diameter 25 as \( x^2 + y^2 = 25 \). Is the correct? Why or why not? This is the incorrect equation for the circle. The values for \( h \) and \( k \) are reversed and a squared and a should be squared. The correct equation is \( (x - 8)^2 + (y - 10)^2 = 162.25 \).

Reasoning How can you determine if the graph of the circle \( x^2 + y^2 = 4 \) is correctly drawn? Check that the center of the circle is \((-6, -9)\) and that the radius of the circle is 7.
ANSWERS
page 25

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Practice

Form K

Write an equation of a circle with the given center and radius. Check your
answers.
1. center (1, 1), radius 4
(x 2 1)2 1 (y 2 1)2 5 16

2. center (22, 0), radius 6
(x 1 2)2 1 y2 5 36

3. center (5, 23), radius 1
(x 2 5)2 1 (y 1 3)2 5 1

4. center (21, 25), radius 5
(x 1 1)2 1 (y 1 5)2 5 25

Practice (continued)

10-3

Circles

Form K

Circles

For each equation, find the center and radius of the circle.
13. (x 1 2)2 1 (y 1 2)2 5 64
(22, 22); 8

14. x2 1 (y 2 5)2 5 16
(0, 5); 4

15. (x 1 6)2 1 y2 5 9
(26, 0); 3

16. (x 2 7)2 1 (y 1 1)2 5 81
(7, 21); 9

Use the center and the radius to graph each circle.

Write an equation for each translation.

17. (x 1 3)2 1 (y 2 3)2 5 16

5. x2 1 y2 5 16; left 2 units and down 1 unit

Translate left 2 units:

Translate down 1 unit:

(x 2 (22))2 1 y2 5 16

(x 1 2)2 1 Q y 2 Q 21 R R 5 16

(x 1 2)2 1 y2 5 16

(x 1 2)2 1 Q y 1 1 R 5 16

z

z

z

6. x2 1 y2 5 81; right 4 units
(x 2 4)2 1 y2 5 81

z

Rewrite the equation in standard form.
(x 2 (23))2 1 (y 2 3)2 5 42
Find the center and radius from the equation. center (23, 3), radius 4
Plot the center and draw a circle with the given radius.

2

y
8
4
x

2

8 4 O

7. x2 1 y2 5 1; left 1 unit and up 1 unit
(x 1 1)2 1 (y 2 1)2 5 1

18. (x 2 2)2 1 (y 2 1)2 5 49

19. (x 1 4)2 1 (y 1 2)2 5 25

y

4

4

8. x2

1

y2

5 4; up 7 units

9. x2

x2 1 (y 2 7)2 5 4

1

y2

4 O
4

(x 2 5)2 1 (y 1 2)2 5 36

Write an equation for each circle. Each interval represents one unit.

Then find the radius.

Use the standard form (x 2
x Scale: 1

y Scale: 1

1 (y 2

k)2

5

Use the given information to write an equation of the circle.
22. center (23, 1), through (23, 21)
(x 1 3)2 1 (y 2 1)2 5 4

12.

x Scale: 1

x Scale: 1

y Scale: 1

(x 1

(x 1 5)2 1 (y 2 1)2 5 16

21. (3, 4)
x2 1 y2 5 25

r 2.

(x 2 4)2 1 (y 1 3)2 5 4

11.

8

O
4
8

20. (22, 0)
x2 1 y 2 5 4

The circle is 4 units wide,
so the radius is 2.
h)2

4

8

Write the equation of the circle that passes through the given point and has a
center at the origin. (Hint: You can use the distance formula to find the radius.)

First, identify the center. (4, 23)

10.

y
x

x

5 36; right 5 units and
down 2 units

4

2)2

y Scale: 1

1 (y 1

4)2

23. radius 6, center (4, 25)
(x 2 4)2 1 (y 1 5)2 5 36

24. Writing Explain how to find the center and radius of the circle x2 1 y2 2 6y 2 16 5 0.
Answers may vary. Sample: Complete the square for y2 2 6y and rewrite the equation
in the form (x 2 h)2 1 (y 2 k)2 5 r 2 : (x 2 0)2 1 (y 2 3)2 5 52 . The center is (h, k), so
the center of this circle is (0, 3). The radius is 5.

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10-3

Standardized Test Prep
Circles

For Exercises 1−5, choose the correct letter.
1. Which is an equation of the circle with center at the origin and radius 3? C

x 2 1 y2 5 9

x2 1 y2 5 81

(x 2 3)2 1 (y 2 3)2 5 9

x2 1 y2 5 16

2x2 1 y2 5 16
(x 1 2)2 1 (y 1 1)2 5 16

Circles

1. Write the equation in standard form of a circle with a center at (3, 4)

with the y-axis tangent to the circle.
(x 2 3)2 1 (y 2 4)2 5 9

2. What is the equation for the translation of x2 1 y2 5 16 two units left and one
unit down? I

(x 2 2)2 1 (y 2 1)2 5 16

Enrichment

In geometry, you learned that a tangent to a circle is a line that
intersects the circle in exactly one point. The tangent is also
perpendicular to the radius at the point of tangency. Use these facts,
along with what you know about the equation of a circle, to solve the
problems below.

Multiple Choice

x 2 1 y2 5 3

2. Write the equation of a circle with radius 2 units with both the x-axis and the

y-axis tangent to the circle. Explain why there is more than one equation to
describe this circle.

3. Which equation represents a circle with a center at (7, 29) and a diameter of 8? C

(x 2 7)2 1 (y 2 9)2 5 64

(x 2 7)2 1 (y 1 9)2 5 16

Answers may vary. Sample: (x 2 2)2 1 (y 2 2)2 5 4; this circle could be
located in any quadrant where the centers could be (2, 2), (2, 22), (22, 2),
or (22, 22).

(x 2 7)2 1 (y 1 9)2 5 64

(x 1 7)2 1 (y 2 9)2 5 16

3. Write the equation in standard form of a circle that is tangent to the y-axis,

tangent to the horizontal line y 5 5, and tangent to the vertical line x 5 24.

4. What is the center of the circle (x 2 3)2 1 (y 1 2)2 5 81? G

(23, 2)

(3, 22)

(3, 2)

(x 1 2)2 1 (y 2 3)2 5 4

(9, 9)

5. What is the radius of the circle (x 1 8)2 1 (y 2 3)2 5 100? A

10

20

50

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4. Write the equation in standard form of a circle with a diameter that has

100

endpoints (21, 23) and (3, 23).
(x 2 1)2 1 (y 1 3)2 5 4

Short Response
6. What are the radius and center of a circle with the equation

5. Write the equation in standard form of a circle with center at (22, 3) that

(x 1 7)2 1 (y 2 8)2 5 144?

passes through the point (21, 7).

[2] The radius is 12; the center is (27, 8).
[1] incorrect radius OR center
[0] no answers given

(x 1 2)2 1 (y 2 3)2 5 17

6. Write the equation in standard form of a circle with center in the fourth

quadrant with the x-axis, the vertical line x 5 4, and the vertical line x 5 6
tangent to the circle.
(x 2 5)2 1 (y 1 1)2 5 1

7. Write the equation of a circle with center on the line y 5 2x, radius 2, and is

tangent to the y-axis.
(x 2 2)2 1 (y 2 4)2 5 4 or (x 1 2)2 1 (y 1 4)2 5 4

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10-3 Reteaching

Circles

- When working with circles, begin by writing the equation in standard form: 
  \((x - h)^2 + (y - k)^2 = r^2\)
- Unlike equations of parabolas, which include either \(x^2\) or \(y^2\), the equation of a circle will include both \(x^2\) and \(y^2\).

**Problem**

What is the center and radius of the circle with the equation 
\[x^2 + y^2 - 4x + 6y = 127\]
To complete each circle, find \(h, k, \) and \(r\) for each group.

\[\frac{x^2 - 4x}{4} + \frac{(y + 3)^2}{9} = 25\]
Add \(\frac{x^2}{4} + \frac{y^2}{9}\) for each group. Write the equation in standard form; simplify.

\((x - 2)^2 + (y + 3)^2 = 25\)
Write the expression as perfect squares; simplify. Write the equation in standard form.

\((x - 2)^2 + (y + 3)^2 = 25\)

The center of the circle is \((2, -3)\). The radius of the circle is \(5\).

**Exercises**

Find the center and radius of each circle.

1. \(x^2 + y^2 - 10y = 0\)  \((0, 5), 5\)
2. \(x^2 + y^2 = 225\)  \((0, 0), 15\)
3. \(x^2 + y^2 + 2x - 6y = 13\) \((-1, 3), 5\)
4. \(x^2 + y^2 + 2x + 4y = 31\) \((-1, -2), 6\)
5. \(x^2 + y^2 - 10x - 11y = -72\) \((0, 8), 3\)
6. \(x^2 + y^2 - 4x + 6y = -25\) \((2, 3), 3\)
7. \(x^2 + y^2 - 8x + 9y = -25\) \((-3, -2), 3\)

10-4 ELL Support

Ellipses

Complete the vocabulary chart by filling in the missing information.

<table>
<thead>
<tr>
<th>Word or Word Phrase</th>
<th>Definition</th>
<th>Picture or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>ellipse</td>
<td>An ellipse is the set of all points in a plane such that the sum of the distances from two fixed points is constant.</td>
<td><img src="image1.png" alt="Ellipse" /></td>
</tr>
<tr>
<td>focus of an ellipse</td>
<td>A focus of an ellipse is one of the fixed points.</td>
<td><img src="image2.png" alt="Focus of an ellipse" /></td>
</tr>
<tr>
<td>major axis</td>
<td>The major axis is the segment that contains the foci and has its endpoints on the ellipse.</td>
<td><img src="image3.png" alt="Major Axis" /></td>
</tr>
<tr>
<td>center of an ellipse</td>
<td>The center of an ellipse is the midpoint of the major axis.</td>
<td><img src="image4.png" alt="Center of an ellipse" /></td>
</tr>
<tr>
<td>minor axis</td>
<td>The minor axis is the axis that is perpendicular to the major axis at the center.</td>
<td><img src="image5.png" alt="Minor Axis" /></td>
</tr>
<tr>
<td>vertices of an ellipse</td>
<td>The vertices of an ellipse are the end points of the major axis.</td>
<td><img src="image6.png" alt="Vertices of an ellipse" /></td>
</tr>
<tr>
<td>co-vertices of an ellipse</td>
<td>The co-vertices of an ellipse are endpoints of the minor axis.</td>
<td><img src="image7.png" alt="Co-vertices of an ellipse" /></td>
</tr>
</tbody>
</table>

10-4 Think About a Plan

Ellipses

**Aerodynamics**

Scientists used the Transonic Tunnel at NASA Langley Research Center, Virginia, to study the dynamics of air flow. The elliptical opening of the Transonic Tunnel is 62 ft wide and 58 ft high. What is an equation for the ellipse?

**Know**

1. The width of the tunnel opening is \(62\) ft.
2. The height of the tunnel opening is \(58\) ft.

**Need**

3. To solve the problem I need to find:
   an equation for the ellipse that represents the opening of the Transonic Tunnel.

**Plan**

4. How can a drawing help you solve this problem?
   **Answers may vary.** Example: A drawing could help me relate the dimensions of the tunnel opening to the features of an ellipse.

5. Make a sketch of an ellipse that represents the tunnel opening. Where should you put the origin?

6. How do the width and height of the tunnel opening relate to the equation of the ellipse?
   The width of the opening is the length of the major axis. The height of the opening is the length of the minor axis.

7. How can you write the equation of the ellipse in standard form?
   Find \(a\) from the length of the major axis: \(a = 31\). Find \(b\) from the length of the minor axis: \(b = 29\). The origin is \((0, 0)\), so \(h = 0\) and \(k = 0\).

8. Write the equation of the ellipse in standard form: \(\frac{x^2}{31^2} + \frac{y^2}{29^2} = 1\)
Find the foci for each equation of an ellipse. Then graph the ellipse.

Write an equation of an ellipse in standard form with center at the origin and with the given vertex and co-vertex listed respectively.

Find the distance between the foci of an ellipse. The lengths of the major and minor axes are listed.

Find the foci for each equation of an ellipse. Then graph the ellipse.

Find the distance between the foci of an ellipse. The lengths of the major and minor axes are listed.

Write an equation of an ellipse in standard form with center at the origin and with the given vertex and co-vertex listed respectively.

Find the distance between the foci of an ellipse. The lengths of the major and minor axes are listed.

Write an equation of an ellipse in standard form with center at the origin and with the given vertex and co-vertex listed respectively.
Find the equation of the ellipse given the height and width. Assume that the center is (0, 0).

What is the equation of an ellipse that is 10 units wide and 8 units high? Assume a

What is the positive y-coordinate of the foci of the ellipse with the equation

An ellipse has foci at (± 7, 0) and vertices at (± 10, 0). What is the value of c?

Suppose you are planning a party at an elliptical park with one game at each

In the equation for a vertical ellipse

What is the positive

Ellipses

Step 3 Identify the information that is known.

Step 2 Identify the information that is unknown.

Step 3 Use

Exercises

What is the equation of an ellipse that is 10 units wide and 8 units high? Assume that the center is (0, 0).

Suppose you are planning a party at an elliptical park with one game at each

The distance between foci is 2

The equation of a horizontal ellipse.

Substitute for

The distance from a planet to the sun is greatest when the planet is at aphelion and is least when the planet is at perihelion. The point at which the planet is closest to the sun is called the perihelion. The point at which the planet is farthest from the sun is called the aphelion.

To find the standard form of the equation of an ellipse with center at (0, 0), major axis of length 2a, and minor axis of length 2b, where a > b, use the following:

When the width is greater than the height, use

When the height is greater than the width, use

Find the equation of the ellipse given the height and width. Assume that the center of the ellipse is (0, 0).

1. Height: 26 ft, width: 24 ft
2. Height: 12 ft, width: 4 ft
3. Height: 10 ft, width: 6 ft
4. Height: 8 ft, width: 18 ft
5. Height: 20 ft, width: 50 m
6. Height: 10 ft, width: 22 ft
7. Height: 16 m, width: 10 m
8. Height: 20 ft, width: 3 ft
9. Height: 3 cm, width: 6 cm
10. Height: 14 m, width: 30 m
11. Height: 12 ft, width: 9 ft
12. Height: 12 ft, width: 4 ft
13. Height: 7 m, width: 8 m
14. Height: 2 m, width: 10 m
15. Height: 0.5 cm, width: 9 cm
16. Australian Rules Football is played on an elliptical field. One of the fields used for this sport is 174 meters long and 146 meters wide. Find an equation of the ellipse.
Choose the word from the list that best matches each sentence.

1. The graph approaches
2. The shape of this graph is guided by its asymptotes
3. The distance between these two points in a hyperbola is 2c.
4. The segment connecting the vertices of a hyperbola
5. The two foci lie on this line in a hyperbola
6. This axis is perpendicular to the transverse axis at the center

Choose the word from the list that best matches each sentence.

7. The equations of the asymptotes of a horizontal hyperbola centered at (0, 0) are
8. The equation \( a^2 = b^2 + c^2 \) is used to find the distance to each
9. The length of the conjugate axis is 2b.
10. A hyperbola is formed by two horizontal or vertical branches.
11. A hyperbola has two axes of symmetry.
12. If the term \( y^2 \) is positive, the transverse axis of the hyperbola is vertical.

Find the equation of a hyperbola with the given foci, or vertices. Assume that the transverse axis is horizontal.

1. \( a = 7, b = 3 \), \( c = \)
2. \( a = 12, b = 11 \), \( c = \)
3. \( a = 10, b = 13 \), \( c = \)
4. \( a = 15, b = 20 \), \( c = \)
5. \( a = 14, b = 20 \), \( c = \)
6. \( a = 30, b = 40 \), \( c = \)
7. \( a = 7.6, b = 4.0 \), vertices \( (\pm 4, 0) \), \( (\pm 2, 0) \), \( c = \)
8. \( a = 7.6, b = 4.0 \), vertices \( (\pm 13, 0) \), \( (\pm 5, 0) \), \( c = \)

Find the vertices, foci, and asymptotes of each hyperbola. Then sketch the graph.

13. \( a^2 = 1, b^2 = 9 \), \( c = \)
14. \( a^2 = 1, b^2 = 9 \), \( c = \)
15. \( a^2 = 1, b^2 = 9 \), \( c = \)
16. \( a^2 = 1, b^2 = 9 \), \( c = \)
17. \( a^2 = 9, b^2 = 4 \), \( c = \)
18. \( a^2 = 9, b^2 = 4 \), \( c = \)

Write an equation of a hyperbola with the given foci and vertices.

19. \( a = 1, b = 2 \), vertices \( (\pm 3, 0) \), \( (\pm 5, 0) \), \( c = \)
20. \( a = 1, b = 2 \), vertices \( (0, \pm 3) \), \( (0, \pm 5) \), \( c = \)

Graph each equation.

21. \( 26x^2 - 9y^2 = -140 \)
22. \( 26x^2 - 9y^2 = -120 \)
23. \( 26x^2 - 9y^2 = -243 \)
24. \( 6x^2 - 20y^2 = -328 \)

Write the equation of a hyperbola with the given foci and vertices. Assume that the transverse axis is horizontal.

25. \( a = 1, b = 2 \), vertices \( (\pm 3, 0) \), \( (\pm 5, 0) \), \( c = \)
26. \( a = 1, b = 2 \), vertices \( (0, \pm 3) \), \( (0, \pm 5) \), \( c = \)

Graph each equation.

27. Write the equation of a hyperbola with the given foci and vertices. Assume that the transverse axis is horizontal.

28. Error Analysis On a test, a student wrote \( 1, 4 \) for the foci of the hyperbola with the equation \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \). The teacher gave the student partial credit. What did the student do right? What did the student do wrong?

29. Reasoning Describe how you can find the asymptotes when you have the \( a \) and \( b \) values for a vertical hyperbola.

Use the equation \( c^2 = a^2 + b^2 \) to find the value of \( c \). Then substitute values of \( a \) and \( b \) in the equation \( y = \pm \frac{b}{a}x \) to find the asymptote.
Find the equation of a hyperbola with the given values, foci, or vertices. Assume that the transverse axis is horizontal.

1. \( b = 2, c = 4 \)
   Use the equation \( c^2 = a^2 + b^2 \) to find \( a^2 \).
   \[ c^2 = a^2 + b^2 \]
   \[ a^2 = \frac{c^2 - b^2}{0} \]
   Substitute values for \( a^2 \) and \( b^2 \): \( b^2 = 1 \)

2. \( b = 3, c = 5 \)
   The foci are given by \((\pm c, 0)\). \( c^2 = a^2 + b^2 \)
   \[ c^2 = a^2 + b^2 \]
   \[ a^2 = \frac{c^2 - b^2}{0} \]
   Substitute values for \( a^2 \) and \( b^2 \): \( b^2 = 1 \)

3. \( c = 6, \) vertices \((\pm 2, 0)\)
   The vertices are given by \((-a, 0), (a, 0)\). \( c^2 = a^2 + b^2 \)
   \[ c^2 = a^2 + b^2 \]
   \[ a^2 = \frac{c^2 - b^2}{0} \]
   Substitute values for \( a^2 \) and \( b^2 \): \( b^2 = 1 \)

4. \( \frac{x^2}{36} - \frac{y^2}{1} = 1 \)
   The standard form of a hyperbola is \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \).
   \[ \frac{x^2}{36} - \frac{y^2}{1} = 1 \]
   \[ a^2 = 36 \]
   \[ b^2 = 1 \]

5. \( \frac{x^2}{4} + \frac{y^2}{9} = 1 \)
   The standard form of an ellipse is \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \).
   \[ \frac{x^2}{4} + \frac{y^2}{9} = 1 \]
   \[ a^2 = 4 \]
   \[ b^2 = 9 \]

6. \( 9x^2 - 16y^2 = 144 \)
   The equation is in standard form.
   \[ 9x^2 - 16y^2 = 144 \]
   \[ a^2 = 16 \]
   \[ b^2 = 9 \]

Multiple Choice

1. A hyperbola has vertices \((\pm 3, 0)\) and one focus at \((8, 0)\). What is the equation of the hyperbola in standard form? \( \Box \)
   \[ \frac{x^2}{9} - \frac{y^2}{1} = 1 \] \( \Box \)
   \[ \frac{y^2}{9} - \frac{x^2}{1} = 1 \] \( \Box \)

2. A hyperbola with a horizontal transverse axis has asymptotes \( y = \pm \frac{2}{3}x \).
   Which of the following could be the equation of the hyperbola in standard form? \( \Box \)
   \[ \frac{x^2}{9} - \frac{y^2}{4} = 1 \] \( \Box \)
   \[ \frac{y^2}{9} - \frac{x^2}{4} = 1 \] \( \Box \)

3. What are the vertices of the hyperbola with the equation \( 4x^2 - 9y^2 = 72? \)
   \( \Box \) \( (\pm 3, 0) \) \( \Box \) \( (\pm \sqrt{\frac{9}{4}}, 0) \) \( \Box \) \( (\pm \sqrt{12}, 0) \)

4. What are the foci of the hyperbola with the equation \( x^2 - y^2 = 17? \)
   \( \Box \) \( (0, \pm \sqrt{17}) \) \( \Box \) \( (0, \pm \sqrt{13}) \) \( \Box \) \( (0, \pm \sqrt{25}) \)

Short Response

5. What are the vertices, foci, and asymptotes of the hyperbola with the equation \( 4x^2 - 16y^2 = 64? \)
   \( \Box \) \( \) \( \) \( \) \( \)
   \( \) \( \) \( \) \( \) \( \)
The conic is a circle.
The conic is a horizontal ellipse.

Identify the conic that has the equation $10 - x^2 = 1$.

Find the foci of each hyperbola. Then draw the graph.

1. $y = y^2 + 1 (a = \sqrt{6}, b = 0)$
2. $y - y^2 = 4 (a = 3, b = 0)$
3. $4y^2 - y^2 = 4 (a = 3, b = 0)$

Find the foci of each hyperbola. Then draw the graph.

The conic is a horizontal ellipse.

Exercise
Identify the conic that has the equation $x^2 + y^2 + 4x - 6y + 4 = 0$.

1. $x^2 + y^2 = 4x - 6y + 4 = 0$

Write the original equation.

2. $x^2 + y^2 = 4x - 6y + 4 = 0$

Group the $x$- and $y$-terms.

3. $x^2 + y^2 = 4x - 6y + 4 = 0$

Complete the square.

4. $x^2 + y^2 = 4x - 6y + 4 = 0$

Simplify.

5. $x^2 + y^2 = 4x - 6y + 4 = 0$

Factor.

The conic is a circle.
Identify each conic section by writing the equation in standard form and sketching the graph. For a parabola, give the vertex and the focus.

1. $x^2 + 2y - 8x = 0$
   - Parabola, vertex: $(4, 0)$, focus: $(2.5, 0)$

2. $(x - 1)^2 + (y - 2)^2 = 9$
   - Circle, center: $(1, 2)$, radius: $3$

3. $9x^2 + 4y^2 - 36x - 8y = 0$
   - Ellipse, center: $(2, 2)$, vertices: $(7, 2)$, $(2, 7)$

4. $y = x^2 - 4x + 5$
   - Parabola, vertex: $(2, -1)$, focus: $(2, 0)$

5. $x^2 + y^2 - 6x + 4y = 0$
   - Circle, center: $(3, -2)$, radius: $5$

6. $4x^2 - 9y^2 + 16x = 0$
   - Hyperbola, center: $(2, 0)$, vertices: $(6, 0)$, $(0, 0)$

7. $x^2 + y^2 + 4x - 2y - 4 = 0$
   - Circle, center: $(-2, 1)$, radius: $3$

8. $x^2 - y^2 + 2x - 4y = 0$
   - Hyperbola, center: $(1, 2)$, vertices: $(3, 2)$, $(-1, 2)$

9. $x^2 + 4y^2 - 16x + 12y = 0$
   - Ellipse, center: $(4, -3)$, vertices: $(8, -3)$, $(0, -3)$

10. $2x^2 + 3y^2 - 12x + 6y = 0$
    - Ellipse, center: $(3, -1)$, vertices: $(6, -1)$, $(0, -1)$

11. $x^2 + y^2 + 2x + 3y = 0$
    - Circle, center: $(-1, -1.5)$, radius: $1.5$

12. $4x^2 + 9y^2 - 32x + 54y = 0$
    - Ellipse, center: $(4, -3)$, vertices: $(8, -3)$, $(0, -3)$

13. $x^2 - y^2 + 2x - 4y = 0$
    - Hyperbola, center: $(1, 2)$, vertices: $(3, 2)$, $(-1, 2)$

14. $x^2 - y^2 + 4x - 6y = 0$
    - Hyperbola, center: $(2, 3)$, vertices: $(4, 3)$, $(0, 3)$

15. $x^2 + 4y^2 - 8x + 16y = 0$
    - Ellipse, center: $(4, -4)$, vertices: $(8, -4)$, $(0, -4)$

16. $2x^2 + 3y^2 - 4x + 9y = 0$
    - Ellipse, center: $(2, -3)$, vertices: $(6, -3)$, $(0, -3)$

17. $x^2 + y^2 + x + y = 0$
    - Circle, center: $(-0.5, -0.5)$, radius: $0.5$

18. $x^2 - y^2 + 2x - 4y = 0$
    - Hyperbola, center: $(1, 2)$, vertices: $(3, 2)$, $(-1, 2)$

19. $x^2 + 4y^2 - 8x + 16y = 0$
    - Ellipse, center: $(4, -4)$, vertices: $(8, -4)$, $(0, -4)$

20. $2x^2 + 3y^2 - 4x + 9y = 0$
    - Ellipse, center: $(2, -3)$, vertices: $(6, -3)$, $(0, -3)$

21. Error Analysis: A student found that the equation of a hyperbola with center $(0, 0)$, vertices $(2, 0)$, and focus $(2, 0)$ was $x^2 - y^2 = 4$. Explain why the student is incorrect.

Identify each conic section as an ellipse or a hyperbola by writing the equation in standard form and sketching the graph. Give the center and the axes for an ellipse or the hyperbola, give the center and the foci.

1. $x^2 + 4y^2 - 4x + 8y = 0$
   - Ellipse, center: $(2, -2)$, vertices: $(4, -2)$, $(0, -2)$

2. $x^2 - y^2 - 2x + 4y = 0$
   - Hyperbola, center: $(1, 2)$, vertices: $(3, 2)$, $(-1, 2)$

3. $x^2 + 4y^2 - 8x + 16y = 0$
   - Ellipse, center: $(4, -4)$, vertices: $(8, -4)$, $(0, -4)$

4. $x^2 - 9y^2 + 4x - 36y = 0$
   - Hyperbola, center: $(2, 0)$, vertices: $(6, 0)$, $(0, 0)$

5. $4x^2 + 4y^2 - 16x - 16y = 0$
   - Ellipse, center: $(4, 4)$, vertices: $(8, 4)$, $(0, 4)$

6. $x^2 + 4y^2 - 4x + 8y = 0$
   - Ellipse, center: $(2, -2)$, vertices: $(4, -2)$, $(0, -2)$

7. $x^2 - y^2 + 2x - 4y = 0$
   - Hyperbola, center: $(1, 2)$, vertices: $(3, 2)$, $(-1, 2)$

8. $x^2 + 4y^2 - 8x + 16y = 0$
   - Ellipse, center: $(4, -4)$, vertices: $(8, -4)$, $(0, -4)$

9. $x^2 - y^2 - 2x + 4y = 0$
   - Hyperbola, center: $(1, 2)$, vertices: $(3, 2)$, $(-1, 2)$

10. $x^2 + 4y^2 - 4x + 8y = 0$
     - Ellipse, center: $(2, -2)$, vertices: $(4, -2)$, $(0, -2)$

The graph of each equation is to be translated 3 units right and 1 unit up. Write each new equation.

11. $x^2 - y^2 - 2x + 4y = 0$
    - Hyperbola, center: $(4, 1)$, vertices: $(6, 1)$, $(2, 1)$

12. $x^2 + 4y^2 - 8x + 16y = 0$
    - Ellipse, center: $(7, 4)$, vertices: $(10, 4)$, $(-3, 4)$

13. $x^2 - y^2 - 2x + 4y = 0$
    - Hyperbola, center: $(4, 1)$, vertices: $(6, 1)$, $(2, 1)$

14. $x^2 + 4y^2 - 8x + 16y = 0$
    - Ellipse, center: $(7, 4)$, vertices: $(10, 4)$, $(-3, 4)$

The graph of each equation is to be translated 3 units right and 5 units down. Write each new equation.

15. $x^2 - y^2 - 2x + 4y = 0$
    - Hyperbola, center: $(4, -4)$, vertices: $(6, -4)$, $(2, -4)$

16. $x^2 + 4y^2 - 8x + 16y = 0$
    - Ellipse, center: $(7, 10)$, vertices: $(10, 10)$, $(-3, 10)$

17. $x^2 - y^2 - 2x + 4y = 0$
    - Hyperbola, center: $(4, -4)$, vertices: $(6, -4)$, $(2, -4)$

18. $x^2 + 4y^2 - 8x + 16y = 0$
    - Ellipse, center: $(7, 10)$, vertices: $(10, 10)$, $(-3, 10)$

The graph of each equation is to be translated 3 units right and 5 units down. Write each new equation.

19. $x^2 - y^2 - 2x + 4y = 0$
    - Hyperbola, center: $(4, -4)$, vertices: $(6, -4)$, $(2, -4)$

20. $x^2 + 4y^2 - 8x + 16y = 0$
    - Ellipse, center: $(7, 10)$, vertices: $(10, 10)$, $(-3, 10)$
5. Identify the conic section represented by each equation by writing the equation in standard form.

\[ 25x^2 + 9y^2 = 225 \]

Given the center and major axis, sketch the graph. Show your work.

[a. Hyperbola, center, and foci are identified correctly. Graph is correct.]
[b. Hyperbola, center, and foci are identified correctly. Graph has minor errors.]
[c. Conic section OR center OR foci identified incorrectly. Graph has minor errors.]
[d. Incorrect answers and no work shown OR no answers given]

Extended Response

5. Identify the conic section represented by each equation by writing the equation in standard form.

\[ 4x^2 + 20x - 10y + 37 = 0 \]

Complete the square. Divide each side by 4. Factor out coefficients of the \( x^2 \) and \( y^2 \) terms.

\[ \left( x + 1 \right)^2 + 4\left( y - 2 \right)^2 = 4 \]

Simplify. Subtract 4 from each side. Complete the square.

\[ x^2 + 4y^2 - 8y + 37 = 0 \]

Set equal to zero. Subtract 37.

\[ 4\left( x + 1 \right)^2 + 4\left( y - 2 \right)^2 = 4 \]

Factor out coefficients of the \( x^2 \) and \( y^2 \) terms.

\[ \left( x + 1 \right)^2 + \left( y - 2 \right)^2 = 4 \]

Divide each side by 4. Graph the circle. Plot point 1 and equation (y - 2)^2 = 1.

There are no denominators, and both \( a \) and \( b \) have squared terms.

The equation has the standard form of a circle \( (x - h)^2 + (y - k)^2 = r^2 \). The center of the circle is \( \left( -1, 2 \right) \) and the radius of the circle is 1.

Exercises

Identify the conic section represented by each equation by writing the equation in standard form. Then sketch the graph.

5. \[ x^3 + 2y - 3 = 0 \]

6. \[ x^3 + 4y^2 = 2x - 8y + 9 = 0 \]

7. \[ 9x^2 - 4y^2 = 18x + 16y = 0 \]

8. \[ x^2 + y^2 = 10x - 6y = 16 \]
Use the given information to write an equation of the circle.

Do you know HOW?

Find the domain and range.

1. $y = x^2 - 3$  
   Domain: all real numbers  
   Range: $y \geq -3$

2. $x = y^2 - 4$  
   Domain: $x \geq -4$  
   Range: all real numbers

3. $y = 2x^2 + 1$  
   Domain: all real numbers  
   Range: $y \geq 1$

4. $x = 3y^2 - 5$  
   Domain: all real numbers  
   Range: $x \geq -5$

Name the vertex, focus, and directrix of each parabola.

5. $y = x^2 + 2$  
   Vertex: $(0, 2)$  
   Focus: $(0, 3)$  
   Directrix: $y = 1$

6. $x = y^2 - 1$  
   Vertex: $(0, 0)$  
   Focus: $(1, 0)$  
   Directrix: $x = -1$

Write an equation of a parabola with vertex at the origin and the given directrix.

7. $y = x^2 + 3$  
   Vertex: $(0, 0)$  
   Directrix: $y = -3$

8. $x = y^2 - 2$  
   Vertex: $(0, 0)$  
   Directrix: $x = 2$

9. $y = -x^2 + 1$  
   Vertex: $(0, 0)$  
   Directrix: $y = 1$

10. $x = -y^2 - 4$  
    Vertex: $(0, 0)$  
    Directrix: $x = 4$

Write an equation of a parabola with a vertex at the origin and the given focus.

11. $y = x^2 + 2$  
    Focus: $(0, 3)$  
    Vertex: $(0, 2)$

12. $x = y^2 - 1$  
    Focus: $(0, 0)$  
    Vertex: $(0, -1)$

13. $y = -x^2 - 4$  
    Focus: $(0, -4)$  
    Vertex: $(0, -3)$

14. $x = -y^2 + 2$  
    Focus: $(0, 2)$  
    Vertex: $(0, 0)$

Write an equation for each translation.

15. $x^2 + y^2 = 16$  
   Moved 4 units left and down 3 units

16. $x^2 + y^2 = 9$  
   Moved 5 units right and up 6 units

Do you UNDERSTAND?

17. Vocabulary  
   Explain how a focus and a directrix are similar and how they are different.

   The focus and directrix are both used to define a parabola. They are both used to find the equation of a parabola.

   Different: The focus is a point on the curve, whereas the directrix is a line.

18. Open-Ended  
   Sketch the graphs of four different conic sections.

   Answers may vary. Conic sections should be drawn and labeled correctly as circle, ellipse, hyperbola, and parabola.

Do you know HOW?

Identify the center and intercepts of each conic section. Give the domain and range of each.

19. $x^2 + y^2 = 25$  
   Domain: all real numbers  
   Range: all real numbers  
   Center: $(0, 0)$  
   Intercepts: $(5, 0)$, $(-5, 0)$, $(0, 5)$, $(0, -5)$

20. $(x - 3)^2 + (y + 2)^2 = 16$  
   Domain: all real numbers  
   Range: all real numbers  
   Center: $(3, -2)$  
   Intercepts: $(7, 0)$, $(-1, 0)$, $(0, 4)$, $(0, -6)$

21. $x^2 + y^2 = 1$  
   Domain: all real numbers  
   Range: all real numbers  
   Center: $(0, 0)$  
   Intercepts: $(1, 0)$, $(-1, 0)$, $(0, 1)$, $(0, -1)$

22. $(x - 2)^2 + (y + 3)^2 = 4$  
   Domain: all real numbers  
   Range: all real numbers  
   Center: $(2, -3)$  
   Intercepts: $(4, 0)$, $(-4, 0)$, $(0, 4)$, $(0, -4)$

Write an equation of a parabola with a vertex at the origin and the given focus.

23. $x = -y^2 - 2$  
   Focus: $(0, -1)$  
   Vertex: $(0, -2)$

24. $y = x^2 + 3$  
   Focus: $(1, 0)$  
   Vertex: $(1, 0)$

25. $y = -x^2 - 4$  
   Focus: $(0, -4)$  
   Vertex: $(0, -3)$

26. $x = y^2 - 4$  
   Focus: $(0, 2)$  
   Vertex: $(0, 2)$

Do you UNDERSTAND?

27. Writing  
   Explain how you can tell whether a parabola opens upward, downward, to the left, or to the right.

   A parabola opens up or down when the standard form of the equation is $y = ax^2 + k$. A parabola opens left or right when the standard form of the equation is $x = ay^2 + k$.

28. Open-Ended  
   Write the equation of a circle with center $(2, -3)$.

   Answers may vary. Sample: $x^2 + (y + 3)^2 = 4$
Find the vertex, focus, and the directrix of each parabola.

1. \( y = x^2 - 2x - 3 \)
2. \( y = x^2 - 4x + 4 \)
3. \( y = 2x^2 + 4x - 1 \)
4. \( y = -x^2 + 6x - 5 \)

Do you UNDERSTAND?

6. Error Analysis: A student says that the domain of the hyperbola is \( x = 2 \). What mistake did the student make? Write the correct domain. Answers may vary. Sample: For the equation \( x^2 - y^2 = 1 \), the domain is all real numbers except \( x = 2 \).

5. Vocabulary: If the directrix of a parabola is 10 units from the focus, how far is the vertex from the focus? 5 units.

10. Compare and Contrast: What is the difference between a parabola with an \( x^2 \) term in its equation and a parabola with an \( y^2 \) term in its equation?

7. \( x - y^2 = 4 \)

8. \( x^2 - y^2 = 9 \)

Do you UNDERSTAND?

8. Writing: How can you find the distance between the vertex of an elliptic cylinder if you know how long the major and minor axes are? Answers may vary. Sample: Find the distance between the center and the point. This is the radius. Then substitute the coordinates of the center and the radius in the equation of the ellipse. Answers may vary. Sample: Compare the equations to \( (x - 2)^2 + (y + 3)^2 = 4 \). Answers may vary. Sample: Draw the graph of the parabola; draw a circle around the center with radius 3.

Chapter 10 Test (continued)

Write an equation of a hyperbola with the given characteristics.

12. Vertices (3, 1), (3, 5); foci (1, 1), (3, 7)
14. Vertices (5, 1), (5, 3); foci (1, 1), (7, 3)

Identify the conic section represented by each equation by writing the equation in standard form. For a parabola, give the vertex. For a circle, give the center and the radius. For an ellipse or a hyperbola, give the center and the focus.

15. \( x^2 + y^2 = 9 \)
16. \( x^2 - y^2 = 1 \)

17. \( x^2 + y^2 + 2x - 4y = 4 \)
18. \( x^2 - y^2 = 244 \)

Do you UNDERSTAND?

19. Reasoning: In the major axis of the ellipse \( 10x^2 + 4y^2 = 44 \), explain why the equation does not have standard form. Answers may vary. Sample: The graph is a horizontal ellipse.

20. Vocabulary: What is the central rectangle of a hyperbola? How does the central rectangle help you draw the graph of a hyperbola? Answers may vary. Sample: The central rectangle of a hyperbola is the rectangle formed by the vertices of the hyperbola. It helps in determining the asymptotes of the hyperbola, which are the lines that the branches approach but never touch.
Chapter 10 Performance Tasks (continued)

Task 3
Suppose you toss a volleyball to a friend who is 12 ft away. Midway between you and your friend is a volleyball net, the top of which is 7 ft high. The coiling of the gymnasium is 21 ft high. When you toss the ball to your friend, the shape of its path forms a parabola. The volleyball is 4 ft off the floor when it leaves your hands, and your friend catches the volleyball 4 ft off the floor as well. Write the equation that models the path of the volleyball if it passes midway between the net and the coiling. Assume the origin is at the base of the volleyball net. Include the vertex, focus, and directrix of the parabola.

\[ y = -\frac{1}{2}x^2 + 10, \text{ vertex } (0, 10), \text{ focus } (0, 15), \text{ directrix at } y = 14 \]

[4] Student writes equation accurately. The vertex, focus, and directrix are identified correctly.
[3] Student writes equation with minor errors, and makes minor errors in identifying the vertex, focus, and directrix.
[2] Student writes the equation with major errors, and makes major errors in identifying the vertex, focus, and directrix.
[1] Student writes only a partial equation and partially identifies the vertex, focus, and directrix.
[0] Student makes no attempt. OR no response is given.

Task 4
a. Write the equation for a conic section correctly.

[3] Student writes the equation for a conic section with a minor error. Student graphs equation with only minor errors. Student translates equation with minor error. Student writes new equation with minor error.
[2] Student writes the equation for a conic section with a major error. Student graphs the equation with a major error. Student translates the equation with a major error. Student writes a new equation with a major error.
[1] Student writes the equation in partial form. Student graphs the equation partially. Student translates the equation partially. Student writes the new equation partially.
[0] Student makes no attempt. OR no response is given.

b. Graph the equation.

c. Translate the conic section two units to the right and three units downward, and write in new equation.

a-c. Student’s work.
[3] Student writes the equation for a conic section with a minor error. Student graphs equation with only minor errors. Student translates equation with minor error. Student writes new equation with minor error.
[2] Student writes the equation for a conic section with a major error. Student graphs the equation with a major error. Student translates the equation with a major error. Student writes a new equation with a major error.
[1] Student writes the equation in partial form. Student graphs the equation partially. Student translates the equation partially. Student writes the new equation partially.
[0] Student makes no attempt. OR no response is given.
Chapter 10 Project: About Face!

About the Project
The Chapter Project gives students an opportunity to use equations of conic sections to design clown faces showing different emotions. They write equations to model face dimensions. The students make a display of their work and invite other classes to tour the exhibit. They explain the designs to visitors.

Introducing the Project
• Ask students to describe clown faces they have seen. 
• Invite students to discuss where they can research the art of creating expressive clown faces.
• Encourage students to discuss features that are common to most clown faces and features that frequently differ among them.

Activity 1: Graphing
Students design clown faces using parts of graphs of parabolas.

Activity 2: Designing
Students design several clown faces conveying various emotions using graphs of conic sections.

Activity 3: Analyzing
Students analyze the dimensions of the faces of class members to find average elliptical face shapes.

Finishing the Project
You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results.
• Have students retrace their methods for finding equations and restrictions on those equations.
• Ask groups to share their insights that resulted from completing the project, such as any shortcuts for writing equations to model their designs.

Finishing the Project
Th e activities should help you complete your project. Plan a class exhibit of your work. With your classmates, you will organize a display of your work.

Getting Started
Read the project. As you work on the project, you will need a graphing calculator, a tape measure, materials on which you can record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist
☐ Activity 1: making designs with parabolas
☐ Activity 2: making designs with conic sections
☐ Activity 3: modeling face shapes
☐ class exhibit

Scoring Rubric
4 Face designs are well defined and show various emotions. Equations and restrictions are correct and can be used by others to reconstruct the faces.
3 Designs show various emotions but could be better. Equations and their restrictions are mostly accurate with minor errors. The class display could be more effective.
2 Designs show some emotions but could be improved. Equations and their restrictions are mostly accurate with some errors. The class display is informative and attractive.
1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project
Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of the Project
Evaluate your work, based on the Scoring Rubric.
Solve.
Evaluate each expression.

5. \(11 - 1\)
6. \(26\)
7. \(22\)
8. \(21\)
9. \(17\)
10. \(12\)
11. \(8\)
12. \(3\)
13. \(2\)
14. \(11 - 1\)

Vegetable combinations.

Example

Number of Combinations

The number of permutations of \(n\) items of a set arranged \(r\) items at a time is

\[P_n^r = \frac{n!}{(n - r)!}\]

for \(0 \leq r \leq n\).

Example

\[P_5^2 = \frac{5!}{(5 - 2)!} = \frac{120}{6} = 20\]

Evaluate each expression.
3. \(P_4^1 = 4\)
4. \(P_5^2 = 20\)

Number of Combinations

The number of combinations of \(n\) items of a set chosen \(r\) items at a time is

\[C_n^r = \frac{n!}{r!(n - r)!}\]

for \(0 \leq r \leq n\).

Example

\[C_7^3 = \frac{7!}{3!(7 - 3)!} = \frac{7!}{3!4!} = \frac{5040}{6} = 840\]

Evaluate each expression.
5. \(C_5^3 = 10\)
6. \(C_5^3 = 10\)

11-1 Practice

For each situation, determine whether to use a permutation or a combination. Then solve the problem.
31. You draw the names of 5 raffle winners from a basket of 50 names. Each person wins the same prize. How many different groups of winners could you draw? combination: 2,118,760
32. A paint store offers 15 different shades of blue. How many different ways could you purchase 3 shades of blue? combination: 455
33. How many different 5-letter codes can you make from the letters in the word \(c\)\(h\)\(r\)\(o\)\(r\)? permutation: 720

Assume \(a\) and \(b\) are positive integers. Determine whether each statement is true or false. If it is true, explain why. If it is false, give a counterexample.
34. \(a\cdot b = b\cdot a\); True; Commutative Property of Multiplication
35. \((a\cdot b)\cdot c = a\cdot (b\cdot c)\); False; let \(a = 2\), \(b = 3\), \(c = 4\); \(2\cdot 3\cdot 4 = 24\), \(2\cdot 12 = 24\)
36. \(a\cdot (b + c) = ab + ac\); True; Distributive Property of Multiplication
37. \(\frac{1}{a} = a\); True; Identity Property of Multiplication
38. \(\frac{a}{b} = \frac{b}{a}\); False; let \(a = 4\) and \(b = 2\); \(\frac{4}{2} = 2\), \(\frac{2}{4} = \frac{1}{2}\)
39. \(a\cdot b\cdot c = abc\); True; Commutative and Associative Properties

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Evaluate each expression.

1. Evaluate the following expression.
   \[ \frac{11}{2} \]

2. Evaluate the following expression.
   \[ 9 \times 6 \]

3. Evaluate the following expression.
   \[ 10 \]

4. Evaluate the following expression.
   \[ 5! \times 4! \]

5. Evaluate the following expression.
   \[ 5! \times 4! \]

Find the number of permutations in the following problems.

6. Your coach has twelve team jerseys numbered from 1 through 12. He plans to give one jersey to each of the twelve members of the basketball team. In how many ways can the jerseys be assigned?

7. The owner of a car lot is lining up 7 cars in the show-room window. In how many ways can the cars be ordered?

Evaluate each expression.

8. \[ \binom{5}{3} \]

9. \[ \binom{6}{2} \]

10. \[ \binom{8}{5} \]

11. Twelve different types of pizzas are being judged in a contest. In how many different ways can the pizzas be judged first, second, third, and fourth?

12. An ice cream parlor offers 14 different types of ice cream. In how many different ways can you select 5 types of ice cream to sample?

13. Eleven groups entered a science fair competition. In how many ways can the groups finish first, second, and third?

14. Your aunt is ordering appetizers for her and her family. The restaurant offers 10 different appetizers. She will select 4 appetizers. How many different combinations of appetizers can your aunt possibly select?

15. Your friend believes that she has 1,028,160 different combinations that she could possibly select. What error did your friend make? How many different combinations could she possibly select?

16. Your friend used a permutation when she should have used a combination. She could possibly select 8568 different combinations.

17. A traveler can choose from three airlines, five hotels, and four rental car companies. How many arrangements of these services are possible?

18. How many different arrangements of these services are possible?

19. An ice cream parlor offers 14 different types of ice cream. In how many different orders can 3 different appetizers be brought to the table?

20. In how many possible ways can this be done?

21. Why is this answer the same as the number of ways that 3 customers can order different appetizers of the 12 be brought to the table?

Dinner at a Chinese Restaurant

A typical Chinese restaurant will often feature a Special Dinner, in which the customer has the choice of ordering one appetizer and one entree.

1. If there are 4 appetizers and 11 entrees, how many different Special Dinners are there? 44

2. If there are 12 appetizers and 7 entrees, how many different Special Dinners are there? 84

3. If there are A appetizers and E entrees, how many different Special Dinners are there? AE

4. There are 12 appetizers, 4 soups; 6 contain meat, and 2 do not. In how many different orders can 3 different appetizers be brought to the table?

5. In how many different orders can 5 different appetizers of the 12 be brought to the table?

6. Do Exercises 1–5 involve permutations or combinations? Permutations

7. Suppose that 7 customers arrive, and each orders a different appetizer to share from a choice of 12 appetizers.
   a. Does this problem involve permutations or combinations? Combinations
   b. Why? Order doesn’t matter
   c. In how many ways can you make the 7 appetizers?

8. Suppose that 5 customers arrive, and each orders a different appetizer to share from a choice of 12 appetizers. In how many ways can this be done?

9. Suppose that 5 customers arrive, and each orders a different appetizer to share from a choice of 12 appetizers.
   a. In how many ways can this be done?
   b. Why is this the answer?
   c. In how many ways can 5 customers order different appetizers?

10. Evaluating the number of permutations in the following problems.

   a. Evaluate the following expression.
   \[ \binom{5}{3} \]

   b. Evaluate the following expression.
   \[ \binom{6}{2} \]

   c. Evaluate the following expression.
   \[ \binom{8}{5} \]

   d. Evaluate the following expression.
   \[ \binom{5}{3} \times \binom{6}{2} \]

   e. Evaluate the following expression.
   \[ \binom{8}{5} \times \binom{5}{3} \]

   f. Evaluate the following expression.
   \[ \binom{6}{2} \times \binom{8}{5} \]

   g. Evaluate the following expression.
   \[ \binom{8}{5} \times \binom{5}{3} \]

   h. Evaluate the following expression.
   \[ \binom{5}{3} \times \binom{6}{2} \]

   i. Evaluate the following expression.
   \[ \binom{8}{5} \times \binom{5}{3} \]

   j. Evaluate the following expression.
   \[ \binom{5}{3} \times \binom{6}{2} \]

   k. Evaluate the following expression.
   \[ \binom{8}{5} \times \binom{5}{3} \]

   l. Evaluate the following expression.
   \[ \binom{5}{3} \times \binom{6}{2} \]

   m. Evaluate the following expression.
   \[ \binom{8}{5} \times \binom{5}{3} \]

   n. Evaluate the following expression.
   \[ \binom{5}{3} \times \binom{6}{2} \]

   o. Evaluate the following expression.
   \[ \binom{8}{5} \times \binom{5}{3} \]
3. Exercises

In how many ways can the judges select the first-prize and second-prize winners.

For example, suppose Ana, Bob, Cal, and Dan enter a local essay contest. Here are some possible ways for the judges to select the first-prize and second-prize winners.

<table>
<thead>
<tr>
<th>First Prize</th>
<th>Second Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ana</td>
<td>Bob</td>
</tr>
<tr>
<td>Bob</td>
<td>Ana</td>
</tr>
<tr>
<td>Dan</td>
<td>Cal</td>
</tr>
<tr>
<td>Cal</td>
<td>Dan</td>
</tr>
</tbody>
</table>

The order of the names in the selection is important. The selection “Ana, Bob” is a permutation of the group of contestants.

There are 4 people in the group of contestants.

Step 1 Substitute for each variable in the formula.

$$P_n^r = \frac{n!}{(n-r)!}$$

Step 2 Describe n and r.

There are 4 people in the group of contestants. n = 4

There are 2 people in each selection of prize winners. r = 2

Step 3 Substitute for each variable in the formula.

$$P_4^2 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4 \times 3 \times 2!}{2!} = 12$$

There are 12 ways for the judges to choose the first-prize and second-prize winners.

Exercises

1. In how many ways can you choose 6 letters for a password from the set A, B, C, D, E, F, G.

2. In how many ways can a club with 15 members elect a president, vice president, and treasurer?

3. In how many ways can a family of 6 line up in 1 row for a photograph? 720

4. How many 3-flavor blends can you create from 10 frozen yogurt flavors?

5. How many 3-flavor blends can you create from 10 frozen yogurt flavors? 120
A group of five cards are numbered 1–5. You choose one card at random. Find each experimental probability.

1. Find each experimental probability.

2. A basketball player attempted 24 shots and made 13. Find the experimental probability that the player will make the next shot she attempts. \[ \approx 0.54 \text{ or } 54\% \]

3. A baseball player attempted to steal a base 70 times and was successful 47 times. Find the experimental probability that the player will be successful on his next attempt to steal a base. \[ \approx 0.67, \text{ or } 67\% \]

Graphing Calculator For Exercises 3–4, define a simulation by telling how you represent correct answers, incorrect answers, and the quiz. Use your simulation to find each experimental probability.

3. If you guess the answers at random, what is the probability of getting at least three correct answers on a five-question true-or-false quiz? Answers may vary. Sample: Let “1” be a correct answer. Let “2” be an incorrect answer. Generate 46 sets of five random 1’s, 2’s, 3’s, 4’s, and 5’s. \( \frac{46}{50} = 0.92, \text{ or } 92\% \)

4. A free-throw multiple-choice quiz has four choices for each answer. If you guess the answers at random, what is the probability of getting at least four correct answers? Answers may vary. Sample: Let “1” be a correct answer. Let “2”, “3”, and “4” be incorrect answers. Generate 64 sets of 5 random 1’s, 2’s, 3’s, 4’s, and 5’s. \( \frac{5}{64} = 0.078125, \text{ or } 0.78\% \)

A group of five cards are numbered 1–5. You choose one card at random. Find each theoretical probability.

5. \( P(\text{card is a} \, 2) = \frac{1}{5} \), or 20\%

6. \( P(\text{even number}) = \frac{3}{5} = 0.60, \text{ or } 60\% \)

7. \( P(\text{prime number}) = \frac{2}{5} = 0.40, \text{ or } 40\% \)

8. \( P(\text{less than 5}) = \frac{5}{5} = 1 \), or 100\%

A bucket contains 15 blue pens, 25 black pens, and 40 red pens. You pick one pen at random. Find each theoretical probability.

9. \( P(\text{black pen}) = \frac{25}{60} = 0.42, \text{ or } 42\% \)

10. \( P(\text{blue pen or red pen}) = \frac{35}{60} = 0.58, \text{ or } 58\% \)

11. \( P(\text{not a blue pen}) = 1 - \frac{15}{60} = \frac{45}{60} = 0.75, \text{ or } 75\% \)

12. \( P(\text{black pen or not a red pen}) \)

13. Use area to find the following theoretical probabilities.

14. Find each theoretical probability.

15. The ball lands in the pool. \[ = 0.00, \text{ or } 0\% \]

16. The ball lands in the garden or the pool. \[ = 0.99, \text{ or } 99\% \]

17. The ball does not land in the pool. \[ = 0.01, \text{ or } 1\% \]

Five people each flip a coin one time. Find each theoretical probability.

18. \( P(\text{at least 3 heads}) = 0.5, \text{ or } 50\% \)

19. \( P(\text{exactly 2 tails}) = 0.375, \text{ or } 37.5\% \)

20. \( P(\text{at least 3 heads}) = 0.625, \text{ or } 62.5\% \)

21. \( P(\text{at least 3 heads}) = 0.03125, \text{ or } 3.125\% \)

22. The spinner shown at the right has four equal-sized sections. Suppose you spin the spinner two times. \( \{1, 1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 1\}, \{2, 2\}, \{2, 3\}, \{2, 4\}, \{3, 1\}, \{3, 2\}, \{3, 3\}, \{3, 4\}, \{4, 1\}, \{4, 2\}, \{4, 3\}, \{4, 4\} \)

a. What is the sample space?

b. How many outcomes are there? 16

c. What is the theoretical probability of getting a sum of 4? \[ \frac{3}{16} = 0.1875, \text{ or } 18.75\% \]

23. If \( x \) is a real number and \( x = 0 \), what is the probability that \( \frac{1}{x} \) is undefined? \[ 0 \]

24. If \( x \) is a real number and \( x \neq 0 \), what is the probability that \( \frac{1}{x} \) is undefined? \[ 0 \]

25. Of the 195 students in the senior class, 104 study Spanish and 86 study French. With 12 studying both Spanish and French. What is the theoretical probability that a student chosen at random is studying Spanish, but not French? \[ = 0.47 \text{ or } 47\% \]

Find each of the following theoretical probabilities.

26. Shawn rolls a pair of fair number cubes. What is the theoretical probability that he will roll a sum of 4? \[ \frac{3}{36} = 0.0833, \text{ or } 8.33\% \]

27. Karen is playing a basketball game. What is the theoretical probability that she will make a free throw? \[ \frac{1}{2} \text{ or } 50\% \]

28. A box contains 24 green markers, 16 red markers, and 10 blue markers. \( a. \frac{10}{30} \text{ or } 33\% \)

b. \( \frac{1}{3}, \text{ or } 33\% \)

c. \( \frac{17}{30} \text{ or } 56.67\% \)

Use combinations to find the following theoretical probability.

29. Six of the 32 players on the football team are left-handed. There are 5 starting offensive linemen. What is the theoretical probability that 2 of the starting offensive linemen are left-handed? \[ \frac{\binom{5}{2} \cdot \binom{27}{0}}{\binom{32}{5}} = \frac{10 \cdot 1}{208,8125} = 0.0048, \text{ or } 0.48\% \]

Use area to find the following theoretical probabilities.

30. The floor in your friend’s house covers 400 ft². The floor in her bedroom is 14 ft by 10 ft. What is the theoretical probability that a randomly selected point on the floor of the house is in your friend’s bedroom? \[ \frac{140}{400} = 0.35, \text{ or } 35\% \]

31. A garden is 15 ft by 12 ft. Tomatoes fill a 5 foot by 4 foot section of the garden. A squirrel leaps from a tree into the garden. What is the theoretical probability that the squirrel will land in the tomato section of the garden? \[ \frac{20}{180} = 0.1111, \text{ or } 11.11\% \]

Find each theoretical probability.

32. A student rolled a six-sided number cube 60 times. She rolled the number 4 16 times. The probability is about 26\%.

33. A baseball player got a hit in 12 of his last 40 at bats. What is the probability that the baseball player will get a hit in his next at bat? \[ \frac{12}{40} = 0.30, \text{ or } 30\% \]

34. A baseball player attempted to steal a base 70 times and was successful 47 times. Find the probability that the player was successful on his next attempt to steal a base. \[ \frac{47}{70} = 0.67, \text{ or } 67\% \]

35. There are 225 juniors and 255 seniors at your school. The school chooses 5 juniors and seniors as Student All-Stars. What is the theoretical probability that exactly 2 of the Student All-Stars will be juniors? \[ \frac{225 \cdot 5}{480} = 0.3625, \text{ or } 36.25\% \]

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For Exercises 1−3, find each theoretical probability based on one roll of two number cubes. Enter each answer in the grid as a whole percent.

1. \( P(\text{sum} = 9) \)
2. \( P(\text{at least one 6}) \)
3. \( P(\text{sum} > 12) \)

For Exercises 4−5, find each theoretical probability based on one marble drawn at random from a bag of 14 red marbles, 10 pink marbles, 18 blue marbles, and 6 gold marbles. Enter each answer in the grid as a fraction in simplest form.

4. \( P(\text{not pink}) \)
5. \( P(\text{blue or gold}) \)

### Answers

1. 3/36
2. 2/36
3. 0
4. 21/36
5. 26/36

### 11-2 Reteaching

#### Probability

Probability is a measure of how likely a specific event is to occur. To find the experimental probability of a specific event, you conduct an experiment, or simulation of the experiment, multiple times. Each time you run the experiment or simulation, you are conducting a trial. You count the number of times the event you are looking for occurs. The experimental probability is the ratio of the number of times the event occurs to the number of trials.

\[ P(\text{event}) = \frac{\# \text{ of times the event occurs}}{\# \text{ of trials}} \]

**Problem**

You toss a coin 12 times and record each result: H, T, T, H, H, T, H, T, H, T, T, H. What is the experimental probability of tails?

**Step 1**

Determine the total number of trials, the event you are looking for, and the number of times the event occurred during the trials.

One toss of the coin is one trial. You did 12 trials. The event you are looking for is tails. Tails occurred 5 times.

**Step 2**

Use the formula for experimental probability.

\[ P(\text{tails}) = \frac{\text{# of tails}}{\text{# of trials}} = \frac{5}{12} = 0.42, \text{ or } 42\% \]

The experimental probability of tails is about 42.

#### Exercises

In a telephone survey of 150 households, 75 people answered “yes” to a particular question, 50 answered “no,” and 25 were “not sure.” Find each experimental probability.

1. \( P(\text{yes}) \)
2. \( P(\text{no}) \)
3. \( P(\text{not sure}) \)
4. \( P(\text{not yes}) \)
5. \( P(\text{yes or no}) \)
6. \( P(\text{yes and not sure}) \)

#### 11-2 Enrichment

Biology uses a Punnett Square to predict the gene combinations that are possible for an offspring when the genes of the parents are known. Each parent organism carries two genes, or alleles, for a particular trait. For example, a parent might have the genotype BB for dimples. A capital B represents the dominant trait, which is having dimples. A lower case b represents the recessive trait, which is not having dimples.

1. Both parents contribute one allele to their offspring. For example, if both parents have the genotype Bb, an offspring could inherit the genotype of Bb with each parent contributing one dominant allele. What are the other possible combinations? Bb, Bb, Bb

2. This information can be displayed in a Punnett Square. Each side of the square represents the genotype of one parent. The Punnett Square for the offspring of the parents who both have the genotype Bb is shown at the right. If the dominant allele is present it will be the trait that appears. What is the probability that this offspring will not have dimples?

3. Create a Punnett Square to show the possible gene combinations for the offspring if one parent has the genotype BB and the other has the genotype BB.

4. What is the probability that the offspring of parents with Bb and Bb will have dimples?

5. You can show more complicated crosses when you consider two or more genes that are independent of each other. For example, pea pods can either be round (R) or wrinkled (r), yellow (Y) or green (y). What are the possible combinations of shape and color? RR, Rr, rr, YY, Yy, yy

6. Create a Punnett Square to show the possible gene combinations for the pea pods.

7. What is the probability that a pea pod with both parents RYy will be round and yellow? In other words, what is the probability that there is an R and a Y present?

8. What is the probability that a pea pod with both parents ryY will be wrinkled and yellow?

9. What is the probability that a pea pod with both parents RryY will be wrinkled and green?

#### Exercises

Use the spinner at the right to find each theoretical probability.

7. \( P(\text{the number is even}) \)
8. \( P(5) \)
9. \( P(\text{the number is prime}) \)
10. \( P(\text{multiple of 3}) \)
11. \( P(\text{the number is less than 6}) \)
12. \( P(7 \text{ and not } 7) \)
For Exercises 1–5, draw a line from each word or phrase in Column A to the matching item in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. dependent events</td>
<td>A. the occurrence of one event does NOT affect how a second event can occur</td>
</tr>
<tr>
<td>2. probability of independent events</td>
<td>B. two events that cannot occur at the same time</td>
</tr>
<tr>
<td>3. independent events</td>
<td>C. ( P(A) \times P(B) )</td>
</tr>
<tr>
<td>4. probability of mutually exclusive events</td>
<td>D. the occurrence of one event affects how a second event can occur</td>
</tr>
<tr>
<td>5. mutually exclusive events</td>
<td>E. ( P(A) + P(B) )</td>
</tr>
</tbody>
</table>

For Exercises 6–8, draw a line from each item in Column A to the matching item in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Flip a coin. Then roll a number cube.</td>
<td>A. independent events</td>
</tr>
<tr>
<td>7. snow and 80°F weather</td>
<td>B. dependent events</td>
</tr>
<tr>
<td>8. Pick a piece from a set of chess pieces. Then pick a second piece.</td>
<td>C. mutually exclusive events</td>
</tr>
</tbody>
</table>

Classify each pair of events as dependent or independent.  
1. A number of the junior class is selected, one of her pets is selected. dependent  
2. A number of the junior class is selected as junior class president; a freshman is selected as freshman class president. independent  
3. An odd-numbered problem is assigned for homework; an even-numbered problem is picked for a test. independent  
4. The sum of two rolls of a number cube is 6; the product of the same two rolls is 8. dependent  
5. \( P(A) = \frac{1}{6}, P(B) = \frac{1}{6} \)  
6. \( P(A) = 0.8, P(B) = 0.2 \)  
7. \( P(A) = \frac{1}{8}, P(B) = \frac{1}{8} \)  
8. \( P(A) = \frac{3}{5}, P(B) = \frac{2}{5} \)  
9. Suppose you have seven CDs in a box. Four are rock, one is jazz, and two are country. Today you choose one CD without looking, put it in the box. Tomorrow you do the same thing. What is the probability that you choose a country CD both days? \( \frac{2}{7} \)  

You randomly select an integer from 1 to 100. State whether the events are mutually exclusive. Explain your reasoning.

10. The integer is less than 40; the integer is greater than 50. Yes; all integers 1–39 are less than 50.
11. The integer is odd; the integer is a multiple of 4. Yes; all multiples of 4 are even.
12. The integer is less than 50; the integer is greater than 40. No; the integers 41–49 are less than 50 and greater than 40.

\( M \) and \( N \) are mutually exclusive events. Find \( P(M \text{ or } N) \).

13. \( P(M) = \frac{1}{36}, P(N) = \frac{1}{6} \)  
14. \( P(M) = 10\% \), \( P(N) = 45\% \)  
15. \( P(M) = 20\% \), \( P(N) = 18\% \)  
16. \( P(M) = 1 \), \( P(N) = \frac{2}{3} \)  

You have a drawer with five pairs of white socks, three pairs of black socks, and one pair of red socks. You choose one pair of socks at random each morning, starting on Monday. You do not put the socks you choose back in the drawer. Find the probability of each event.

17. A bicycle is a 1-speed. \( 1\% \)  
18. A bicycle is a 1-speed or a 3-speed. \( 11\% \)  
19. A bicycle is not a 1-, 3-, or 10-speed. \( 70\% \)  
20. A bicycle is a 1-speed or a 3-speed. \( 17\% \)  

A fair number cube is tossed. Find each probability.

21. \( P(\text{even or 3}) = \frac{1}{2} \)  
22. \( P(\text{less than 2 or even}) = \frac{1}{2} \)  
23. \( P(\text{prime or 4}) = \frac{1}{3} \)  

You randomly choose a natural number from 1 to 10. What is the probability that you choose a number divisible by 2 or 3? \( P(2 \text{ or } 3) = \frac{3}{5} \)  

The graph at the right shows the types of bicycles in a bike rack. Find each probability.

24. A bicycle is a 1-speed. \( 1\% \)  
25. A bicycle is not a 1- or 3-speed. \( 70\% \)  
26. A bicycle is a 1-speed or a 3-speed. \( 11\% \)  
27. A bicycle is not a 1- or 3-speed. \( 86\% \)  

You have a deck of 52 cards. You draw a card from the deck. Replace it, shuffle the card, and draw again. Find the probability of each event.

28. You draw a spade and then a spade. \( 16\% \)  
29. You draw a spade and then a club. \( 10\% \)  

You randomly select an integer from 1 to 100. You randomly select an integer from 1 to 100. Find each probability.

30. A number of the airplane parts being examined pass inspection. What is the probability that all of the next 5 parts examined will pass inspection? \( \frac{1}{8} \)
Classify each pair of events as dependent or independent.
1. Roll a number cube. Then roll it again. independent
2. Pull a card from a deck of playing cards. Then pull a second card. dependent
3. Randomly choose a student from your class. Then choose another student. dependent
4. Flip a coin. Then spin a spinner. independent

Use the table shown below to answer the following questions.

<table>
<thead>
<tr>
<th>Movie Collection</th>
<th>Video</th>
<th>DVD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>12</td>
<td>26</td>
</tr>
<tr>
<td>Comedy</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>Drama</td>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

5. You randomly pick a video and a Dvd. What is the probability that you pick an action video and a comedy Dvd? about 6%

6. Your friend randomly picks a video and a Dvd. What is the probability that she picks a comedy video and an action Dvd? about 24%

7. What is the probability of randomly picking a drama video and a comedy Dvd? about 2%

8. Writing Explain the difference between independent events and dependent events. Answers may vary. Sample: Events are dependent when the occurrence of one event affects the occurrence of another event that can occur. Events are independent when the occurrence of one event does not affect the occurrence of other events that can occur.

9. The sum is odd. The sum is less than 5. not mutually exclusive
10. The difference is 1. The sum is even. mutually exclusive
11. The sum is a multiple of 4. The sum is odd. mutually exclusive

Find the probability for the following mutually exclusive events.
12. Students can either participate in track and field or play baseball. About 13% of students participate in track and field. About 8% play baseball. What is the probability that a student chosen at random either participates in track and field or plays baseball? about 21%

13. About 4 of a town’s population has black hair. About 5 of the population has blonde hair. What is the probability that a person chosen at random from this town will have either black hair or blonde hair? about 49%

Use the diagram at the right to answer the following questions.
14. Suppose you randomly select a shape from this circle. What is the probability that the shape is black or has five points? 70%

15. What is the probability of randomly selecting a shape that is black or has five points? 80%

Multiple Choice

For Exercises 1−4, choose the correct letter.

A store display shows two red shirts, one blue shirt, and three shirts with red and white stripes. The display also shows two pairs of blue jeans, one pair of white pants, and one pair of white shorts.

1. What is the probability of randomly selecting an item with white or red on it? D

2. What is the probability of randomly selecting two items and getting a pair of blue jeans, putting them back to the display, and then randomly selecting a blue shirt? F

3. What is the probability of randomly selecting a complete outfit (one shirt and one pair of jeans, pants, or shorts) on two picks? C

4. What is the probability of selecting an item with red or blue on it? I

Short Response

5. There is a 50% chance of thunderstorms on Monday, a 50% chance on Tuesday, and a 50% chance on Wednesday. Assume these are independent events. What is the probability that there will be thunderstorms on Monday, Tuesday, and Wednesday? Show your work.

There is a 50% chance of thunderstorms on Monday, Tuesday, and Wednesday.

Answer: 0.50 × 0.50 × 0.50 = 0.125

There is a 12.5% probability of thunderstorms on Monday, Tuesday, and Wednesday.

ANSWERS
11-3 Reteaching
Probability of Multiple Events

- If Event A can change the way Event B occurs, then the events are dependent.
- If Event A cannot change the way Event B occurs, then the events are independent.

If Event A and Event B are independent, the probability of Event A and Event B both occurring is the product of their individual probabilities.

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

**Problem**

A pet store has two cages of mice. The first cage has 10 white mice—4 females and 6 males. Suppose you randomly choose 1 mouse from each cage. What is the probability that you choose a female mouse from each cage? What is the probability that you choose 2 female mice?

- **Step 1** Determine the events.
  - Event A: "you choose a white female mouse."
  - Event B: "you choose a gray female mouse."
- **Step 2** Decide if the events are independent.
  - Your choice of a white mouse does not affect your choice of a gray mouse. The events are independent.
- **Step 3** Use the formula.
  - \[ P(\text{white female and gray female}) = P(\text{white female}) \cdot P(\text{gray female}) \]
  - \[ P(\text{white female}) = \frac{4}{10} = 0.4 \]
  - \[ P(\text{gray female}) = \frac{6}{10} = 0.6 \]
  - \[ P(\text{white female and gray female}) = 0.4 \cdot 0.6 = 0.24 \]

The probability of choosing 2 female mice is 24%.

**Exercises**

Use the information from the problem above. You choose one mouse at random from each cage. Find each probability.

1. \( P(\text{white male and gray male}) \)
2. \( P(\text{white mouse and gray male}) \)
3. \( P(\text{white male and white female}) \)
4. \( P(\text{white female and gray male}) \)
5. \( P(\text{white mouse and white mouse}) \)
6. \( P(\text{gray female}) \)
7. \( P(\text{gray female and gray male}) \)
8. \( P(\text{white male or gray male}) \)
9. \( P(\text{male or gray}) \)
10. \( P(\text{female or gray}) \)

11-4 ELL Support
Conditional Probability

**Problem**

Amsc asked 30 of his classmates whether they prefer soccer or basketball. What is the probability that a student prefers soccer given that the student is female? Explain your work.

\[ P(\text{female and soccer}) = P(\text{female}) \cdot P(\text{soccer}) \]

\[ P(\text{female and soccer}) = \frac{14}{30} \cdot \frac{15}{30} = 0.24 \]

The probability that a student is female and prefers soccer is 24%.

**Exercises**

Use the information from the problem above. You choose one mouse at random. Find each probability.

1. \( P(\text{white male and gray male}) \)
2. \( P(\text{white mouse and gray male}) \)
3. \( P(\text{white male and white female}) \)
4. \( P(\text{white female and gray male}) \)
5. \( P(\text{white mouse and white mouse}) \)
6. \( P(\text{gray female}) \)
7. \( P(\text{gray female and gray male}) \)
8. \( P(\text{male or white}) \)
9. \( P(\text{female or gray}) \)
10. \( P(\text{white male or gray female}) \)

11. \( P(\text{female or gray}) \)
12. \( P(\text{white male or gray male}) \)

11-4 Think About a Plan
Conditional Probability

**Transportation**

You can take Bus 65 or Bus 79. You take the first bus that arrives. The probability that Bus 65 arrives first is 0.75. There is a 40% chance that Bus 65 picks up passengers along the way. There is a 60% chance that Bus 79 picks up passengers. Your bus picked up passengers. What is the probability that it was Bus 65?

1. What is the probability that Bus 65 arrives first? 75%
2. What is the probability that Bus 65 picks up passengers? 40%
3. What is the probability that Bus 79 picks up passengers? 60%
4. What is the probability asking you to determine? the probability that Bus 65 arrived first

**Planning the Solution**

5. Let B65 = Bus 65 arrived first, B79 = Bus 79 arrived first, P = passengers, NP = no passengers. What conditional probability are you looking for? \( P(B65 | P) \)

6. How can a tree diagram help you solve the problem?

Answers may vary. Sample: A tree diagram can help me organize the information in the problem and understand how the probabilities interact.

7. Write an equation you can use to find the probability that your bus was Bus 65. \( P(B65 | P) = \frac{0.75 \times 0.4}{0.75 \times 0.4 + 0.25 \times 0.6} \)

8. Make a tree diagram for this problem.

9. Which two branches of the tree diagram show a bus picking up passengers? 0.75 \( \times \) 0.4 and 0.25 \( \times \) 0.6

10. What is the probability your bus was Bus 65? \( \frac{0.75 \times 0.4}{0.75 \times 0.4 + 0.25 \times 0.6} \) or 66.67%
Each probability.

10. $P(\text{less than high school education}) = 0.55$

7. $P(\text{less than $30,000}) = 0.35$

4. $P(\text{less than $30,000}) \text{ or } (\text{less than high school education}) = 0.25$

4. $P(\text{less than high school education or less | earns over $30,000}) = 0.4$

Use the table below to find each probability. The table gives information about students at one school.

<table>
<thead>
<tr>
<th>Favorite Leisure Activities</th>
<th>Female</th>
<th>Male</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sports</td>
<td>39</td>
<td>67</td>
<td>106</td>
</tr>
<tr>
<td>Hiking</td>
<td>48</td>
<td>58</td>
<td>106</td>
</tr>
<tr>
<td>Reading</td>
<td>85</td>
<td>76</td>
<td>161</td>
</tr>
<tr>
<td>Phoning</td>
<td>62</td>
<td>54</td>
<td>116</td>
</tr>
<tr>
<td>Shopping</td>
<td>71</td>
<td>68</td>
<td>139</td>
</tr>
<tr>
<td>Other</td>
<td>25</td>
<td>29</td>
<td>54</td>
</tr>
</tbody>
</table>

5. $P(\text{sports | female}) = 0.37$

6. $P(\text{female | sports}) = 0.36$

7. $P(\text{reading | male}) = 0.25$

8. $P(\text{male | reading}) = 0.47$

9. $P(\text{hiking | female}) = 0.35$

10. $P(\text{female | hiking}) = 0.55$

11. The senior class is 55% female, and 32% of the class are females who play a sport. What is the probability that a student plays a competitive sport, given that the student is female? 0.58

12. A school's colors are blue and gold. At a pep rally, 65% of the students are wearing blue. What is the probability that a student is wearing gold? 0.35

13. What is the probability that a student prefers novels, given that the student is in high school? 0.51

14. A softball game has an 80% chance of being cancelled if it rains and a 30% chance of being cancelled if there is fog when there is no rain. There is a 70% chance of fog with no rain and a 30% chance of rain.

a. Make a tree diagram based on the information above.

b. Find the probability that there will be fog and the game will be cancelled.

15. The population of a high school is 51% male. 45% of the males and 48% of the females attend concerts.

a. Make a tree diagram based on the information above.

b. Find the probability that a student is male and attends concerts.

16. Why might Tony want to know the probability that his classmate is left-handed, given that she is female?

17. A school's colors are blue and gold. At a pep rally, 65% of the students are wearing blue.

18. $P(\text{blue | female})$ = 0.37

19. $P(\text{female | blue})$ = 0.35

20. $P(\text{blue | female})$ = 0.35

21. $P(\text{female | blue})$ = 0.35

22. $P(\text{blue | female})$ = 0.35

23. $P(\text{female | blue})$ = 0.35

The following table shows national employment statistics. Use the table to find each probability.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Professionals</th>
<th>Sales People</th>
<th>Laborers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men</td>
<td>487,900</td>
<td>203,300</td>
<td>281,000</td>
</tr>
<tr>
<td>Women</td>
<td>474,713</td>
<td>321,313</td>
<td>1462</td>
</tr>
</tbody>
</table>

7. $P(\text{male | professional}) = 0.47$

8. $P(\text{female | professional}) = 0.53$

9. $P(\text{male | sales}) = 0.55$

10. $P(\text{male | professional}) = 0.35$

11. $P(\text{sales | male}) = 0.27$

12. $P(\text{male | laborer}) = 0.67$
11-4 Standardized Test Prep

Conditional Probability

Multiple Choice

For Exercises 1–2, choose the correct letter.

A local bookstore classifies its books by type of reader, type of book, and cost. Use the table at the right for Exercises 1–2.

1. What is the probability that a book selected at random is a child’s book, given that it costs $15? C

2. What is the probability that a book selected at random is fiction, given that it costs $10? A

Extended Response

3. Of the photographs produced in one day at a photo shop, 25% are black-and-white and 75% are color. Portraits make up 60% of the black-and-white photos and 45% of the color photos. Let B, C, P, and N represent black-and-white, color, portrait, and not a portrait, respectively. Draw a tree diagram to represent this situation. What is the probability that a photo chosen at random is not a portrait?

4. [Student used correct tree diagram and conditional probability formula to show P(B | P) = 50%.

5. [Student used diagram and formula, but misunderstood part of the problem or ignored important information.

6. [Student attempted to use diagram and formula, but did so incorrectly. OR student used inappropriate strategy, but showed some understanding of the problem.

7. [Student attempted to solve problem, but used inappropriate strategy and made little progress toward solution.

8. Incorrect answers and no work shown OR no answers given

11-4 Enrichment

Conditional Probability

The Probability of Receiving a Defective Item in a Shipment

Many manufactured items look interchangeable. Examples are ball bearings, light bulbs, and transistors. However, an individual ball bearing may be too large or too small, and a light bulb or a transistor that looks fine may prove to be defective. The following exercises require the computation of certain probabilities based on the number of defective items and the size of the sample. Answers can be given in terms of combination symbols.

1. A shipment contains 50 transistors, 3 of which are defective. What is the probability that a randomly chosen transistor from this shipment works? What is the probability that it is defective?

2. A sample consisting of 2 transistors is chosen from this shipment. What is the probability that both transistors work? What is the probability that both transistors are defective?

3. A shipment contains 80 ball bearings, 5 of which are defective. What is the probability that a randomly selected ball bearing is of an acceptable size? What is the probability that it is defective?

4. A sample of three ball bearings is chosen from the above shipment.

5. A shipment of toy cars contains 40 red cars and 45 blue cars. Two cars of each color are defective. A sample of three toy cars is chosen from the shipment.

6. What is the probability that all three cars are blue?

7. What is the probability that an odd number of ball bearings are defective?

8. What is the probability that all three ball bearings are defective?

9. What is the probability that all three ball bearings are acceptable?

11-4 Reteaching

Conditional Probability

When events A and B are dependent, the probability of B occurring depends on whether A has already occurred. This kind of probability is called conditional probability. The probability of B given that A has occurred is written as P(B | A).

Problem

A computer lab has 10 computers. Some have CD drives and some have DVD drives. Some are new and some are used. A student picks a computer at random. Use the table to find each probability.

<table>
<thead>
<tr>
<th>CD</th>
<th>DVD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Used</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

P(computer is new) = 10/10

P(computer has a CD drive) = 7/10

P(computer is new and has a CD drive) = 4/10

P(computer is new given it has a CD drive) = 4/7

Note the difference between the last two probabilities. The conditional probability is based only on the number of computers that meet the condition, not on the total number of computers in the lab.

Exercises

1. P(red) = 1/2

2. P(with dots) = 3/10

3. P(red and with dots) = 3/20

4. P(red with dots) = 3/20

5. P(white no dots) = 1/10

6. P(no dots | white) = 5/10

11-4 Reteaching (continued)

Conditional Probability

You can use a tree diagram to help you find the probabilities of dependent events.

Problem

Suppose a class is 55% male. Of the males, 34% are at least 68 in. tall. Of the females, 36.3% are at least 68 in. tall. What is the probability that a randomly chosen student is a female at least 68 in. tall?

Step 1 Organize the information in a tree diagram.

Step 2 Determine the probability you want to find.

Step 3 Rewrite the conditional probability formula to find P(F | G).

Step 4 Substitute information from the tree diagram.

The probability that a randomly chosen student is a female at least 68 in. tall is 3.4%.

Exercises

Use the tree diagram above to find each probability.

7. P(L and M) = 16.3%

8. P(L and M) = 18.7%

9. P(L and F) = 20.6%
**11-5 Think About a Plan**

### Meteorology

On May 3, 1999, 59 tornadoes hit Oklahoma in the largest tornado outbreak ever recorded in the state. Sixteen of these were classified as strong (F2 or F3) or violent (F4 or F5).

- Make a box-and-whisker plot of the data for length of path.
- Identify the outliers. Remove them from the data set and make a revised box-and-whisker plot.
- Writing How does the removal of the outliers affect the box-and-whisker plot? How does it affect the mean of the data set?

1. Arrange the data in increasing order.
   1, 2, 4, 6, 7, 8, 8, 9, 12, 13, 15, 17, 22, 27, 39, 50, 53, 54, 55, 56, 57, 60, 62

2. Minimum value = 1
   Minimum value = 1
   Q1 = 5
   Q3 = 27
   Interquartile range = 22

3. Use your previous answers to make a box-and-whisker plot of the data for length of path.

4. How can you identify the outliers in the data set?
   Answers may vary. Sample: Look at the ends of the ordered data for values that are substantially different.

5. What are the outliers in the data set?
   22, 27, 39

6. Remove the outliers from the data set and make a revised box-and-whisker plot.

7. How does the removal of the outliers affect the box-and-whisker plot?
   Answers may vary. Sample: The box is shifted left and the median moves to almost the center of the box. The whiskers become shorter, especially the right one.

8. How does the removal of the outliers affect the median of the data set?
   The median shifts from 8.5 to 8.

### Major Tornadoes in Oklahoma, May 3, 1999

<table>
<thead>
<tr>
<th>Length of path (miles)</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>F3</td>
</tr>
<tr>
<td>9</td>
<td>F3</td>
</tr>
<tr>
<td>6</td>
<td>F2</td>
</tr>
<tr>
<td>4</td>
<td>F2</td>
</tr>
<tr>
<td>7</td>
<td>F2</td>
</tr>
<tr>
<td>7</td>
<td>F2</td>
</tr>
<tr>
<td>15</td>
<td>F4</td>
</tr>
<tr>
<td>30</td>
<td>F4</td>
</tr>
<tr>
<td>1</td>
<td>F1</td>
</tr>
<tr>
<td>22</td>
<td>F3</td>
</tr>
<tr>
<td>15</td>
<td>F3</td>
</tr>
<tr>
<td>8</td>
<td>F2</td>
</tr>
<tr>
<td>8</td>
<td>F2</td>
</tr>
<tr>
<td>13</td>
<td>F3</td>
</tr>
<tr>
<td>2</td>
<td>F1</td>
</tr>
</tbody>
</table>

### Practice (continued)

Find the values at the 20th and 80th percentiles for each set of values.

10. 15 15 16 17 18 17 17 18 19 18 19
   11. 37 39 34 38 36 35 36 37 35 38 40 42 34 3
   12. 15 16 17 18 19 18 19 20 20 21 20 21 20 19

Identify the outlier in each data set. Then find the mean, median, and mode of the data set when the outlier is included and when it is not.

13. 23 76 79 76 74 75 73; about 68.6, 76, 76; about 76.2, 76, 76
14. 43 46 49 50 52 54 78 47 78; about 52.4, 49.5, no mode; about 48.7, 48, no mode
15. The table shows the number of shares-of-ice servings sold during the first week of July. See below.

<table>
<thead>
<tr>
<th>Date</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share sold</td>
<td>50</td>
<td>55</td>
<td>55</td>
<td>57</td>
<td>62</td>
<td>67</td>
<td>64</td>
<td>56</td>
<td>53</td>
</tr>
</tbody>
</table>

### Practice

Find the mean, median, and mode of each set of values.

1. Customers per day: 98 97 79 92 101 99 97 102 93
   about 97; 97.7; 97

2. Weight (g): 2.3 2.5 2.6 2.8 2.3
   2.5

3. Length (m): 12 13 14 15 16 17 18
   15.2, 15; 17

### Answers

1. Determine the maximum, minimum, and range of each set of values.
2. Make a box-and-whisker plot for each set of values.
3. Determine what a box-and-whisker plot shows about the set of values.
4. Determine if the outliers in a data set are errors and what can be done about them.
5. Make a box-and-whisker plot and determine what the box, whiskers, and median tell about the data.
6. Identify the outlier in each data set. Then find the mean, median, and mode of the data set when the outlier is included and when it is not.
7. Find the values at the 20th and 80th percentiles for each set of values.
8. Identify the outlier in each data set. Then find the mean, median, and mode of the data set when the outlier is included and when it is not.
9. The table shows the number of shares-of-ice servings sold during the first week of July. See below.

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11-5 Practice Form K

## Analyzing Data

### Multiple Choice

For Exercises 1–5, choose the correct letter. Use the data set below.

<table>
<thead>
<tr>
<th>Day</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deliveries</td>
<td>24</td>
<td>26</td>
<td>33</td>
<td>56</td>
<td>68</td>
<td>72</td>
<td>70</td>
<td>67</td>
<td>50</td>
<td>64</td>
<td>48</td>
<td>54</td>
</tr>
</tbody>
</table>

1. What is the mean of the data set? A) 12  B) 15  C) 16.5  D) 18

2. How many modes does the data set have? A) 0  B) 1  C) 2  D) 3

3. What is the interquartile range of the data? A) 0.5  B) 3  C) 4.5  D) 15

4. What is the median value of the data set without the outlier? A) 16  B) 17  C) 19  D) 29

5. What value is at the 50th percentile? A) 12  B) 15  C) 16  D) 17

### Short Response

6. Make a box-and-whisker plot of the data set. Label the median, minimum, maximum, first quartile, and third quartile.

[Graph showing a box-and-whisker plot with specified values]

### Enrichment

Analyzing Data

#### Moments to Remember

You can convey statistical information in several different ways. If you obtain more than 20 or so scores, listing the scores becomes unwieldy. In such cases, you can present the information in a frequency distribution.

Suppose scores range from \( x_i \) through \( x_n \). The frequency of score \( x_i \), written as \( f_i \), is the number of times that score appears in the distribution.

1. Using this notation, express the total of the scores corresponding to \( x_i \), \( \sum x_i \).

2. Using sigma notation, express the total of the data represented in the distribution. \( \sum x_i f_i \).

3. How would you express the mean of the distribution in terms of \( x_i \) and \( f_i \)? \( \frac{\sum x_i f_i}{\sum f_i} \) if \( x_i \) is a score in a data set and \( f_i \) is the frequency.

4. If \( x_i \) is a score in a data set and \( f_i \) is any number, the first moment of \( x_i \) is defined as the difference \( x_i \) \( = \) \( m \). For example, if \( x_i = 41 \) is a score in a data set, the first moment of 73 about 60 is \( 73 \) \(-\) 60 

5. Suppose a frequency distribution has scores \( x_i \) through \( x_n \) and associated frequencies \( f_i \) through \( f_n \). Express the total of the first moments of \( x_i \) about \( m \), \( \sum (x_i - m) f_i \).

#### Practice

11-5 Practice (continued) Form K

Make a box-and-whisker plot for each set of values.

1. 15, 19, 24, 16, 12, 18, 20, 22, 16, 17

2. 11, 26, 32, 27, 36, 28, 30, 31, 28

Find the following percentiles of the data set displayed below.

<table>
<thead>
<tr>
<th>Values</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>32</th>
<th>34</th>
<th>36</th>
<th>37</th>
<th>38</th>
<th>40</th>
<th>41</th>
<th>42</th>
<th>43</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentile</td>
<td>25th</td>
<td>50th</td>
<td>75th</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>28</td>
<td>30</td>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12. 45th percentile 35  13. 75th percentile 39  14. 25th percentile 31

15. 95th percentile 43  16. 90th percentile 40  17. 15th percentile 29

#### Enrichment

Describe a situation in which the median would be a more useful measure of central tendency than the mean. Answers may vary. Sample: If there are outliers in a set of data, then the median will give you a better sense of the data’s central tendency.
The mean of the middle two values is 3.

Find the mean, median, and mode of each set of values.

Exercises

Step 1
What are the mean, median, and mode for the data set below?

1 2 3 4 5 6

The mean is about 3.65, the median is 3.5, and the mode is 3.

The most common value is 3.

1: three times 2: four times 3: ten times

4: five times 5: nine times 6: three times

11-6

Think About a Plan

Standard Deviation

Energy
The data for daily energy usage of a small town during ten days in January is shown.

Plan
a. Find the mean and the standard deviation of the data.

b. How many values in the data set fall within one standard deviation of the mean? Within two standard deviations? Within three standard deviations?

Use the note cards to write the steps in order.

1. First, find the mean of the n values in the data set

2. Second, find the difference, \( x_i - \bar{x} \), between each value \( x_i \) and the mean

3. Next, square each difference, (\( x_i - \bar{x} \))^2

4. Then, calculate the variance by finding the mean of these squares.

5. Finally, take the square root of the variance.

The data for daily energy usage of a small town during ten days in January is shown.

\begin{align*}
\text{Day 1:} & \quad 83.8 \text{ MWh} \\
\text{Day 2:} & \quad 87.1 \text{ MWh} \\
\text{Day 3:} & \quad 92.5 \text{ MWh} \\
\text{Day 4:} & \quad 81.0 \text{ MWh} \\
\text{Day 5:} & \quad 82.4 \text{ MWh} \\
\text{Day 6:} & \quad 77.6 \text{ MWh} \\
\text{Day 7:} & \quad 78.9 \text{ MWh} \\
\text{Day 8:} & \quad 81.8 \text{ MWh} \\
\text{Day 9:} & \quad 80.1 \text{ MWh} \\
\text{Day 10:} & \quad 81.8 \text{ MWh}
\end{align*}

\[ s = \sqrt{\frac{\sum(x - \bar{x})^2}{n}} \]
Find the mean, variance, and standard deviation for each data set.

1. 232 254 204 274 298 298 312 342 398 about 295.7; about 2246.8; about 47.4
2. 26 27 28 28 29 30 30 32 35 35 35 about 30.2; about 3.2
3. 2.2 2.2 2.4 2.4 2.4 2.5 2.5 2.6 about 2.4; about 0.2; about 0.1
4. 75 73 77 79 74 83 74 68 70 72 about 74.2; about 15.1; about 3.9

Determine the whole number of standard deviations that includes all data values.

7. The hours students in your study group study is 66 min, the standard deviation is 2.9 min. 3
8. The mean weight of your pets is 18.25 lb; the standard deviation is 30.1 lb. 3
9. Use the data for average daily water usage of a family during the past 10 months. Find the mean and the standard deviation of the data. How many items in the data set fall within one standard deviation of the mean? Two standard deviations of the mean? Within two standard deviations? 249.4; about 162.5; 7; 10
10. Use the chart at the right for Exercises 13–17. Use the standard deviation for each year to describe how school fundraising varied from 2006–2007 to 2007–2008.

Find the mean and standard deviation of the data.

11. 6, 12, 12, 8, 10

Mean

\[ \bar{x} = \frac{6 + 12 + 12 + 8 + 10}{5} = 10 \]

Variance

\[ s^2 = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{50}{5} = 10 \]

Standard Deviation

\[ s = \sqrt{s^2} = \sqrt{10} \]

12. 8, 16, 12, 15, 4

Mean: 11

Variance: 20

Standard deviation: \( \sqrt{20} \approx 4.5 \)

13. 27, 34, 45, 30, 26, 42

Mean: 34

Variance: 52.3

Standard deviation: 7.2

Use a graphing calculator to solve the following problems.

5. The most recent test scores for a math class are displayed in the table below. What are the mean and the standard deviation for this data set?

<table>
<thead>
<tr>
<th>Score</th>
<th>77</th>
<th>86</th>
<th>79</th>
<th>94</th>
<th>80</th>
<th>82</th>
<th>76</th>
<th>97</th>
<th>85</th>
<th>78</th>
<th>91</th>
<th>79</th>
<th>88</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>81</td>
<td>Standard deviation</td>
<td>8.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Your sister’s bowling scores for the last 12 games are displayed in the table below. What are the mean and standard deviation for this data?

<table>
<thead>
<tr>
<th>Game</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>212</td>
<td>187</td>
<td>176</td>
<td>215</td>
<td>198</td>
<td>229</td>
<td>205</td>
<td>175</td>
<td>201</td>
<td>216</td>
<td>227</td>
<td>215</td>
</tr>
<tr>
<td>Mean</td>
<td>205</td>
<td>Standard deviation</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The mean weight of your favorite type of fruit is 6.2 oz. The standard deviation is 0.4 oz.

Find the mean and standard deviation of the data set shown below and got 46.53. What error did she make? What is the correct standard deviation?

<table>
<thead>
<tr>
<th>Day</th>
<th>10</th>
<th>12</th>
<th>8</th>
<th>14</th>
<th>6</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Temperature</td>
<td>77</td>
<td>92</td>
<td>88</td>
<td>80</td>
<td>75</td>
<td>84</td>
</tr>
</tbody>
</table>

She calculated the variance rather than the standard deviation. She needs to find the square root of the variance. The correct standard deviation is about 6.82.
ANSWERS

11-6 Standardized Test Prep

Multiple Choice

For Exercises 1-4, choose the correct letter.

1. Of the 25 students who take a standardized test, the minimum score is 98 and the maximum score is 155. The mean score is 121, and the standard deviation is 16.5. What is the number of standard deviations from the mean? Consider the following reasoning.

Within how many standard deviations of the mean do all the costs fall?

Within how many standard deviations of the mean do all the costs fall?

2. What is the standard deviation of the data set below?

What are the variance and standard deviation for the data set \{100, 158, 170, 192\}? What is the number of standard deviations from the mean? Consider the following reasoning.

Within how many standard deviations of the mean do all the scores fall?

What are the variance and standard deviation for the data set \{100, 158, 170, 192\}? What is the number of standard deviations from the mean? Consider the following reasoning.

Within how many standard deviations of the mean do all the scores fall?

3. A data set has a mean of 125 and a standard deviation of 15.8. All the data values are within two standard deviations of the mean. Which could be the data values?

A data set has a mean of 255 and a standard deviation of 12. All the data values are within two standard deviations of the mean. Which could be the data values?

4. The scores on a math test are:

What are the variance and standard deviation for the data set \{5, 9, 15, 24, 25, 27\}? What is the number of standard deviations from the mean? Consider the following reasoning.

Within how many standard deviations of the mean do all the scores fall?

What are the variance and standard deviation for the data set \{5, 9, 15, 24, 25, 27\}? What is the number of standard deviations from the mean? Consider the following reasoning.

Within how many standard deviations of the mean do all the scores fall?

5. The ages of students in a club are:

Within how many standard deviations of the mean do all the students’ ages fall?

Within how many standard deviations of the mean do all the students’ ages fall?

Short Response

5. The ages of students in a club are: 13, 17, 18, 15, 16, 14, 17, 10, 15, 16, 13. Calculate the mean and standard deviation. What is the number of standard deviations that includes all the data values? Show your work.

What formula results when these substitutions are made?

What is the corresponding formula for the standard deviation?

Notice that the middle term has two constants, 2 and \(\sigma\). Why are they factored outside the sigma symbol? The last term is equivalent to adding the square of the mean \(\mu\) times.

Notice that only the number of scores, the sum of the scores, and the sum of the squares of the scores are needed to find the standard deviation and variance with these formulas. Suppose the data is presented in terms of the scores \(x_i\) through \(x_n\), their associated frequencies \(f_i\) through \(f_n\).

Within how many standard deviations of the mean do all the students’ ages fall?

Within how many standard deviations of the mean do all the scores fall?

6. The scores on a math test are:

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

7. Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

8. Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

9. Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

10. Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?

Within how many standard deviations of the mean do all the scores fall?
Think About a Plan
Samples and Surveys

Entertainment A magazine publisher mails a survey to every tenth person on a subscriber list that is alphabetized by last name. The survey asks for three favorite leisure-time activities. What sampling method is the survey using? Identify any bias in the sampling method.

Know
1. The company sending out the survey is a
2. The surveys are mailed to every tenth person on a subscriber list, alphabetized by last name
3. The survey asks for three favorite leisure-time activities

Need
4. To solve the problem I need to find the sampling method used by the survey and any bias in the sampling method

Plan
5. What sampling method is the survey using? systematic sampling

Answers may vary. Sample: Yes; the people who receive the survey subscribe to the magazine, so they are likely to list reading magazines as a favorite leisure-time activity. The sample is also self-selected, depending on who returns the survey. These people may overrepresent or underrepresent some choices of activities.

systematic sampling
6. Do the people who receive the survey represent the general population? Explain.
7. Do the people who return the survey represent the general population? Explain.

Answers may vary. Sample: No; only people who choose to return the survey are represented

Answers may vary. Sample: Yes; the people who receive the survey subscribe to magazines, so they are likely to list reading magazines as a favorite leisure-time activity. The sample is also self-selected, depending on who returns the survey. These people may overrepresent or underrepresent some choices of activities.

Conclusion
8. Is there any bias in the sampling method? Explain.

Answers may vary. Sample: The people who receive the survey subscribe to magazines, so they are likely to list reading magazines as a favorite leisure-time activity. The sample is also self-selected, depending on who returns the survey. These people may overrepresent or underrepresent some choices of activities.

11. a. Write a survey question to find out the number of students in your class who plan to travel out of state after graduation. Check student’s work.
   b. Describe the sampling method you would use. Check student’s work.
   c. Conduct your survey. Check student’s work.

12. A television show’s website asks every 25th person who visits the site to name their favorite TV show.
   a. What sampling method is the survey using? convenience
   b. Describe any bias in the sampling method. People who visit the show’s website are more likely than general television viewers to pick the show’s stars as their favorites. When you take a random sample of size n from a large population, the sample has a margin of error of approximately \( \pm \frac{2}{\sqrt{n}} \). Approximate the margin of error for each sample.
   c. In a traffic survey, 42% of the 1287 drivers passing through the checkpoint were traveling more than 105 mi from home. ±3% ±5%
   d. In one lake, 30% of the last 323 fish caught have a certain chemical present in their body. ±6%

You can use the margin of error ME to find an interval that is likely to contain the result you would get if you asked the entire population. If the percentage found from a survey is \( p \), the percentage from the total population is likely to be between \( p - ME \) and \( p + ME \). For each margin of error, find a small interval that is likely to contain the result from the total population given that the result from the survey is \( p = 61\% \).

15. ME = ±0.6% ±1% ±2%
16. ME = ±0.7% ±1% ±2%
17. ME = ±1.4% ±2% ±3%
18. ME = ±3.7% ±5% ±6%

19. Reasoning A certain survey has a margin of error of ±3%. About how many people participated in the survey? = 1111 people

20. Writing Describe the relationship between a change in the sample size and the change in the margin of error. Answers may vary. Sample: They vary inversely. As one increases, the other decreases. If sample size increases by a factor of \( a \), the margin of error decreases by a factor of \( a^{-3} \).

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4. **Multiple Choice**

5. **Writing**

6. **Multiple Choice**

7. **Open-Ended**

**Answers may vary. Sample: Your classroom's sample is not biased if the survey deals only with issues pertaining to the high school.**

**Multiple Choice**

**For Exercises 1–4, choose the correct letter.**

1. The School Dance Committee conducts a survey to find what type of music students would like to hear at the next dance. Which is an example of a random sample? 🟤 🟦 Call 30% of the people in the senior class directory. 🟥 🟧 Interview every 10th student as they enter the school. 🟨 Ask every 5th person leaving a school orchestra concert. 🟩 Set up a jazz website where students can list their 5 favorite songs.

2. **Which is a characteristic of a biased survey question?**

- It is about a controverted issue.
- It produces inaccurate results.
- It is about a well-known person.
- It is about a very unpopular person.

3. In a survey, 30% of 1000 students said they spent at least 5 h online during the past week. What is the approximate margin of error for this sample? 🟤 ≈ 2.5%  🟦 ≈ 6%  🟧 ≈ 12.5%  🟨 ≈ 26%

4. A newspaper surveys a sample of 2000 people and finds that 64% agree with a certain political position. What interval is most likely to contain the percentage of the total population who agree with the position? 🟤 62%–68%  🟦 63%–65%  🟧 64%–66%  🟨 66%–70%

**Short Response**

5. A city council surveys a sample of citizens about a new law. The survey finds that 30% of citizens think the law should be repealed. The survey has a margin of error of about 4%. About how many people did the council survey? Show your work.

- [4] 488
- [4] \( \sqrt{\frac{p(1-p)}{n}} \)
- [4] \( n = \frac{p(1-p)}{\epsilon^2} \)
- [4] \( n = \frac{0.3(1-0.3)}{0.04} = 116 \)
- [1] incorrect or incomplete work shown
- [1] incorrect answer and no work shown OR no answer given

**Identify and describe the bias in the following survey questions.**

8. Isn't summer a much more pleasant season than winter?

This is a leading question. The question suggests that summer is more pleasant than winter.

9. Are college students better off studying useful subjects such as math or impractical subjects such as art history?

This is a loaded question. Positive terms are used to describe one option and negative terms are used to describe the other option.

10. Do you believe that this year's class field trip was fun and educational?

This question combines two issues. Whether or not the field trip was fun and whether or not it was educational are two separate issues.

11. Do you agree that Mrs. Regis's class is more interesting than Mr. Wright's class?

This is a leading question. The question suggests that Mrs. Regis's class is more interesting than Mr. Wright's class.

**Rewrite the following survey questions so that they are no longer biased.**

12. Do you prefer the excitement of rock and roll or the tediousness of classical music?

Answers may vary. Sample: Do you prefer rock and roll or classical music?

13. Would you agree that dogs make better pets than cats?

Answers may vary. Sample: Which pet would you prefer, a dog or a cat?

14. Do you believe that Mayor Johnson is friendly and effective?

Answers may vary. Sample: Do you believe that Mayor Johnson is effective?

15. **Writing**

A supervisor wants to determine what percent of people in his office building believe it is important to have an Internet connection at home. What sampling method can he use to gather an unbiased sample? What is an example of a survey question that is likely to yield unbiased information?

He could ask every eighth person leaving the building at the end of the day whether or not he or she believes it is important to have an Internet connection at home: "Do you believe that it is important to have an Internet connection at home?"
An athletic shoe company wants to learn which shoe features are important to local high-school students. Is there any bias in any of these survey questions? Explain.

Exercises

A politician wants to know what issues are most important to the voters in his district. Identify the type of bias in each survey question.

1. How do you feel about the toxic pollution being released into the air by the local manufacturing plants? Loaded question; using the words toxic pollution rather than emissions can cause strong reactions.
2. Isn’t a school bond not the right way to raise money for local education? Confusing question; it asks about more than one issue.
3. How do you feel about the toxic pollution being released into the air by the local manufacturing plants? Loaded question; using the words toxic pollution rather than emissions can cause strong reactions.
4. Do you think your local library should offer videos and Internet access? Confusing question; it asks about more than one issue.
5. Which city service is most important to you? Loaded question; using the words toxic pollution rather than emissions can cause strong reactions.
6. An athletic shoe company wants to learn which brand of athletic shoes is worn most often by local high-school students. The company sets up a booth in a local mall and offers a coupon for a free pair of their athletic shoes to anyone who answers the question, “What is your favorite brand of athletic shoes?”
   a. People in the mall are readily available to the booth. Also, people must volunteer to participate. The sample is a convenience sample and is self-selected. People in the mall are readily available to the booth. Also, people must volunteer to participate. The sample is a convenience sample and is self-selected.
   b. The survey is biased in several ways:
      - Only people who choose to walk up to the booth participate in the survey.
      - Only people who choose to walk up to the booth participate in the survey.
      - People may be more likely to say this company makes their favorite shoes when they are offered a free pair.
   c. How important is it to you that shoe materials have not been tested on laboratory animals?
      - People who are not high-school students may participate in the survey.
      - People who are not high-school students may participate in the survey.
      - Only people who choose to walk up to the booth participate in the survey.

Answers may vary. Sample: I need the first three terms because they represent having 3, 2, or 1 successes. A success is a morning with suitable weather. Answers may vary. Sample: the probability that there will be one, two, or three mornings with suitable weather.

Planning the Solution

5. What binomial can help you find the binomial distribution for this problem? I need the first three terms because they represent having 3, 2, or 1 successes. A success is a morning with suitable weather. Answers may vary. Sample: the probability that there will be one, two, or three mornings with suitable weather.

6. Expand your binomial. p^2 + 3pq + 3q^2 + q^3

7. Which terms of your binomial expansion do you need to solve the problem? p = 0.6, q = 0.4

8. What term can you substitute for the variables in your binomial expansion? Answers may vary. Sample: I need the first three terms because they represent having 3, 2, or 1 successes. A success is a morning with suitable weather.

Getting an Answer

9. Use your binomial expansion to find the probability that there will be at least one morning with suitable weather.Answers may vary. Sample: the probability that there will be one, two, or three mornings with suitable weather.

10.4

0.6

0.06
Find the probability of successes in trials for the given probability of success $p$ on each trial.

1. $x = 5, n = 8, p = 0.4$; about 16%

2. $x = 2, n = 6, p = 0.3$; about 13%

3. $x = 3, n = 10, p = 0.25$; about 23%

4. $x = 1, n = 3, p = 0.2$; about 38%

A light fixture contains 6 light bulbs. With normal use, each bulb has an 85% chance of lasting for 4 months. What is the probability that all 6 bulbs will last for 4 months? about 38%

Expand each binomial.

6. $(2a + 4b)^1 = 2a + 4b$

7. $(a + 3b)^2 = a^2 + 6ab + 9b^2$

8. $(2c + 3d)^3 = 8c^3 + 36c^2d + 54cd^2 + 27d^3$

9. $(2x + 5y)^4 = 16x^4 + 80x^3y + 240x^2y^2 + 300xy^3 + 125y^4$

Find the indicated term of each binomial expansion.

10. third term of $(2a - 3b)^6 = 192a^4b^2$

11. fifth term of $(r + 3s)^6 = 405r^3s^3$

12. fourth term of $(2x - 3y)^4 = -432x^3y$

13. first term of $(8g + 6h)^5 = 512g^5$

Use the binomial expansion of $(x + y)^6$ to calculate each binomial distribution.

14. $n = 3, p = 0.7$

15. $n = 3, p = 0.2$

16. The probability that the weather will be acceptable for a launch of a satellite over the next 3 days is 10% each day. What is the probability that the weather will be acceptable at least 1 of the next 3 days? 97.3%

17. A poll shows that 80% of a school district’s home owners favor an increase in property tax to fund a new high school. What is the probability that exactly 4 of 5 people chosen at random favor the tax increase? about 20%

Find the probability of $x$ successes in $n$ trials for the given probability of success $p$ on each trial.

1. $x = 1, n = 8, p = 0.6$

2. $x = 3, n = 9, p = 0.5$

3. $x = 6, n = 12, p = 0.1$

4. $x = 2, n = 7, p = 0.25$

5. $x = 4, n = 10, p = 0.45$

6. $x = 5, n = 14, p = 0.2$

7. At a pet shop, 50% of the cats have short hair. The owner of the pet shop will randomly choose 3 cats to take to an animal show. What is the probability that 3 of the cats will have short hair? 0.19

8. Your brother baked a large batch of cookies. He put chocolate chips in 45% of the cookies. He randomly selects 10 cookies to give to a friend. What is the probability that 6 of the cookies contain chocolate chips? 0.18

9. Reasoning Does rolling a number cube 10 times fit all of the conditions for a binomial experiment? Explain why or why not. Answers may vary. Sample No, it does not fit all of the conditions for a binomial experiment. Each trial has 6 possible outcomes, not two.

10. Multiple Choice Which of the following is not one of the conditions for a binomial experiment?

a. There is a fixed number of trials.

b. Each trial has two possible outcomes.

c. The trials are independent.

d. The probability of each outcome is constant throughout the trials.

Find each probability.

In one neighborhood the probability of a power outage during a rainstorm is 4%.

a. What is the probability of at least 1 power outage in the next 10 rainstorms? 71.4%

b. What is the probability of at least 2 power outages in the next 10 rainstorms? 20.4%

There is a 60% probability of rain each of the next 5 days. Find each probability. Round to the nearest whole percent.

10. It will rain on at least 3 of the next 5 days. 68%

11. It will rain on at least 1 of the next 5 days. 99%

12. It will rain on at least 1 of the next 4 days. 97%

13. It will rain on at least 1 of the next 2 days. 84%

22. Open Ended Describe a situation with a 20% probability of success in each of 4 trials. Graph the binomial distribution. Check students’ work. See sample graph above.

23. The probability that an egg from one farm is small is 10%. What is the probability that exactly 1 egg in a sample of 4 eggs is too small? about 29%

In one neighborhood the probability of a power outage during a rainstorm is 4%. Find each probability.

24. $P$(at least 1 power outage in the next 5 rainstorms) about 68%

25. $P$(at least 2 power outages in the next 10 rainstorms) about 0.6%

26. $P$(at least 1 power outage in the next 20 rainstorms) about 56%

27. Writing Explain the relationship between the expansion of $(x + y)^6$ and the 12th row of Pascal’s triangle. The numbers in the 12th row of Pascal’s triangle are the coefficients of the expansion of $(x + y)^6$.

28. A newspaper carrier can throw the paper and have it land on a customer’s porch 80% of the time. Use the Binomial Theorem to calculate each probability for the delivery’s first 3 throws of the morning.

a. The carrier does not land any papers on a porch. about 0.34%

b. The carrier lands only 1 paper on a porch. about 5.7%

c. The carrier lands exactly 2 papers on a porch. about 32.5%

d. The carrier lands all 3 papers on a porch. about 61.4%

29. Reasoning The probability that a baby born in Scotland has red hair is 13%. A certain Scottish hospital has an average of 20 babies born per week. At the beginning of the week, the hospital has 3 stickers available to put on the babies’ cribs. Does this seem to be an adequate amount? Justify your answer. Yes; the probability that the hospital will have 3 or fewer redheads out of the 20 babies born is about 74%.
Multiple Choice

For Exercises 1−5, choose the correct letter.

1. The probability that a newborn baby at a certain hospital is male is 50%.
   What is the probability that exactly 2 of 3 babies born on a certain day are male?
   \[ \text{A} \] 37.5% \[ \text{B} \] 50% \[ \text{C} \] 66.7% \[ \text{D} \] 77%

2. The probability that a newborn baby at the hospital is female is 50%. What is the probability that at least 2 of 3 children born on a certain day are female?
   \[ \text{A} \] 33.3% \[ \text{B} \] 37.5% \[ \text{C} \] 50% \[ \text{D} \] 66.7%

3. What is the fifth term of the expansion of \((2x - y)^5\)?
   \[ \text{A} \] \(-1792x^3y^2\) \[ \text{B} \] \(-448x^2y^3\) \[ \text{C} \] 256x^3y^2 \[ \text{D} \] 1120x^4y^4\]

4. A poll shows that 30% of voters favor an earlier curfew. Find the probability that all of five voters chosen at random favor an earlier curfew.
   \[ \text{A} \] 0.244 \[ \text{B} \] 1.5% \[ \text{C} \] 4.1% \[ \text{D} \] 16.7%

5. The probability that a machine part is defective is 10%. Find the probability that no more than 2 out of 12 parts tested are defective.
   \[ \text{A} \] 28% \[ \text{B} \] 55% \[ \text{C} \] 89% \[ \text{D} \] 98%

Short Response

6. A scientist runs an experiment 4 times. Each run has a 60% chance of success. Calculate and graph the distribution of binomial probabilities for the experiment.

7. Suppose you repeat an experiment \(n\) times, and each time you run the experiment it has a probability of success \(p\) and a probability of failure \(q\). Then, the probability of \(x\) successes in \(n\) trials is:
   \[ E_x = \binom{n}{x} p^x q^{n-x} \]
   where \(q = 1 - p\).

   a. \(E_1 = \binom{10}{1} 0.3^1 0.7^9\)
   b. \(E_2 = \binom{10}{2} 0.3^2 0.7^8\)
   c. \(E_3 = \binom{10}{3} 0.3^3 0.7^7\)
   d. \(E_4 = \binom{10}{4} 0.3^4 0.7^6\)

Exercises

Find the probability of \(x\) successes in \(n\) trials for the given probability of success \(p\) on each trial. Round to the nearest tenth of a percent.

1. \(x = 3, n = 4, p = 0.3\)
2. \(x = 4, n = 6, p = 0.1\)
3. \(x = 7, n = 9, p = 0.2\)
4. \(x = 5, n = 6, p = 0.3\)
5. A light fixture contains six light bulbs. With normal use, each bulb has a 95% chance of lasting for 2 yr. What is the probability that all six bulbs last for 2 yr? About 73.2%
6. Use the information from Exercise 5. What is the probability that free of the six bulbs will last for 2 yr? About 22.2%
7. Suppose the bulbs have an 85% chance of lasting for 2 yr. Find the probability that three of the six bulbs will last for 2 yr. About 18.6%
A normal distribution has a mean of 50 and a standard deviation of 6. Find the
probability that a value selected at random is in the given interval.
10. from 44 to 50
11. at least 50
12. 56
13. 78
14. 20. at most 56
15. 64
16. from 50 to 62
17. 58
18. 81.5%
19. 43.5%
20. 64%
21. at least 38
22. No, the data do not fit a normal curve.
23. Sunflower Heights
40
50
60
70
80
90
100
110
120
130
140
150

Weight (in pounds)

Frequency

Height (in.)

127

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6. For Exercises 1−5, choose the correct letter.

Multiple Choice

1. The mean number of pairs of shoes sold daily by a shoe store is 36, with a standard deviation of 9. What is the probability that a value selected at random is at least 39? A 6.4% B 13.8% C 30.6% D 64%

2. What is the standard deviation for the normal distribution shown at the right? A 30 B 60 C 120 D 180

3. A normal distribution has a mean of 700 and a standard deviation of 35. What is the probability that a value selected at random is at most 660? A 0.2525 B 0.3233 C 0.6767 D 0.975

4. Scores on an exam are distributed normally with a mean of 78 and a standard deviation of 10. Out of 250 tests, about how many students score above 88? A 20 B 40 C 60 D 80

5. A hardware store sells bags of mixed nails. The number of nails of a given length is distributed normally with a mean length of 3 in. and a standard deviation of 0.25 in. About how many nails in a bag of 120 are between 2.875 in. and 3.075 in.? A 23 B 46 C 68 D 82

Short Response

6. The heights of the girls in a school choir are distributed normally, with a mean of 64 and a standard deviation of 1.75. If 36 girls are between 62.5 in. and 67.5 in. tall, how many girls are in the choir? Show your work.

8. a. 64 - 3.4 = 60.6; 64 + 3.4 = 67.4
   b. The range 60.6 - 67.4 is within two standard deviations of the mean, 64.3. 95% of the data is within the range 60.6 - 67.4.

9. If k = 1, do you learn anything about the data? Yes, it says that at least 0 measurements lie within 65−65.

10. The average score on a math test is 76. The standard deviation is 6.2. Sketch a normal curve showing the test scores at one, two, and three standard deviations from the mean.

11. A local bakery makes chocolate chip cookies. The number of chocolate chips in the cookies is approximately normally distributed, with mean 14 and standard deviation 2. What percent of the cookies have between 12 and 16 chocolate chips? 95%

12. If k = 3, what is the least number of measurements in the sample that will lie in the interval for k = 3? A 23 B 46 C 68 D 82

Tchebychef’s Theorem

Tchebychef’s Theorem states that given a number k greater than or equal to 1 and a set of n measurements, at least \( \frac{1}{k^2} \) of the measurements will lie within k standard deviations of the mean. Note that the theorem is true for any number you wish to choose for k as long as it is greater than or equal to 1.

The mean and standard deviation of a sample of n = 25 measurements are 75 and 10, respectively.

1. Using Tchebychef’s theorem and k = 2, what can you assume?

2. What is the least number of measurements in the sample that will lie in the interval for k = 3? A 23 B 46 C 68 D 82

3. Using Tchebychef’s theorem and k = 3, what can you assume?

4. What is the least number of measurements in the sample that will lie in the interval for k = 3? A 23 B 46 C 68 D 82

5. If k = 1, do you learn anything about the data? Yes, it says that at least 0 measurements lie within 65−65.

6. Calculate the mean and standard deviation. Round to the nearest whole number.

7. Using Tchebychef’s theorem and k = 2, what is the least number of measurements in the sample that will lie within 2 standard deviations of the mean? What interval corresponds to within 2 standard deviations of the mean? B 57 between 56 and 58

8. How many measurements in the sample actually lie in that interval? 13

9. Using Tchebychef’s theorem and k = 3, what is the least number of measurements in the sample that will lie within 3 standard deviations of the mean? What interval corresponds to within 3 standard deviations of the mean? A 23 between 22 and 24

10. How many measurements in the sample actually lie in that interval? 6

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Find the theoretical probability of each event when rolling a fair number cube.

Find the experimental probability of each event.

1. 11.
2. 10.
3. 8.
4. 4.

Lessons 11-1 through 11-4
Chapter 11 Quiz 1

About 82% of newborn calves will weigh 77 lb–89 lb.

Th e graph of a normal distribution is a normal curve.

A normal curve is shaped like a bell, with the highest point at the mean and tapering down evenly on either side of the bell.

Problem

The weight in pounds of newborn calves on a farm is distributed normally, with a mean of 85 and a standard deviation of 4. What percent of newborn calves on the farm weigh between 77 lb and 89 lb?

Step 1

Draw a normal curve. Label the mean.

Step 2

Divide the graph into 6 equal sections. Each section should be one standard deviation wide, which is 4 lb in this problem. Label each section with the appropriate percent for a normal distribution.

Step 3

Add the percents for the sections with weights 77 lb–81 lb, 81 lb–85 lb, and 85 lb–89 lb.

About 82% of newborn calves will weigh 77 lb–89 lb.

Exercises

Use the graph above to find the percent of calf weights within each interval.

1. 73 lb to 81 lb about 16%
2. 81 lb–85 lb about 84%
3. 77 lb–81 lb about 97%
4. less than 85 lb about 50%
5. at most 89 lb about 95%
6. at least 85 lb about 2.5%

Chapter 11 Quiz 2

Do you know HOW?

1. 1 to how many different orders can 6 numbered blocks be chosen from a set of 23 blocks? 72,881,840
2. Find the experimental probability of each event.
3. a glass broken by a dishwasher who broke 3 of 25 glasses
4. Find the theoretical probability of each event when rolling a fair number cube.
5. (P, 3, or 6) 0.50
6. (P or odd) 0.67
7. (P or even) 0.17
8. C and D are independent events, P(C) = \( \frac{1}{3} \) and P(D) = \( \frac{1}{3} \). What is P(C and D)? \( \frac{1}{9} \)
9. X and Y are mutually exclusive events, P(X) = \( \frac{2}{3} \) and P(Y) = \( \frac{1}{3} \). What is P(X or Y)? \( \frac{1}{2} \)
10. A certain county court can assign any one of its 50 lawyers to a case. Of the lawyers, 5 are female and over 40 and 25 are male. What is the probability that the lawyer is under age 40, given that the lawyer is female?

Do you UNDERSTAND?

11. Explain the difference between \( P(A \text{ and } B) \) and \( P(A) \cdot P(B) \).

12. Writing: How do \( P(A) \cdot P(B) \) and \( P(A \text{ and } B) \) differ?

Do you know HOW?

1. You must complete the following chores: take out the trash, wash the dishes, vacuum the carpet, clean your room, make your bed, and feed the fish. Does this situation involve a permutation or combination? In how many ways can you do the chores? permutation: 720

You randomly select a number from the sample space {5, 7, 9, 13, 15, 17}.

Find each theoretical probability.

2. \( P(\text{less than 13}) = \frac{3}{6} \)

3. \( P(\text{odd}) = \frac{3}{6} = 1 \)

4. \( P(\text{multiple of 5}) = \frac{0}{6} \)

5. A basketball player made 27 free throws in her last 45 tries. What is the experimental probability that she will make her next free throw? 0.6

6. You have a CD with 8 rock songs, 3 blues songs, and 2 jazz songs. Today you hit the shuffle button on your CD player, which plays the songs in a random order. Tomorrow you do the same thing. What is the probability that the CD player plays a blues song first each day? \( \frac{3}{13} \)

Use the results of the survey below to find each conditional probability.

How many pets do you have in your home?

<table>
<thead>
<tr>
<th>Pets</th>
<th>Male respondents</th>
<th>Female respondents</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

7. \( P(\text{male} | 2 \text{ pets}) = \frac{10}{26} \)

8. \( P(\text{pets} | 2 \text{ males}) = \frac{10}{12} \)

Use the following set of values for Exercises 11–16. Round answers to the nearest hundredth, if necessary.

0.3 0.6 0.8 0.9 1.3 0.4 0.6 1.2 1.4 1.1 0.2 0.2

11. Find the mean, median, and mode(s).

12. Find the range.


14. Find the interquartile range.

15. Find the standard deviation.

Find the probability of:

16. Find the 25th and 75th percentiles.

Do you UNDERSTAND?

13. Find the interquartile range.

14. Find the range.

15. Find the standard deviation.

16. Find the 25th and 75th percentiles.

Find the following probabilities.

17. A number from 1 to 25 is randomly chosen.

a. What is the probability that the number chosen is a multiple of 2 or 7? 0.48 or 48% 

b. What is the probability that the number chosen is a multiple of 2 and 7? 0.08 or 8% 

c. What is the probability that the number chosen is a multiple of 2 or 7? 0.6 or 60% 

Do you UNDERSTAND?

18. Reasoning When using a simulation, are you finding the experimental probability or the theoretical probability of an event? Why? Answers may vary. Sample: You are finding the experimental probability. When you simulate an event, you use the data from that simulation (experiment) to calculate probability.

19. Writing Explain why the probability of two mutually exclusive events occurring at the same time is zero. Answers may vary. Sample: Mutually exclusive events are events that cannot happen at the same time. If they cannot happen, then the probability is zero.

20. An alumni association compiled the following information about its recent graduates.

- 20% graduated with a B average or better
- 95% of those students who graduated with a B average or better were employed within 6 months of graduation
- 50% of those that graduated with less than a B average were employed within 6 months of graduation
- 20% graduated with a B average or better

a. What is the probability that someone is employed within 6 months of graduation, given that he had less than a B average? 0.50 or 50%

b. What is the probability that someone is not employed within 6 months of graduation, given that she had a B average or better? 0.05 or 5%
Chapter 11 Performance Tasks

**Task 1**

a. Cassie has seven skirts, five blouses, and ten pairs of shoes. How many possible outfits can she wear? 350 outfits
b. Cassie decides that four of her skirts should not be worn to school. How many possible outfits can she wear to school today? 150 outfits
c. Two of Cassie’s friends come over and share her clothes. In how many different ways can the three girls wear the seven skirts? 30 ways
d. Cassie has six bracelets. In how many different ways can she wear three bracelets at a time? 20 ways

[4] Student uses appropriate methods to calculate combinations and permutations. The only errors are minor computational or copying errors.

[3] Student uses appropriate methods to calculate combinations and permutations, with several errors.

[2] Student uses appropriate methods to calculate at least one of the combinations or permutations correctly OR student gives correct answers, without work shown.

[1] Student attempts a solution, but shows little understanding of the problem.

[0] Student makes no attempt or no response is given.

**Task 2**

You evaluate six overhead lights to find the intensity of light at work stations that are about 2 m from the light. Use the data below. Round to the nearest whole percent, if necessary.

<table>
<thead>
<tr>
<th>Trial Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity of Light (lux)</td>
<td>102</td>
<td>20</td>
<td>105</td>
<td>92</td>
<td>103</td>
<td>99</td>
</tr>
<tr>
<td>Distance from the Light (m)</td>
<td>2.1</td>
<td>2.0</td>
<td>1.7</td>
<td>2.2</td>
<td>2.1</td>
<td>1.9</td>
</tr>
</tbody>
</table>

a. What is the probability that a random light is more than 2 m from a work station? 50% or 50% success.

[4] Student uses appropriate methods to find probabilities. The only errors are minor computational or copying errors.

[3] Student uses appropriate methods to find probabilities, with several errors.

[2] Student uses appropriate methods to find at least one of the probabilities correctly OR student gives correct answers, without work shown.

[1] Student attempts a solution, but shows little understanding of problem.

[0] Student makes no attempt or no response is given.

b. Find the mean, median, and mode of the following data set.

\[ \text{Mean} = \frac{102 + 20 + 105 + 92 + 103 + 99}{6} = 88 \]
\[ \text{Median} = \frac{92 + 99}{2} = 95.5 \]
\[ \text{Mode} = 92 \]

c. Use the data below. Round to the nearest whole.

<table>
<thead>
<tr>
<th>Distance from the Light (m)</th>
<th>2.1</th>
<th>2.0</th>
<th>1.7</th>
<th>2.2</th>
<th>2.1</th>
<th>1.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity of Light (lux)</td>
<td>102</td>
<td>20</td>
<td>105</td>
<td>92</td>
<td>103</td>
<td>99</td>
</tr>
</tbody>
</table>

[9] Student uses appropriate methods to find the indicated term of each binomial expansion.

**Task 3**

The heights of seven students in a class are 5’2”, 5’4”, 5’7”, 5’3”, 5’8”, 5’10”, and 5’7”.

[4] Student uses appropriate methods to find the mean, median, and mode of the data set. About 5’7”, 5’7”, 5’7”.

[3] Student finds the standard deviation of the data set. About 0.30.

[2] Student makes a bar graph of the data set.

[1] Student selects an unbiased sample, gives a sufficient explanation, and uses appropriate methods to find the data that falls off as you move away from the mean.

[0] Student applies methods correctly OR student gives correct answers, without work shown.

[4] Student selects an unbiased sample, gives a sufficient explanation, and uses appropriate methods to find the data that falls off as you move away from the mean.

[3] Student finds the standard deviation of the data set. About 0.30.

[2] Student makes a bar graph of the data set.

[1] Student selects an unbiased sample, gives a sufficient explanation, and uses appropriate methods to find the data that falls off as you move away from the mean.

[0] Student makes a bar graph of the data set.

**Task 4**

Determine the number of students in your high school who plan to attend college.

a. Survey a sample proportion that is least likely to be biased. Explain your choice.

[4] Student selects a sample proportion that is least likely to be biased. Explain your choice.

[3] Student uses the percent of your sample who plan to attend college to find each probability.

[2] Student selects an unbiased sample, gives a sufficient explanation, and uses appropriate methods to find the data that falls off as you move away from the mean.

[1] Student selects an unbiased sample, gives a sufficient explanation, and uses appropriate methods to find the data that falls off as you move away from the mean.

[0] Student makes a bar graph of the data set.
Multiple Choice
For Exercises 1–12, choose the correct letter.
1. Which is equivalent to \( \frac{x}{y} \)? A
   \( \frac{x}{y} = \frac{1}{y} \)  B \( \frac{x}{y} = \frac{x}{y} \)  C \( \frac{x}{y} = \frac{x}{y} \)  D \( \frac{x}{y} = \frac{x}{y} \)

2. Solve \( (x + 3) - x = 0 \). G
   \( x = -7 \) or \( x = 5 \)  B \( x = 5 \) or \( x = -7 \)  C \( x = 0 \) or \( x = 0 \)  D \( x = 0 \) or \( x = 0 \)

3. Which graph best models exponential decay? A
   A.  
   B.  
   C.  
   D.  

4. Which is true is the equation of a hyperbola with foci at \( (5, 0) \) and \( (0, 0) \)? A
   \( x^2/25 - y^2/7 = 1 \)  B \( x^2/7 - y^2/25 = 1 \)  C \( x^2/25 - y^2/7 = 7 \)  D \( x^2/7 - y^2/25 = 1 \)

5. Which of these parabolas opens to the left? J
   \( y = x^2 - 4x + 4 \)  B \( y = x^2 - 4x + 4 \)  C \( y = -x^2 + 4x - 4 \)  D \( y = x^2 - 4x + 4 \)

6. Which of these parabolas opens to the left? J
   \( y = x^2 - 4x + 4 \)  B \( y = x^2 - 4x + 4 \)  C \( y = -x^2 + 4x - 4 \)  D \( y = x^2 - 4x + 4 \)

7. Simplify \( \frac{3(2 + 1)}{2 - 1} \).
   \( 3 \)  B \( 3 \)  C \( 3 \)  D \( 3 \)

8. A and B are two independent events. \( P(A) = \frac{1}{3} \) and \( P(B) = \frac{1}{5} \).
   \[ \text{What is } P(A \text{ and } B)? \]  A \( \frac{1}{15} \)  B \( \frac{1}{15} \)  C \( \frac{1}{15} \)  D \( \frac{1}{15} \)

9. Which of the following is the margin of error for a sample of 500? A
   \( \pm \frac{1}{4} \)  B \( \pm \frac{1}{4} \)  C \( \pm \frac{1}{4} \)  D \( \pm \frac{1}{4} \)

10. Which value is the smallest? G
    \( \log_2 12 \)  B \( \log_2 12 \)  C \( \log_9 \)  D \( \log_9 \)

11. Which of the following is the sum of series \( \sum_{n=1}^{10} \). A
    \( -5 \)  B \( -3 \)  C \( -3 \)  D \( -3 \)

12. Which of the solutions to \( \sqrt{2x - 3} = 4 + 0 \)? G
    \( 0 \)  B \( \frac{7}{2} \)  C \( \frac{7}{2} \)  D \( \frac{7}{2} \)

13. Find the slope of a line perpendicular to \( y = 3x + 2 \). Show your work.
    The slope of a line perpendicular to it is equal to the opposite reciprocal of the slope of the original line, or \( m = \frac{-1}{3} \).

14. Solve \( 5x^2 - 7 = 18 \). 5x^2 - 7 = 18, 5x^2 = 25, x^2 = 5, x = \pm \frac{\sqrt{5}}{5} \)

Extended Response
15. The heights of dogs at the City Animal Shelter are distributed normally, with a mean of 25.4 in. and a standard deviation of 4.8 in.
    a. Sketch a normal curve and divide the area under the curve into sections that are one, two, and three standard deviations from the mean.
    b. Of the 75 dogs at the shelter, what percent of dogs would you expect to be less than 20.6 in. tall?
    c. The shelter has one dog that is 40 in. tall. Would you consider this height to be an outlier? Explain.

[4] Student sketches curve and labels standard deviations as shown. Student finds: 50 — 34% = 16% would be less than 22.6 in., and a height of 40 in. is likely to be an outlier because it is more than three standard deviations from the mean.

[3] Student uses appropriate strategies, but misunderstands part of the problem or ignores a condition in the problem.

[2] Student attempts to use appropriate strategies, but applies them incorrectly or incompletely.

[1] Student work contains significant errors and little evidence of correct strategies used.

[0] Incorrect answers and no work shown or all answers given.

Chapter 11 Project: On the Move

About the Project
The Chapter Project gives students an opportunity to conduct research to identify transportation problems. They develop products or processes to solve these problems. They conduct marketing surveys to see how their inventions meet the market’s needs and they make marketing decisions based on the results of the surveys. Then they develop presentations that introduce their new products.

Introducing the Project
• Ask students: Have you ever been included in a market survey? How can market surveys be used to help develop something new to sell?
• Ask students: Have you ever been included in a market survey? How can market surveys be used to help develop something new to sell?
• Instruct students to begin to make lists of questions they can use in their initial surveys.

Activity 1: Interviewing
Students conduct surveys to identify transportation problems.

Activity 2: Analyzing
Students make graphs and calculate summary statistics to identify problems and issues.

Activity 3: Designing
Students propose and develop details for products or services to solve the problems they identified.

Activity 4: Interviewing
Students conduct market surveys for their inventions, analyze the results, and make marketing decisions based on the results.

Finishing the Project
You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results.
• Have students review their methods for conducting market research, making graphs, and calculating summary statistics for the project.
• Ask groups to share insights that resulted from completing the project, such as any shortcuts they found for calculating summary statistics or making graphs.

Beginning the Chapter Project
Surveys show that many people list traffic as one of their top problems. Creative people in the transportation industry are designing faster, safer, less expensive, and environmentally cleaner ways to get around. In Curitiba, Brazil, a highly efficient bus system uses features of modern subway to move people faster and more smoothly.

In this project, you will identify a transportation problem. Then, you will design a new product or service to solve this problem. Finally, you will conduct a survey to decide whether your product or service is practical and marketable.

List of Materials
• Calculator
• Graph paper

Activities
Activity 1: Interviewing
Conduct a survey to identify a transportation problem in your community. Check students’ work.
• Choose the group of people you want to survey.
• Design the survey. Before you write questions, decide what data you want to collect. You can collect data about types of transportation people use, how far or how often they use each type, and how satisfied they are.
• Test the survey on a few people to make sure the questions are clear. Revise it if necessary.
• Collect the data.

Activity 2: Analyzing
Organize the data you gathered in the survey in Activity 1. Check students’ work.
• Make graphs of the data.
• Calculate summary statistics.
• Use your graphs and summary statistics to analyze the data.
• List problems or issues revealed by your data.

Prentice Hall Algebra 2 • Teaching Resources
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Chapter 11 Project Manager: On the Move

Getting Started
Read the project. As you work on the project, you will need a calculator, materials on which you can record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist
☐ Activity 1: designing and conducting a survey
☐ Activity 2: graphing and analyzing data
☐ Activity 3: designing a product or service
☐ Activity 4: analyzing a market survey
☐ marketing decision

Scoring Rubric
4 Surveys are well-designed. Graphs and summary statistics are accurate, and clearly show the results of your surveys. Explanations, decisions, and presentation are based on your research. All work is presented in a clear and organized manner.
3 Graphs and summary statistics are mostly correct, with some minor errors. Decisions are mostly based on the research. Some work could be presented more clearly.
2 Graphs and summary statistics contain major errors. Decisions are not based on sound research.
1 Major concepts are misunderstood. Project satisfies few of the requirements and shows poor organization and effort.
0 Major elements of the project are incomplete or missing.

Your Evaluation of Project
Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of Project

Activity 3: Designing
Suppose the members of your survey population are potential customers for your business. Check students’ work.
• What problem seems the most important to the people you surveyed?
• Propose a product or service that could solve this problem. Be sure your idea is practical. Make a drawing, scale model, or written description of your new transportation product or service. Include a price or charge that you think is appropriate.

Activity 4: Interviewing
Conduct a market survey for the transportation product or service you proposed. Check students’ work.
• In a series of interviews, identify your potential customers, what they want the product to do, and what changes they would like for the product.
• Graph the data. Analyze your results. Should your business market this new product or service? If so, what changes should you make first, if any? Be sure you can defend your marketing decisions on the basis of the data you collected.

Finishing the Project
The answers to the four activities should help you complete your project. Prepare a presentation that unveils the new product or service you invented and describes the results of your surveys. Present it to your classmates. Then discuss with them the marketing decision you made on the basis of your survey. Do they agree with your decision?

Reflect and Revise
Before giving a presentation, review your analysis of the market survey. Are your graphs clear and correct? Have your summary statistics been calculated correctly? Are your decisions and conclusions supported by the data? Practice your presentation in front of at least two people before presenting it to the class. Ask for suggestions for improvement.

Extending the Project
Market research plays an important role in many business decisions. Find out about some of the survey techniques used by market researchers.
Solve each matrix equation.

11. 

9.

7.

5.

1.

Complete this page.

Use the chart below to review vocabulary. These vocabulary words will help you complete this page.

<table>
<thead>
<tr>
<th>Words</th>
<th>Explanations</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding elements</td>
<td>Elements in the same position in a pair of matrices</td>
<td>$K \begin{bmatrix} 1 &amp; 3 \ 2 &amp; 4 \end{bmatrix}$, $K \begin{bmatrix} 1 &amp; 3 \ 2 &amp; 4 \end{bmatrix}$</td>
</tr>
<tr>
<td>Zero matrix</td>
<td>A matrix in which all elements are zero</td>
<td>$K \begin{bmatrix} 0 &amp; 0 \ 0 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>Additive inverse</td>
<td>A matrix in which each element is the opposite of the corresponding element in another matrix</td>
<td>$-K \begin{bmatrix} 1 &amp; 2 \ 3 &amp; 4 \end{bmatrix}$, $-K \begin{bmatrix} 1 &amp; 2 \ 3 &amp; 4 \end{bmatrix}$</td>
</tr>
<tr>
<td>Equal matrices</td>
<td>Matrices with the same dimensions and equal corresponding elements</td>
<td>$K \begin{bmatrix} 1 &amp; 2 \ 3 &amp; 4 \end{bmatrix}$, $K \begin{bmatrix} 5 &amp; 6 \ 7 &amp; 8 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Use the vocabulary to fill in the blanks.

1. The matrices had the same corresponding elements, so they were **equal matrices**.

2. The sum of a matrix and its additive inverse is a **zero matrix**.

3. The **corresponding elements** are in the same position in a pair of matrices with equal dimensions.

Circle the correct answer.

4. What is the additive inverse of $K \begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$?

   - $K \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$
   - $K \begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$
   - $K \begin{bmatrix} -2 & -3 \\ 4 & -5 \end{bmatrix}$

   **Correct Answer:** $K \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

5. Which matrix is equal to $K \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$?

   - $K \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
   - $K \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$
   - $K \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$

   **Correct Answer:** $K \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Find the sum.

6. $K \begin{bmatrix} 4 & -6 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $+$ $K \begin{bmatrix} 0 & 4 \\ 2 & 5 \end{bmatrix}$

   - $K \begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$
   - $K \begin{bmatrix} 0 & 4 \\ 2 & 5 \end{bmatrix}$
   - $K \begin{bmatrix} 4 & 0 \\ 4 & 5 \end{bmatrix}$

   **Correct Answer:** $K \begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$

**Teaching Resources**

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12-1 Practice (continued) Form K

Adding and Subtracting Matrices

Find each sum or difference.
11. \( \begin{bmatrix} 3 & -1 \\ -5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -8 \end{bmatrix} \)
12. \( \begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 0 & 6 \\ 0 & 0 \end{bmatrix} \)
13. \( \begin{bmatrix} 7 & 4 \\ -7 & -4 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 3 & -5 \end{bmatrix} \)

Find the value of each variable.
14. \( \begin{bmatrix} 1 \end{bmatrix} x + \begin{bmatrix} -5 \end{bmatrix} = \begin{bmatrix} 10 \end{bmatrix} \)
15. \( \begin{bmatrix} 2x \\ 7 \end{bmatrix} - \begin{bmatrix} -6 \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 7 \end{bmatrix} \)
16. \( \begin{bmatrix} -8 \\ -2 \\ -5 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ -7 \end{bmatrix} = \begin{bmatrix} 19 \\ 13 \\ 6 \end{bmatrix} \)

10. Writing Describe the Commutative and Associative Properties of Matrix Addition.

Here are some properties similar to the Commutative and Associative Properties of Real Number Addition. The Commutative Property of Matrix Addition states that matrices can be added in any order and the sum will remain the same. The Associative Property of Matrix Addition states that matrices can be grouped in any way and the sum will remain the same. The properties of matrix addition are the same as those of real numbers, except the properties apply to the elements of a matrix.

19. Reasoning Is it possible to find the value of \( x \) in the following equation? Why or why not?

\[ 2x - 1 = 3 - 2 \cdot x \]

Yes, the variable appears in the same pair of corresponding elements. Solving the equation \( 2x + 2 = 4 - 5x \) gives the answer \( x = \frac{2}{3} \).

Appendix A

ANSWERS

12-1 Practice Form K

Adding and Subtracting Matrices

Find each sum.
1. \( \begin{bmatrix} 2 \\ 4 \\ -1 \\ 5 \end{bmatrix} + \begin{bmatrix} -2 \\ -4 \\ 1 \\ 2 \end{bmatrix} \)
2. \( \begin{bmatrix} 0 \\ -1 \\ 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ -3 \\ -1 \end{bmatrix} \)

Find the value of each variable.
3. \( 2x + 1 = 2y + 1 \)
4. \( x - 3 = y - 3 \)
5. \( x = -3 \) or \( y = -3 \)
6. Explain how the sieve of Sundaram helps determine if a number will be prime or not. Answers may vary.
12-1 Reaching
Adding and Subtracting Matrices

A matrix is like a table without the row and column labels.

To add or subtract matrices of the same size, combine the corresponding elements, the numbers in the same positions in each matrix. The sum or difference of matrices will have the same dimensions, the number of rows and columns, as the matrices you combined.

To help you keep track of your work, draw lines between the rows and columns and cross off elements as you combine them.

a. Find each sum or difference.

b. Repeat for the remaining matrix elements.

c. Add row 1, column 2 elements. Cross them off.

d. Add row 1, column 2 elements. Cross them off.

e. Repeat for the remaining matrix elements.

Exercises

Find each sum or difference.

1. -3 5
   9 -2
   7 -1
   4 -8

2. 3 1 4
   2 0 5
   1 1 1
   -3 3 -5

3. 1 -2
   5 -3
   6 5
   -1 -8

4. -2 0
   3 1
   4 1 3
   -7 -3 -4

5. -10 4
   0 -10 4
   4 4 4
   7 -7 -4

6. -3 5
   9 -2
   7 -1
   4 -8

12-2 ELL Support
Matrix Multiplication

Kerri is learning how to multiply matrices. She wrote the steps to multiply

\[ A = \begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix} \] by \[ B = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \]
on note cards, but the cards got mixed up.

1. First, multiply the elements in the first row of \( A \) by the elements in the first column of \( B \). Place the sum in the first row, first column of \( AB \).
2. Second, multiply the elements in the first row of \( A \) by the elements in the second column of \( B \). Place the sum in the first row, second column of \( AB \).
3. Then, multiply the elements in the second row of \( A \) by the elements in the first column of \( B \). Place the sum in the second row, first column of \( AB \).
4. Finally, multiply the elements in the second row of \( A \) by the elements in the second column of \( B \). Place the sum in the second row, second column of \( AB \).

Use the note cards to write the steps in order.

a. Multiply the elements in the first row of \( A \) by the elements in the first column of \( B \). Place the sum in the first row, first column of \( AB \).

b. Multiply the elements in the second row of \( A \) by the elements in the second column of \( B \). Place the sum in the second row, second column of \( AB \).

12-2 Think about a Plan
Matrix Multiplication

Sport: Two teams are competing in a track meet. Points for individual events are awarded as follows: 5 points for first place, 3 points for second place, and 1 point for third place. Points for team relays are awarded as follows: 5 points for first place, 2 points for second place, and 1 point for third place.

a. Use matrix operations to determine the scores of the track meet.

b. Who would win if the scoring was changed to 5 points for first place, 1 point for second place, and 1 point for third place for each individual event and relay scoring remaining 5 points for first place?

Know

1. The number of each place for each school and the point value of each place

Need

2. To solve the problem I need to write the number of wins and the point values as matrices, then multiply the matrices:

Plan

3. Write the number of wins as a 2 \( \times \) 3 matrix and the original and alternate point values as 3 \( \times \) 1 matrices.

4. Use matrix multiplication to find the original total team scores and the alternate total team scores for the track meet.

5. What was the score in the track meet?

6. Who would win if the scoring were changed? West River
12-2 Practice
Matrix Multiplication

Use matrices A, B, C, and D. Find each product, sum, or difference.

1. $A = \begin{bmatrix} 1 & -1 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ -1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -3 & -1 \\ 2 & -2 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. $2D + 3C - 2B$

3. $A + 2C - 3B$

4. $DC + BD$

5. $RD + DR$

6. $A + D - 2(2C)$

Find each product. Solve each matrix equation. Check your answers.

7. $[\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} - 2] = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

8. $5x - 3 = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$

9. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

10. $[\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix} - 2] = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

11. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

12. $A - 2B + 3C$

13. $A - B + C$

14. $2A - 3B + 4C$

15. $A - 2B + 3C$

16. $2A - 3B + 4C$

17. $5x - 3 = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

18. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

19. $[\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix} - 2] = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

20. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

21. $A - 2B + 3C$

22. $A - B + C$

23. $2A - 3B + 4C$

24. $A - 2B + 3C$

25. $2A - 3B + 4C$

26. $5x - 3 = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

27. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

28. $[\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix} - 2] = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

29. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

30. $A - 2B + 3C$

31. $A - B + C$

32. $2A - 3B + 4C$

33. $A - 2B + 3C$

34. $2A - 3B + 4C$

35. $5x - 3 = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

36. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

37. $A - 2B + 3C$

38. $A - B + C$

39. $2A - 3B + 4C$

40. $A - 2B + 3C$

41. $2A - 3B + 4C$

42. $5x - 3 = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

43. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

44. $A - 2B + 3C$

45. $A - B + C$

46. $2A - 3B + 4C$

47. $A - 2B + 3C$

48. $2A - 3B + 4C$

49. $5x - 3 = \begin{bmatrix} 1 & -6 \\ 2 & 0 \end{bmatrix}$

50. $\frac{5}{2}x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

51. $A - 2B + 3C$

52. $A - B + C$

53. $2A - 3B + 4C$

54. $A - 2B + 3C$

55. $2A - 3B + 4C$
**ANSWERS**

**12-2 Standardized Test Prep**

**Matrix Multiplication**

**Extended Response**

1. Solve each matrix equation.

2. Find each product.

**Nilpotent Matrices**

A matrix $A$ is said to be nilpotent if there is an integer $n$ such that $A^n = 0$, the zero matrix.

1. What can you say about the dimensions of a nilpotent matrix? Why?

2. What is the order of the nilpotent matrix?

3. What can you conclude about $A$ if $A^n = 0$? What is the order of $A$?

4. Find two possible solutions.

5. Use this information to solve matrix $A$. Verify that this matrix is nilpotent of order 2.

6. If $b = 6$, what can you conclude from equation (1)? From equation (2, 2)?

7. Find the possible solutions.

8. Using equations (2, 1), (1, 1), and (2, 2), and substituting in matrix $A$, find another nilpotent matrix of order 2.

**Reteaching**

**Matrix Multiplication**

- To multiply a matrix by a real number, multiply each element in the matrix by the real number. This is called scalar multiplication. The real number is the scalar.

- Solving matrix equations with scalars is like solving other kinds of equations. Isolate the variable on one side of the equal sign and simplify the other side.

**Problem**

What is the solution of \( \begin{bmatrix} x & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 5 & 4 \end{bmatrix} \)?

**Exercises**

1. Solve each matrix equation.

2. Find each product.
12-3 Determinants and Inverses

Choose the word from the list that best matches each sentence.

1. The \[
\begin{vmatrix}
2 & 3 \\
4 & 5
\end{vmatrix}
\]
is a determinant.

2. A square matrix has the same number of rows and columns.

3. The multiplicative inverse matrix of \[
\begin{vmatrix}
2 & 3 \\
4 & 5
\end{vmatrix}
\]
can be written as \[
A^{-1}
\].

4. A matrix with a determinant of zero is called a singular matrix.

5. A matrix multiplied by its inverse produces the multiplicative identity matrix.

Circle the determinant of the following matrices.

6. \[
\begin{vmatrix}
1 & 2 \\
3 & 4
\end{vmatrix}
\]

7. \[
\begin{vmatrix}
2 & 3 \\
4 & 5
\end{vmatrix}
\]

8. \[
\begin{vmatrix}
2 & -1 \\
-1 & 2
\end{vmatrix}
\]

Determine whether the following matrices are inverses.

9. \[
A = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}, \quad B = \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}
\]

10. \[
A = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}, \quad B = \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}
\]

11. \[
A = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}, \quad B = \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}
\]

12. \[
A = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}, \quad B = \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}
\]

Evaluate the determinant of each matrix.

13. \[
\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}
\]

14. \[
\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}
\]

Graphing Calculator Evaluate the determinant of each 3 × 3 matrix.

15. \[
\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}
\]

16. \[
\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}
\]

17. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

18. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

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25. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

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28. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

29. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

30. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

31. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

32. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

33. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

34. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

35. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².

36. The area between the North Carolina cities of Raleigh, Durham, and Chapel Hill is called the Research Triangle. Use the map to determine the approximate area of the Research Triangle. The coordinates are given in miles. Answers may vary. Sample: about 52 mi².
12-3 Practice (continued)

Determinants and Inverses

Find the inverse of each matrix, if one exists.

To start, find the determinant of the matrix.

12. \( \begin{pmatrix} 6 & 2 \\ 2 & 1 \end{pmatrix} \)

Area = \( \frac{1}{2} \text{det} A \)

Area = \( \frac{1}{2} \text{det} \begin{pmatrix} 6 & 4 \\ 5 & 1 \end{pmatrix} \)

Area = \( \frac{3}{2} \text{ units}^2 \)

Find the inverse of each matrix, if one exists.

To start, find the determinant of the matrix.

13. \( A = \begin{pmatrix} 0 & 5 \\ 2 & 4 \end{pmatrix} \)

\( \det A = \frac{1}{2} \)

\( A^{-1} = -\frac{2}{5} \)

\( A^{-1} = -\frac{1}{3} \)

To find the inverse of a matrix, if one exists.

17. Your aunt’s checking account number is 0434-0572-2945-3072. Use the coding matrix \( C = \begin{pmatrix} -2 & 1 & 3 \\ -1 & 0 & 6 \end{pmatrix} \) to encode the account number.

\( \begin{pmatrix} -10 & 7 & -4 & -3 & 10 & -7 & 0 \\ 9 & 28 & 8 & 6 & 9 & -5 & 14 \end{pmatrix} \)

\( \begin{pmatrix} 10 & 5 & 3 & 10 & 7 & -4 & 0 \\ 28 & 9 & 6 & 8 & 9 & -5 & 14 \end{pmatrix} \)

12-3 Enrichment

Determinants and Inverses

Suppose \( A \) and \( B \) are \( 2 \times 2 \) matrices as follows:

\( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \)

\( B = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \)

1. What is the value of the determinant of \( A \)? \( \det A = ad - bc \)

2. Evaluate \( \det B \). \( ah - fg \)

3. Evaluate \( \det A \cdot \det B \). \( adh + bcf - abg - cdf \)

4. Compute matrix \( A_B \):

\( \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + gh \end{pmatrix} \)

5. Evaluate \( \det(AB) \). \( (ae + bg)(cf + dh) - (af + bh)(ce + dg) = adhe + bcfh - abcg - cdgh \)

6. What can you conclude? \( \det(AB) = \det A \cdot \det B \)

7. Explain your results.

The determinant of the product of two matrices is equal to the product of the determinants.

8. Suppose that a \( 2 \times 2 \) matrix \( A \) has an inverse \( A^{-1} \). Use the product rule to investigate how the determinant of \( A^{-1} \) is related to the determinant of \( A \).

\( \det(A \cdot A^{-1}) = \det A \cdot \det A^{-1} \)

\( = \det A \cdot \det A^{-1} \)

\( = \det A \cdot \det A^{-1} \)

\( = \det A^{-1} \)

9. Explain your results.

The determinant of the inverse of a matrix is equal to the reciprocal (inverse) of the determinant of the original matrix.
What is the solution of the matrix equation
\[\begin{pmatrix} 12 & -4 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.
\]

The determinant of a matrix combines elements according to a pattern.

Let \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \) be a 2 \times 2 matrix. The determinant is
\[\det(A) = ad - bc \]

Exercise
Find the determinant of each matrix.
1. \( \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} \)
2. \( \begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} \)
3. \( \begin{vmatrix} 3 & 0 \\ -1 & 2 \end{vmatrix} \)
4. \( \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} \)

Nutrition Suppose you are making a trail mix for your friends and want to fill three 1-lb bags. Almonds cost $2.25/lb, peanuts cost $1.30/lb, and raisins cost $0.90/lb. You want each bag to contain twice as much nuts as raisins by weight. If you spent $4.45, how much of each ingredient did you buy?

Know
1. Each bag will contain twice as much nuts as raisins by weight.
2. Each bag will contain twice as much nuts as raisins by weight.
3. You spent $4.45.

Need
4. To solve the problem I need to: write and solve 3 equations for three unknowns.

Plan
5. Let \( x \) = the number of pounds of almonds, \( y \) = the number of pounds of peanuts, and \( z \) = the number of pounds of raisins. Write a system of equations that solves the problem.
6. Solve the system of equations.
7. Use a calculator. Solve for the variable matrix.
8. How much of each ingredient did you buy?
9. How can you check your solution? Does your solution check?
Inverse Matrices and Systems

Practice Form G
Solve each matrix equation. If an equation cannot be solved, explain why.
1. \[
\begin{bmatrix}
0.25 & -0.75 \\
3.5 & 2.25
\end{bmatrix} X = \begin{bmatrix}
-1.5 \\
-17
\end{bmatrix}
\]
2. \[X \begin{bmatrix}
3 & -9 \\
1 & -6
\end{bmatrix} = \begin{bmatrix}
12 \\
8
\end{bmatrix}
\]
3. \[X \begin{bmatrix}
3 & -6 \\
-1 & 2
\end{bmatrix} = \begin{bmatrix}
4 \\
9
\end{bmatrix}
\]

No solution; det \( A = 0 \)

Write each system as a matrix equation. Identify the coefficient matrix, the variable matrix, and the constant matrix.
5. \[
\begin{bmatrix}
4x + 13y & = -36 \\
9x - 5y & = 26
\end{bmatrix}
\]
6. \[
\begin{bmatrix}
5x + 4y - z & = 0 \\
7y - 2z & = x
\end{bmatrix}
\]
7. \[
\begin{bmatrix}
3z & = 2 \\
6x + 3y & = 3
\end{bmatrix}
\]

Solve each system of equations using a matrix equation. Check your answers.
9. \[
\begin{align*}
x + 3y & = 2, \quad y + 6 = 10 \\
2x + 3y & = 12, \quad x + 7y = 2
\end{align*}
\]
10. \[
\begin{align*}
x + 3y & = -1, \quad y - 2z = 1 \\
x + 2y & = -2, \quad 3z = 4
\end{align*}
\]

Inverse Matrices and Systems

Practice Form G

Solve each system.
21. \[
\begin{align*}
x + 2y & = 18 \\
x + 3y & = 25
\end{align*}
\]
22. \[
\begin{align*}
x + y & = 3 \\
y + z & = -3
\end{align*}
\]

Determine whether each system has a unique solution.
27. \[
\begin{align*}
x + 2y & = 4 \\
x + y & = 3
\end{align*}
\]
28. \[
\begin{align*}
x + 2y & = 4 \\
x + y & = 3
\end{align*}
\]

Reasoning: Explain how you could use a matrix equation to show that the lines represented by \( y = -3x + 4 \) and \( y = -4x + 8 \) are parallel. Write the matrix equation \[
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

to solve the system. If the equation has a solution \((c, d)\), then the lines intersect at that point. A solution exists if the determinant of the coefficient matrix \( \begin{vmatrix} 1 & 4 \\ -3 & 1 \end{vmatrix} \)

does not equal zero. ...

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12-4 Standardized Test Prep
Inverse Matrices and Systems

Multiple Choice
For Exercises 1–4, choose the correct letter.

1. Which matrix equation represents the system $2x - y = 11$?
   - $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$
   - $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$
   - $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$
   - $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$

2. Let $\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$.
   What values of $x$ and $y$ make the equation true?
   - $(12, -3)$
   - $(4, -2)$
   - $(-3, 2)$
   - $(2, -3)$

3. Which system has a unique solution?
   - $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 6 \end{bmatrix}$
   - $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \end{bmatrix}$
   - $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
   - $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

4. Let $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.
   What value of $x$ makes the equation true?
   - $\begin{bmatrix} 10 \\ -6 \\ 0 \end{bmatrix}$
   - $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
   - $\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$
   - $\begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix}$

Short Response
5. The Spirit Club sold buttons for $51, hats for $4, and t-shirts for $8. They sold 3 times as many buttons as hats. Together, the number of hats and t-shirts sold was equal to the number of buttons sold. They earned a total of $480.
   Write a 2 by 3 matrix equation to find how many buttons, hats, and t-shirts the club sold.
   - $\begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} B \\ H \\ T \end{bmatrix} = \begin{bmatrix} 480 \\ 8 \end{bmatrix}$

6. What do you think the determinant of an $n \times n$ matrix with two identical rows will be? Its determinant is zero.

12-4 Reteaching
Inverse Matrices and Systems

- You can write the system $\begin{bmatrix} x + b & y & b \\ a & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$
- $B = \begin{bmatrix} x & y & z \end{bmatrix}$ is the coefficient matrix, $x$ is the variable matrix, and $p$ is the constant matrix.
- Solve the matrix equation by multiplying both sides by the inverse of the coefficient matrix, if it exists.

Problem
What is the solution of the system $\begin{bmatrix} 3x + 2y = 14 \\ 5x + 6y = -3 \end{bmatrix}$? Solve using matrices.

Write the system as a matrix equation.

$\begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ -3 \end{bmatrix}$

Find the inverse of the coefficient matrix.

$\begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}^{-1} = \frac{1}{3(6) - 2(5)} \begin{bmatrix} 6 & -2 \\ -5 & 3 \end{bmatrix}$

Isolate the variable matrix by multiplying both sides by the inverse of the coefficient matrix.

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3(6) - 2(5)} \begin{bmatrix} 6 & -2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 14 \\ -3 \end{bmatrix}$

Simplify.

The solution is $(x, y)$. Substitute the values in the original system to check your work.

Exercises
Solve each system of equations using a matrix equation. Check your answers.

1. $\begin{cases} 2x + 3y = 5 \\ x + 2y = 4 \end{cases}$
2. $\begin{cases} 3x - y = 1 \\ 2x + 3y = 7 \end{cases}$
3. $\begin{cases} 4x - 2y = 3 \\ 2x + 5y = -9 \end{cases}$

12-4 Enrichment
Inverse Matrices and Systems

You can find the determinant of a $3 \times 3$ matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ by using submatrices that you form by removing rows and columns.

You can form the submatrix $M_{ij}$ by removing the $i$th row and the $j$th column. For example, $M_{ij}$ is the submatrix formed by removing the first row and the second column.

$M_{ij} = \begin{bmatrix} a & b \\ d & e \end{bmatrix}$

The determinant of a $3 \times 3$ matrix $A$ is $det A = a \cdot det M_{13} - b \cdot det M_{23} + c \cdot det M_{33}$.

1. Find the determinant of $C = \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix}$ using the formula above.
2. Find the determinant of $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{bmatrix}$ using the formula above.
3. The matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ has two identical rows. Calculate $det A$.

Answers may vary. Sample: $120$

4. Write a $2 \times 2$ matrix with two identical rows. Calculate its determinant.

5. Write a $2 \times 2$ matrix with two identical columns. Calculate its determinant.

6. What do you think the determinant of an $n \times n$ matrix with two identical rows will be? Its determinant is zero.

12-4 Reteaching (continued)
Inverse Matrices and Systems

You can use a graphing calculator and matrices to solve a linear system.

Problem
$ax + by = c$
$dx + ey = f$

What is the solution of the system $\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$?

Solve using a graphing calculator and matrices.

Write the system as a matrix equation.

$\begin{bmatrix} 4 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 2 \end{bmatrix}$

Enter the coefficient matrix as matrix $A$.

Multiply $A^{-1}$B to find the variable matrix.

The solution is $(x, y)$. Substitute the values in the original system to check your work.

Exercises
Solve each system of equations using a graphing calculator and matrices. Check your answers.

4. $\begin{cases} 4x + y = 2 \\ 3x - 2y = -1 \end{cases}$
5. $\begin{cases} 6x + 3y = 1 \\ 5x - 4y = 2 \end{cases}$
6. $\begin{cases} 2x - 3y = 10 \\ 5x - 2y = 8 \end{cases}$

7. $\begin{cases} 4x + 2y = 12 \\ 3x - 5y = -7 \end{cases}$
9. Use matrix addition to find the coordinates of each image after a translation in the same coordinate plane.

3. Find the coordinates of each image after the given dilation.

2. For Exercises 6–9, draw a line from each matrix in Column A to the rotation that it represents in Column B.

For Exercises 6–9, draw a line from each matrix in Column A to the rotation that it represents in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
</table>
| 6. \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\] | A. 90° rotation |
| 7. \[
\begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}
\] | B. 180° rotation |
| 8. \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\] | C. 270° rotation |
| 9. \[
\begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}
\] | D. 360° rotation |

For Exercises 1–5, draw a line from each word in Column A to its definition in Column B.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. image</td>
<td>A. a transformation that enlarges or reduces an image</td>
</tr>
<tr>
<td>2. preimage</td>
<td>B. the point around which an image is rotated</td>
</tr>
<tr>
<td>3. dilation</td>
<td>C. a transformed figure</td>
</tr>
<tr>
<td>4. rotation</td>
<td>D. the original figure</td>
</tr>
<tr>
<td>5. center of rotation</td>
<td>E. a transformation that turns an image around a fixed point</td>
</tr>
</tbody>
</table>

Find the coordinates of each image in matrix form.

Graph each figure and its image after the given rotation.

Graph each figure and its image after the given rotation.

Find the coordinates of each image after the given dilation.

Find the coordinates of each image after the translation.

Graph each figure and its image after reflection in the given line.

Graph each figure and its image after reflection in the given line.

Find the coordinates of each image after the given transformation.

Each pair of matrices represents the coordinates of the vertices of the preimage and image of a polygon. Describe the transformation.

21. \[
\begin{bmatrix}
1 & 2 & 4 \\
3 & 2 & 10 \\
6 & 2 & -3
\end{bmatrix}
\] to \[
\begin{bmatrix}
5 & 2 & 9 \\
3 & 2 & -3 \\
6 & 2 & -3
\end{bmatrix}
\] translation 3 units left and 1 unit down

22. \[
\begin{bmatrix}
1 & 2 & 4 \\
5 & 2 & 5 \\
6 & 2 & 9
\end{bmatrix}
\] to \[
\begin{bmatrix}
3 & 4 & 5 \\
1 & 5 & 6 \\
9 & 5 & 2
\end{bmatrix}
\] a dilation of 2:1

23. Writing The matrices \[
\begin{bmatrix}
4 & 5 \\
-2 & 3
\end{bmatrix}
\] and \[
\begin{bmatrix}
-1 & -1 \\
4 & 5
\end{bmatrix}
\] represent the coordinates of the vertices of a triangle before and after a reflection in the line \( y = x \). Describe the relationship between the coordinates of the corresponding vertices.

Answers may vary. Sample: The values of the x- and y-coordinates have been exchanged.
12-5 Practice Geometric Transformations

Use matrix addition to find the coordinates of each image after a translation of 2 units right and 4 units down. Then graph each image on the same coordinate plane as its preimage.

1. A(3, 0), B(2, 2), C(1, 4)
2. A(3, 0), B(2, 5), C(1, 7)
3. A(3, 0), B(2, 1), C(1, 4)
4. A(3, 0), B(2, 1), C(1, 2)

4. Error Analysis Triangle ABC has vertices (2, 2), (6, 4), and (4, 5). Angela translated triangle ABC 4 units right and 7 units up. She found the coordinates (6, 11), (10, 12), and (8, 10).

What error did Angela make? What are the correct coordinates?

5. Error Analysis A triangle has vertices (2, 3), (3, 1), and (4, 5). Angela translated the x-coordinates by 7 and the y-coordinates by 4 when she should have increased the x-coordinates by 7 and the y-coordinates by 4, 8, 10, and 12.

Find the coordinates of each image after the given dilation.

6. A(3, 1), B(5, 3), C(4, 5)
7. A(3, 1), B(5, 3), C(4, 5)
8. A(3, 1), B(5, 3), C(4, 5)

8. Reasoning Your classmate said that diluting an image by a factor of 0.25 will increase the size of the image. Is he correct? Explain.

Your classmate is not correct. Diluting an image by a factor of 0.25 will decrease the size of the image. Multiplying by 0.25 is like dividing by 4.

12-5 Enrichment Geometric Transformations

Rotations, translations, and reflections are called Euclidean transformations. They are isometric: the size and shape of a transformed figure does not change, but the location and orientation may change.

Affine transformations are generalizations of these Euclidean transformations. Affine transformations are nonisometric and do not preserve length and angle measure. As a result, the image may be a different shape than the preimage.

One type of affine transformation is a shear transformation. A shear transformation takes a shape and “pushes” it in a direction that is parallel to either the x-axis or the y-axis. For example, in the figures below, the image of rectangle ABCD is a parallelogram ABFE under a shear transformation.

1. The matrix that represents rectangle ABCD above to

\[
\begin{pmatrix}
1 & 2 & \frac{1}{2} \\
0 & 1 & 0
\end{pmatrix}
\]

2. How did the shear matrix change rectangle ABCD? Answers may vary. Sample: Rectangle ABCD was pushed parallel to the x-axis to form a parallelogram.

3. Multiply \[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \] How does this shear matrix affect rectangle ABCD?

4. Write a shear matrix that would push a figure parallel to the y-axis. Answers may vary. Sample: any matrix of the form \[\begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \] or \[\begin{pmatrix} x & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \]

5. Write a shear matrix that would push a figure parallel to the y-axis. Answers may vary. Sample: any matrix of the form \[\begin{pmatrix} 1 & y & 0 \\ 0 & 1 & 0 \end{pmatrix} \]

6. Draw a rectangle on a coordinate grid. Use your shear matrices from exercises 4 and 5 to transform the rectangle. Check students’ work.
12-5 Re-teaching
Geometric Transformations

• A translation changes the position of a geometric figure without changing its size, shape, or orientation.
• You can use matrix addition to find the coordinates of a polygon’s vertices after a translation.

Problem
A polygon has vertices A(0, 0), B(2, 3), C(1, 4), and D(3, 2). Write a matrix to represent the vertices of a polygon after a dilation of 0.5.

Exercises
A polygon has vertices A(0, 0), B(3, 1), C(1, 4), and D(3, 2). Write a matrix to represent the vertices of A'B'C'D' after each transformation.

1. dilation of 0.5
2. rotation of 180°
3. reflection across x-axis
4. reflection across y-axis
5. dilation of 0.5
6. reflection across x-axis

12-5 Re-teaching (continued)

Problem
A polygon has vertices A(0, 0), B(2, 3), C(1, 4), and D(3, 2). Write a matrix to represent the vertices of A'B'C'D' after each transformation.

Exercises
A polygon has vertices A(0, 0), B(2, 3), C(1, 4), and D(3, 2). Write a matrix to represent the vertices of A'B'C'D' after each transformation.

12-6 ELL Support
Vectors

Choose the word from the list that best matches each sentence.

- dot product
- initial point
- normal vectors
- magnitude
- terminal point

1. A vector begins at the initial point.
2. The dot product of vectors \( \vec{v} \) and \( \vec{w} \) is \( \vec{v} \cdot \vec{w} = \ |
3. A vector has both distance and direction.
4. Normal vectors are perpendicular to each other.
5. A vector ends at the terminal point.
6. The magnitude of a vector is the length of the arrow.

Circle the dot product of the following vectors. Then tell whether the vectors are normal.

1. \( \vec{u} = (3, 1), \vec{v} = (2, 5) \)
2. \( \vec{u} = (-2, 4), \vec{v} = (6, 3) \)
3. \( \vec{u} = (3, 4), \vec{v} = (-8, 4) \)
4. \( \vec{u} = (-2, 7), \vec{v} = (8, 4) \)

12-6 Think About a Plan
Vectors

Aviation A twin-engine airplane has a speed of 300 mi/h in still air. Suppose the airplane heads south and encounters a wind blowing 50 mi/h due east. What is the resultant speed of the airplane?

Know
1. The airplane is traveling south at a speed of 300 mi/h.
2. The wind is blowing east at a speed of 50 mi/h.

Need
2. To solve the problem I need to find the sum of the vectors that represent the speed of the airplane and the speed of the wind.

Plan
4. Sketch the speed of the airplane and the speed of the wind as vectors. Then use the tip-to-tail method to sketch \( \vec{a} + \vec{w} \).
5. What is the component form of the vector for the speed of the airplane? \( \vec{a} = (-300, 0) \)
6. What is the component form of the vector for the speed of the wind? \( \vec{w} = (50, 0) \)
7. Express \( \vec{a} + \vec{w} \) in component form: \( \vec{a} + \vec{w} = (-250, -300) \)
8. What equation can you use to find the magnitude of \( \vec{a} + \vec{w} \)? \( |\vec{a} + \vec{w}| = \sqrt{(-250)^2 + (-300)^2} \)
9. What is the resultant speed of the airplane? about 304 mi/h in a S10°E direction.
9. Find the component forms of the unknown vector \( v \).

30. \( b = v = x = (3, -1) \)

31. \( v = a = b = (3, -1) \)

32. \( a + b = v = c = (3, -1) \)

33. \( a + b = v = c = (3, -1) \)

34. A train leaves Dawson station and travels 360 mi due north. Then it turns and travels 120 mi due west to reach New Port. If the train travels 75 mi/h on a straight route directly back to Dawson, how long will the return trip take? Round your answer to the nearest hour.

35. Reasoning (Identify the additive identity vector \( u \)) if it exists. Explain your reasoning.

Let \( u = \begin{pmatrix} 2 \end{pmatrix} \) and \( v = \begin{pmatrix} 3 \end{pmatrix} \) Graph the following vectors.

36. \( u \)

37. \( v \)

38. \( \frac{1}{2}u \)

**NOTES**

12-6 Practice (continued) Form G

**Vectors**

Let \( u = \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} \) and \( v = \begin{pmatrix} 2 \\ 7 \\ 2 \end{pmatrix} \) Find the following vectors.

24. \( 3u + 2v = \begin{pmatrix} 9 \\ 16 \\ 6 \end{pmatrix} \)

25. \( v + 3w = \begin{pmatrix} 10 \\ 13 \\ 14 \end{pmatrix} \)

26. \( w - 2v = \begin{pmatrix} -8 \\ -5 \\ 6 \end{pmatrix} \)

27. A bird flies 16 mi/h in still air. Suppose the bird flies due south with a wind blowing 15 mi/h due east. What is the resultant speed of the bird roundtrip to the nearest mile per hour?

28. A model rocket lands 245 ft west and 102 ft south of the point from which it was launched. How far did the rocket fly? Round your answer to the nearest foot.

29. Consider a polygon with vertices at \( A(-3, 5), B(2, 3), C(4, 0), \) and \( D(6, -3) \). Express the sides of the polygon as vectors \( \overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \) and \( \overrightarrow{DA} \).

30. Let \( a = (7, 5), b = (4, -1), \) and \( v = (0, -3) \). Solve each of the following for the unknown vector \( v \).

31. \( b + v = x \)

32. \( a + b = v = c \)

33. \( a + v = b = (3, 1) \)

34. A train leaves Dawson station and travels 360 mi due north. Then it turns and travels 120 mi due west to reach New Port. If the train travels 75 mi/h on a straight route directly back to Dawson, how long will the return trip take? Round your answer to the nearest hour.

35. Reasoning (Identify the additive identity vector \( u \)) if it exists. Explain your reasoning.

Let \( u = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \) and \( v = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \)

Graph the following vectors.

36. \( u \)

37. \( v \)

38. \( \frac{1}{2}u \)

**NOTES**

12-6 Practice (continued) Form K

**Vectors**

Let \( u = (1, -3), v = (3, 1), \) and \( w = (-1, -1) \) Find the component forms of the following vectors. Then graph each vector on the coordinate plane.

11. \( -2v = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \)

12. \( 3u = (3, -9) \)

13. \( 5w = (5, -5) \)

14. Reasoning You graphed the vector \( 4v \) You r风俗d graphed the vector \( -6v \) Whose vector will have the greater magnitude? Explain. Both vectors will have the same magnitude, but they will be pointing in opposite directions because one is positive and the other is negative.

Find the dot products of the following vectors.

15. \( t = (4, 7), u = (-2, 5) \) \( \overrightarrow{t} \cdot \overrightarrow{u} = (4)(-2) + (7)(5) = 28 \)

16. \( v = (2, 6), w = (5, 3) \) \( \overrightarrow{v} \cdot \overrightarrow{w} = (2)(5) + (6)(3) = 28 \)

17. \( a = (-1, -1), b = (1, 7) \) \( \overrightarrow{a} \cdot \overrightarrow{b} = (-1)(1) + (-1)(7) = -8 \)

18. Multiple Choice Which of the following pairs of vectors is normal? E

(6, 3), (3, 6)
(5, 7), (0, 8)
(7, 3), (4, 6)
(5, 3), (5, 8)
12-6 Standardized Test Prep

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Let \( u = (4, -7) \) and \( v = (-1, 3) \). What is \( u + v \)?
   a. \( (3, 4) \)
   b. \( (5, 10) \)
   c. \( (1, 10) \)
   d. \( (5, -4) \)
   Answer: d

2. What is the component form of vector \( \mathbf{v} \)?
   a. \( (2, 4) \)
   b. \( (-1, 3) \)
   c. \( (3, -3) \)
   d. \( (-3, -3) \)
   Answer: b

3. Which represents the vector \( u = (16, -9) \) rotated \( 180^\circ \)?
   a. \( (16, 9) \)
   b. \( (-16, 9) \)
   c. \( (-16, -9) \)
   d. \( (16, -9) \)
   Answer: c

4. Let \( \mathbf{u} = \begin{pmatrix} 5 \\ 10 \\ 8 \end{pmatrix} \) and \( \mathbf{w} = \begin{pmatrix} -4 \\ 5 \\ 9 \end{pmatrix} \). What is the vector \( -2\mathbf{u} + \frac{1}{2}\mathbf{w} \)?
   a. \( \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \)
   b. \( \begin{pmatrix} 0 \\ 5 \\ 17 \end{pmatrix} \)
   c. \( \begin{pmatrix} 10 \quad 10 \\ 16 \end{pmatrix} \)
   d. \( \begin{pmatrix} 10 \quad 10 \\ 14 \end{pmatrix} \)
   Answer: b

5. Which represents the vector \( \mathbf{w} = \begin{pmatrix} 14 \\ 14 \\ 14 \end{pmatrix} \) reflected across \( y = x \)?
   a. \( \begin{pmatrix} 14 \\ -14 \\ -14 \end{pmatrix} \)
   b. \( \begin{pmatrix} -14 \\ 14 \\ 14 \end{pmatrix} \)
   c. \( \begin{pmatrix} 14 \\ -14 \\ 14 \end{pmatrix} \)
   d. \( \begin{pmatrix} -14 \\ 14 \\ -14 \end{pmatrix} \)
   Answer: d

Short Response

6. Let \( \mathbf{p} = (5, -1) \) and \( \mathbf{q} = (3, 5) \). Are \( \mathbf{p} \) and \( \mathbf{q} \) normal? Show your work.
   Answer: No, the dot product is \( 5(3) + (-1)(5) = 10 \), so they are not normal.

1.1 incorrect or incomplete work shown
2.0 incorrect answer and no work shown OR no answer given

12-6 Enrichment

Vectors

A unit vector, \( \mathbf{u} \), is a vector of length 1. Any vector \( \mathbf{v} \) can be made into a unit vector by dividing by its length. This is called normalizing the vector.

\[ \mathbf{u} = \frac{\mathbf{v}}{\| \mathbf{v} \|} \]

1. The first step in normalizing a vector is to find its magnitude, \( |\mathbf{v}| \). Find the magnitude of vector \( (3, 4) \).
   \[ |(3, 4)| = \sqrt{3^2 + 4^2} = 5 \]

2. Next evaluate \( \frac{1}{|\mathbf{v}|} \) to determine the unit vector.
   \[ \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \]

3. Normalize the vector \( (2, 5) \).
   \[ \frac{1}{\sqrt{2^2 + 5^2}} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \frac{1}{\sqrt{29}} \begin{pmatrix} 2 \\ 5 \end{pmatrix} \]

4. Given the unit vector \( \frac{1}{3} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \), which vector of the same direction has a magnitude of 10 units?
   \[ \begin{pmatrix} 4 \\ 8 \end{pmatrix} \]

5. Normalize the vector \( (4, -5) \).
   \[ \frac{1}{\sqrt{4^2 + (-5)^2}} \begin{pmatrix} 4 \\ -5 \end{pmatrix} = \frac{1}{\sqrt{41}} \begin{pmatrix} 4 \\ -5 \end{pmatrix} \]

6. Which vector is 4 times the magnitude of the unit vector in Exercise 3?
   \[ 4 \cdot \frac{1}{\sqrt{2^2 + 5^2}} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \frac{4}{\sqrt{29}} \begin{pmatrix} 2 \\ 5 \end{pmatrix} \]

12-6 Reteaching (continued)

Vectors

If vectors are normal, or perpendicular, then their dot product equals zero. For vectors \( \mathbf{v} = (v_1, v_2) \) and \( \mathbf{w} = (w_1, w_2) \), the dot product is \( \mathbf{v} \cdot \mathbf{w} = v_1w_1 + v_2w_2 \).

Problem

Are the vectors \( \mathbf{r} = (4, 8) \) and \( \mathbf{s} = (-6, 5) \) normal?
\[ \mathbf{r} \cdot \mathbf{s} = (4)(-6) + (8)(5) = 20 \]
\[ \| \mathbf{r} \| = \sqrt{4^2 + 8^2} = 4\sqrt{5} \]
\[ \| \mathbf{s} \| = \sqrt{(-6)^2 + 5^2} = \sqrt{41} \]
\[ \frac{\mathbf{r} \cdot \mathbf{s}}{\| \mathbf{r} \| \| \mathbf{s} \|} = \frac{20}{4\sqrt{5} \sqrt{41}} \neq 0 \]
These vectors are not normal.

Exercises

Determine whether the vectors in each pair are normal.

4. \( \mathbf{p} = (3, -2) \) and \( \mathbf{q} = (4, 6) \) yes
5. \( \mathbf{u} = (2, 2) \) and \( \mathbf{v} = (4, -1) \) no

Transform each vector as described. Write your answer in component form.

6. \( (3, 10) \); reflect across \( y = x \) \( (10, 3) \)
7. \( (4, -9) \); rotate \( 270^\circ \) \( (-9, 4) \)
Find each product if it exists. If it does not exist, write undefined.

1. \( \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \)
2. \( \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \)
3. \( \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \)
4. \( \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)
5. \( \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \)

Use matrices A and B. Find each product, sum, or difference.

3. \( A = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \)
4. \( A = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \)
5. \( A = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \)

For each matrix, find the determinant. Then find the inverse, if it exists.

5. \( \begin{pmatrix} 2 \\ 3 \\ 0 \\ -1 \end{pmatrix} \)
6. \( \begin{pmatrix} 1 \\ 0 \\ 2 \\ -1 \end{pmatrix} \)
7. \( \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \)

Do you UNDERSTAND?

13. Writing Describe how to find the dimensions of the matrix \( \begin{pmatrix} 1 & 3 & 5 \\ 2 & 6 & 4 \end{pmatrix} \).
The dimensions of a matrix are number of rows \( \times \) number of columns, so the dimensions of this matrix are \( 2 \times 3 \).

14. Error Analysis A student says that matrix multiplication is commutative for matrices with the same dimensions. Provide a counter-example to show the student’s error. Any pair of non-zero matrices that are not inverses.

15. Reasoning For what values of \( x \) will the matrix \( \begin{pmatrix} x & -3 \\ -4 & 6 \end{pmatrix} \) have an inverse? \( x \neq -4 \)
Chapter 12 Quiz 1

Lessons 12–1 and 12–2

Do you UNDERSTAND?

4. Find each sum or difference. Do you know HOW?

5. Solve each matrix equation. Then find the value of each variable.

6. Do you UNDERSTAND?

8. Writing Describe the process used to multiply matrices. Then find the product of \( \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \) and \( \begin{bmatrix} 5 \\ 6 \end{bmatrix} \). First, multiply the elements in the first row of \( A \) by the elements in the first column of \( B \) and find the sum of the products. Then find the sum of the product of the elements in the first row of \( A \) and the elements in the second column of \( B \). Next, multiply the second row of \( A \) by the first column of \( B \) and add. Finally, multiply the second row of \( A \) by the second column of \( B \) and add. \( \begin{bmatrix} 5 \\ 10 \end{bmatrix} \)

Chapter 12 Test

Lessons 12–3 and 12–4

Do you know HOW?

Evaluate the determinant of each matrix.

Do you know HOW?

4. Find the inverse of each matrix, if one exists.

5. Write each system as a matrix equation. Then solve the system of equations.

6. Do you UNDERSTAND?

10. Error Analysis Your classmate solved the system of equations \( x + 2y = 11 \) and \( 2x + 3y = 19 \). He used the matrix equation below. What error did he make? What are the values of \( x \) and \( y \)?

Chapter 12 Test [continued]

Determine whether the following matrices are multiplicative inverses.

Use inverse matrices to find the solution of each matrix equation.

Use matrices to solve the following systems of equations.

Do you UNDERSTAND?

19. Roger and Clarissa each sold boxes of cookies for a fundraiser. They sold large and small boxes for different prices. Roger sold 12 large boxes and 8 small boxes for a total of $192. Clarissa sold 10 large boxes and 15 small boxes for a total of $235. Use a system of two equations and matrices to find the price of a large box and a small box.

Large box: \( $12.00 \); small box: \( $2.50 \)
Use a matrix equation to solve the system. Show your work.

**Task 1**

1. Student attempts a solution, but shows little understanding of the problem.
2. Student makes no attempt OR no response is given.
3. Student uses appropriate strategies, but misunderstood the application of matrix operations. Student writes an appropriate matrix equation, but multiplies the inverse of the coefficient matrix incompletely. Inverse is correct or non-existence is partially correct.
4. Discussion shows a clear understanding of matrix operations, but may be incomplete. Student writes an appropriate matrix. Operations show appropriate strategies, but are implemented incorrectly. Inverse is correct or non-existence is at least partly correctly explained.

**Extended Response**

10. Solve \( X = \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \) for \( X \).

11. At which vertex is the objective function \( C = -3x + 7y \) maximized?

**Short Response**

13. What is the z-intercept of the line \( 3z = 4y + 127 \)?

14. The coordinates of the vertices of a triangle are \((2, -3, 2), (5, 0, 10), \) and \((2, 7, 20)\). Find the area of the triangle.

15. A hiker starts at the trailhead, walks 3 mi due east, then turns and walks 4 mi due north. What is the direct distance from the hiker's current position to the trailhead?

**Extended Response**

16. A boat travels 15 mi in still water. A current flows due east at 15 mi/h and the boat travels due north across the current.

a. Draw a sketch with vectors to represent the resultant speed of the boat.

b. Solve an equation to find the resultant speed of the boat to the nearest integer. Show your work.

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Chapter 12 Project: Munching Microbes

About the Project

The Chapter Project gives students an opportunity to use matrices to organize data from a bioremediation project. They use the matrices to calculate totals and changes in the amount of wastes present. Then, they research other bioremediation projects and summarize and display their findings.

Introducing the Project

Encourage students to keep all project-related materials in a separate folder.

• Ask students if they have been near sites that were being cleaned up after oil spills or chemical spills. Explain that the field of bioremediation uses naturally-occurring bacteria to degrade hazardous wastes.

• Have students make a list of materials they need to begin the project.

Activity 1: Organizing

Students write matrices with data from soil tests for hazardous wastes.

Activity 2: Calculating

Students use their previously-written matrices to calculate the total amounts of the hazardous waste components and to show by how much the amount of each component has decreased over 12 months.

Activity 3: Researching

Students research a hazardous waste clean-up that includes bioremediation. They then write a report on the clean-up.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage students to explain their processes as well as their results. Ask students to review their project work and update their folders.

• Have students review their methods for organizing the data in matrices, calculating totals and changes in the component amounts, and for researching bioremediation for the project.

• Ask groups to share their insights that resulted from completing the project, such as any shortcuts they found for organizing the data or conducting research.

Extending the Project

Bioremediation is a fairly new field. You can do more research into the field by Extending the Project.

• Ask a classmate to review your project with you. After you have seen each other's presentations, decide if your work is complete, clear, and convincing. Make sure your presentation is neat and accurate. Explainations are thorough and well thought out. The presentation is accurate and thoroughly explains the information.

• Calculations are mostly correct, with some minor errors. Matrices are neat and mostly accurate with minor errors. The explanations lack detail. The presentation has minor errors or lacks important information.

• Calculations contain both minor and major errors. Matrices are not accurate. The explanations and presentation are inaccurate or incomplete.

• Major elements of the project are incomplete or missing.

• Project is not turned in or shows no effort.

Your Evaluation of Project

Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of the Project

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